CHAPTER - 6
CHARACTERIZATION OF INTUITIONISTIC UNIFORM* $AB$-BORDER SPACES
CHAPTER-6

CHARACTERIZATION OF INTUITIONISTIC UNIFORM* $AB$-BORDER SPACES

The concept of border of a set was introduced by M. Caldas, S. Jafari and T. Noiri [9]. It has many applications in geometric networking, arc map editing and network processing. Motivated by these applications, the concepts of intuitionistic uniform* topological spaces, intuitionistic uniform* $AB$-border of an intuitionistic symmetric member, intuitionistic uniform* $AB$-boundary of an intuitionistic symmetric member, intuitionistic uniform* $AB$-derived symmetric members, intuitionistic uniform* $AB$-border irresolute functions, intuitionistic uniform* $AB$-border open functions and intuitionistic uniform* $AB$-border spaces are introduced and studied. In this connection, some interesting properties are established.

6.1 PROPERTIES OF INTUITIONISTIC UNIFORM* $AB$-BORDER OF AN INTUITIONISTIC SYMMETRIC MEMBER

In this section, the concepts of intuitionistic uniform* topological spaces, intuitionistic uniform* $AB$-border of an intuitionistic symmetric member, intuitionistic uniform* $AB$-boundary of an intuitionistic symmetric mem-
ber, intuitionistic uniform* $AB$-derived symmetric members and intuitionistic uniform* $AB$-border irresolute functions are introduced and studied. Some interesting properties and characterization are established.

**Definition 6.1.1.** Let $X$ be a non empty set. An intuitionistic symmetric member $S$ is an object having the form $S = ((x_1, x_2), S^1, S^2)$, where $S^1$ and $S^2$ are subsets of $X \times X$ satisfying $S^1 \cap S^2 = \emptyset$. The set $S^1$ is called the set of members of $S$, while $S^2$ is called the set of nonmembers of $S$.

**Notation 6.1.1.** Let $X$ be non empty set

(i) $X$ denotes $X \times X$.

(ii) $X_\sim = ((x_1, x_2), X, \emptyset)$. $\emptyset_\sim = ((x_1, x_2), \emptyset, X)$.

**Definition 6.1.2.** An intuitionistic uniform* topology on a non-empty set $X$ is a collection $T$ of intuitionistic symmetric members in $X$ satisfying the following axioms:

(i) $\emptyset_\sim, X_\sim \in T$.

(ii) $S_1 \cap S_2 \in T$ for any $S_1 = ((x_1, x_2), S^1_1, S^2_1)$ and $S_2 = ((x_1, x_2), S^1_2, S^2_2) \in T$.

(iii) $\cup S_i \in T$ for any arbitrary family $\{S_i = ((x_1, x_2), S^1_i, S^2_i) : i \in J\} \subseteq T$.

In this case the pair $(X, T)$ is called an intuitionistic uniform* topological space and any intuitionistic symmetric member in $T$ is called an intuitionistic open symmetric member. The complement of an intuitionistic open symmetric member is called an intuitionistic closed symmetric member.
Definition 6.1.3. Let $(X, T)$ be an intuitionistic uniform* topological space. Let $A = (x_1, x_2, A^1, A^2)$ be an intuitionistic symmetric member of $X$. Then the intuitionistic uniform* closure and intuitionistic uniform* interior of $A$ are denoted and defined by

$IU^*cl(A) = \cap\{K = ((x_1, x_2), K^1, K^2) : K$ is an intuitionistic closed symmetric member and $A \subseteq K\}.$

$IU^*int(A) = \cup\{G = ((x_1, x_2), G^1, G^2) : G$ is an intuitionistic open symmetric member and $G \subseteq A\}.$

Definition 6.1.4. Let $(X, T)$ be an intuitionistic uniform* topological space. An intuitionistic symmetric member $A = (x_1, x_2, A^1, A^2)$ of $X$ is said to be an intuitionistic semi-open symmetric member if $A \subseteq IU^*cl(IU^*int(A))$. The complement of an intuitionistic semi-open symmetric member is called an intuitionistic semi-closed symmetric member.

Definition 6.1.5. Let $(X, T)$ be an intuitionistic uniform* topological space. An intuitionistic symmetric member $A = (x_1, x_2, A^1, A^2)$ of $X$ is said to be an intuitionistic semi-regular symmetric member if it is both intuitionistic semi-open and intuitionistic semi-closed.

Definition 6.1.6. Let $(X, T)$ be an intuitionistic uniform* topological space. An intuitionistic symmetric member $A = (x_1, x_2, A^1, A^2)$ of $X$ is said to be an intuitionistic $AB$-open symmetric member if $A = U \cap V$, where $U = ((x_1, x_2), U^1,
$U^2 \in T$ and $V$ is intuitionistic semi-regular. The complement of an intuitionistic $AB$-open symmetric member is called an intuitionistic $AB$-closed symmetric member.

**Definition 6.1.7.** Let $(X, T)$ be an intuitionistic uniform* topological space. Let $A = ((x_1, x_2), A^1, A^2)$ be an intuitionistic symmetric member of $X$. Then the intuitionistic uniform* $AB$ interior and intuitionistic uniform* $AB$ closure of $A$ are denoted and defined by

$$IU^*ABint(A) = \bigcup \{ G = ((x_1, x_2), G^1, G^2) : G \text{ is an intuitionistic } AB\text{-open symmetric member and } G \subseteq A \}. $$

$$IU^*ABcl(A) = \bigcap \{ K = ((x_1, x_2), K^1, K^2) : K \text{ is an intuitionistic } AB\text{-closed symmetric member and } A \subseteq K \}. $$

**Remark 6.1.1.** Let $(X, T)$ be an intuitionistic uniform* topological space. Let $A = ((x_1, x_2), A^1, A^2)$ be an intuitionistic symmetric member of $X$. Then the following conditions hold:

(i) $IU^*ABcl(A) = A$ if and only if $A$ is an intuitionistic $AB$-closed symmetric member.

(ii) $IU^*ABint(A) = A$ if and only if $A$ is an intuitionistic $AB$-open symmetric member.

(iii) $IU^*ABint(A) \subseteq A \subseteq IU^*ABcl(A)$. 

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(iv) \( IU^*ABcl(\overline{A}) = IU^*ABint(\overline{A}) \).

(v) \( IU^*ABint(\overline{A}) = TU^*ABcl(\overline{A}) \).

(vi) \( IU^*ABint(\varphi_\sim) = \varphi_\sim \) and \( IU^*ABint(X_\sim) = X_\sim \).

(vii) \( IU^*ABcl(\varphi_\sim) = \varphi_\sim \) and \( IU^*ABcl(X_\sim) = X_\sim \).

**Notation 6.1.2.** Let \( A = \langle (x_1, x_2), A^1, A^2 \rangle \) be an intuitionistic symmetric member of \( X \). \((x_1, x_2) \in A \) denotes \((x_1, x_2) \in A^1 \) and \((x_1, x_2) \not\in A^2 \).

**Definition 6.1.8.** Let \((X, T)\) be an intuitionistic uniform* topological space. An intuitionistic symmetric member \( A = \langle (x_1, x_2), A^1, A^2 \rangle \) of \( X \) is said to be an intuitionistic uniform* \( AB \)-neighborhood of \((x_1, x_2)\) if there exists an intuitionistic \( AB \)-open symmetric member \( U = \langle (x_1, x_2), U^1, U^2 \rangle \) such that \((x_1, x_2) \in U \subseteq A \).

**Definition 6.1.9.** Let \((X, T)\) be an intuitionistic uniform* topological space. Let \( A = \langle (x_1, x_2), A^1, A^2 \rangle \) be an intuitionistic symmetric member of \( X \). We say that \((x_1, x_2)\) is an intuitionistic uniform* \( AB \)-limit point of \( A \) if for every intuitionistic uniform* \( AB \)-neighborhood of \((x_1, x_2)\) intersects \( A \) in some point other than \((x_1, x_2)\) itself. The set of all intuitionistic uniform* \( AB \)-limit point of \( A \) is called an intuitionistic uniform* \( AB \)-derived symmetric member of \( A \) and is denoted by \( IU^*D_{AB}(U) \).

**Proposition 6.1.1.** Let \((X, T)\) be an intuitionistic uniform* topological space. If \( A = \langle (x_1, x_2), A^1, A^2 \rangle \) is an intuitionistic symmetric member of \( X \) then
\( \mathcal{I}U^* \mathcal{D}_{AB}(A) \subseteq \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A) \).

**Proof**

The proof is simple.

**Notation 6.1.3.** Let \((X, T)\) be an intuitionistic uniform* topological space. \( \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{O}(X, T) \) denotes the family of all intuitionistic \( A \mathcal{B} \)-open symmetric members in \((X, T)\).

**Proposition 6.1.2.** Let \((X, T)\) be an intuitionistic uniform* topological space. If \( A = ((x_1, x_2), A^1, A^2) \) is an intuitionistic symmetric member of \( X \) then \( \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A) = A \cup \mathcal{I}U^* \mathcal{D}_{AB}(A) \).

**Proof**

Let \((x_1, x_2) \in \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A)\). If \((x_1, x_2) \notin A\), then the proof is complete. If \((x_1, x_2) \notin A\) and \( G \in \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{O}(X, T) \) with \((x_1, x_2) \in G\) then \( G \cap (A \# ((x_1, x_2))) = \varnothing \). Hence, \((x_1, x_2) \in \mathcal{I}U^* \mathcal{D}_{AB}(A)\). Which implies that, \( \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A) \subseteq \mathcal{I}U^* \mathcal{D}_{AB}(A) \).

Thus,

\[ \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A) \subseteq A \cup \mathcal{I}U^* \mathcal{D}_{AB}(A). \]  

\text{(6.1.1)}

By Proposition 6.1.1., \( \mathcal{I}U^* \mathcal{D}_{AB}(A) \subseteq \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A) \). Hence,

\[ A \cup \mathcal{I}U^* \mathcal{D}_{AB}(A) \subseteq \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A). \]  

\text{(6.1.2)}

From (6.1.1) and (6.1.2), \( \mathcal{I}U^* \mathcal{A} \mathcal{B} \mathcal{C}l(A) = A \cup \mathcal{I}U^* \mathcal{D}_{AB}(A) \).

**Definition 6.1.10.** Let \((X, T)\) be an intuitionistic uniform* topological space. An intuitionistic uniform* \( A \mathcal{B} \)-border of an intuitionistic symmetric mem-
ber $U = ((x_1, x_2), U^1, U^2)$ of $X$ is denoted and defined by $IU^*Bd_{AB}(U) = U \cap IU^*ABcl(U)$ where $U = ((x_1, x_2), U^2, U^1)$.

**Definition 6.1.11.** Let $(X, T)$ be an intuitionistic uniform* topological space. An intuitionistic uniform* $AB$-boundary of an intuitionistic symmetric member $U = ((x_1, x_2), U^1, U^2)$ of $X$ is denoted and defined by $IU^*\partial_{AB}(U) = IU^*ABcl(U) \cap IU^*ABcl(U)$ where $U = ((x_1, x_2), U^2, U^1)$.

**Proposition 6.1.3.** Let $(X, T)$ be an intuitionistic uniform* topological space. If $A = ((x_1, x_2), A^1, A^2)$ is an intuitionistic symmetric member of $X$ then $IU^*Bd_{AB}(A) \subseteq IU^*\partial_{AB}(A)$.

**Proof**

Let $IU^*Bd_{AB}(A) = A \cap IU^*ABcl(A)$. Since $A \subseteq IU^*ABcl(A)$, $IU^*Bd_{AB}(A) \subseteq IU^*ABcl(A) \cap IU^*ABcl(A)$. By Definition 6.1.11., $IU^*Bd_{AB}(A) \subseteq IU^*\partial_{AB}(A)$. Hence, $IU^*Bd_{AB}(A) \subseteq IU^*\partial_{AB}(A)$.

**Notation 6.1.4.** Let $(X, T)$ be an intuitionistic uniform* topological space. $IU^*ABC(X, T)$ denotes the family of all intuitionistic $AB$-closed symmetric members in $(X, T)$.

**Proposition 6.1.4.** Let $(X, T)$ be an intuitionistic uniform* topological space. Let $A = ((x_1, x_2), A^1, A^2)$ be an intuitionistic symmetric member of $X$. If $A \in IU^*ABC(X, T)$ then $IU^*\partial_{AB}(A) \subseteq A$.

**Proof**

Let $A \in IU^*ABC(X, T)$. Since $A$ is an intuitionistic $AB$-closed symmetric
member, \( IU^*ABcl(A) = A \). Then

\[
IU^*\partial_{AB}(A) = IU^*ABcl(A) \cap IU^*ABcl(\overline{A})
\]

\[
= A \cap IU^*ABcl(\overline{A})
\]

\[
IU^*\partial_{AB}(A) \subseteq A.
\]

Hence, \( IU^*\partial_{AB}(A) \subseteq A \).

**Proposition 6.1.5.** Let \((X, T)\) be an intuitionistic uniform\(^*\) topological space. Let \( A = (\langle x_1, x_2 \rangle, A^1, A^2) \) be an intuitionistic symmetric member of \( X \). If \( IU^*D_{AB}(A) \subseteq A \) then \( A \in IU^*ABC(X, T) \).

**Proof**

Let \( IU^*D_{AB}(A) \subseteq A \). That is,

\[
A \cup IU^*D_{AB}(A) \subseteq A. \tag{6.1.3}
\]

By Proposition 6.1.2.,

\[
A \cup IU^*D_{AB}(A) = IU^*ABcl(A). \tag{6.1.4}
\]

From (6.1.3) and (6.1.4), \( A = IU^*ABcl(A) \). Therefore, \( A \) is an intuitionistic \( AB \)-closed symmetric member. Hence, \( A \in IU^*ABC(X, T) \).

**Proposition 6.1.6.** Let \((X, T)\) be an intuitionistic uniform\(^*\) topological space. Let \( A = (\langle x_1, x_2 \rangle, A^1, A^2) \) be an intuitionistic symmetric member of \( X \). If \( A \in IU^*ABC(X, T) \) then \( IU^*Bd_{AB}(A) \subseteq A \).
Proof

Let $A$ be an intuitionistic $AB$-closed symmetric member. By Proposition 6.1.4., $IU^*\partial_{AB}(A) \subseteq A$ and by Proposition 6.1.3., $IU^*B_{AB}(A) \subseteq IU^*\partial_{AB}(A)$. This implies that, $IU^*B_{AB}(A) \subseteq A$.

Definition 6.1.12. Let $(X, T)$ be an intuitionistic uniform* topological space and $U = ((x_1, x_2), U^1, U^2)$ be an intuitionistic symmetric member of $X$. Let $IU^*B_{AB}(U)$ be an intuitionistic uniform* $AB$-border of $U$. Then the intuitionistic uniform* $AB$ interior and intuitionistic uniform* $AB$ closure of $IU^*B_{AB}(U)$ are denoted and defined by

$$IU^*AB^I(IU^*Bd(U)) = \bigcup\{K = ((x_1, x_2), K^1, K^2) : K \text{ is an intuitionistic } AB\text{-open symmetric member and } K \not\subseteq IU^*Bd(U)\}$$

$$IU^*AB^C(IU^*Bd(U)) = \bigcap\{K = ((x_1, x_2), K^1, K^2) : K \text{ is an intuitionistic } AB\text{-closed symmetric member and } IU^*Bd(U) \not\subseteq K\}$$

Remark 6.1.2. Let $(X, T)$ be an intuitionistic uniform* topological space and $U = ((x_1, x_2), U^1, U^2)$ be an intuitionistic symmetric member of $X$. Let $IU^*Bd(U)$ be an intuitionistic uniform* border of $U$. Then the following conditions hold:

(i) $IU^*AB^I(IU^*Bd(U)) = IU^*AB^C(IU^*Bd(U))$.

(ii) $IU^*AB^C(IU^*Bd(U)) = IU^*AB^I(IU^*Bd(U))$. 

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(iii) \( IU^*AB^\#(IU^*Bd(U)) = IU^*Bd(U) \) if and only if \( IU^*Bd(U) \) is an intuitionistic \( AB \)-closed symmetric member.

(iv) \( IU^*AB^\#(IU^*Bd(U)) = IU^*Bd(U) \) if and only if \( IU^*Bd(U) \) is an intuitionistic \( AB \)-open symmetric member.

Definition 6.1.13. Let \((X, T)\) and \((Y, S)\) be any two intuitionistic uniform* topological spaces. Let \( IU^*Bd(U) \) be an intuitionistic uniform* border of an intuitionistic symmetric member \( U = ((x_1, x_2), U^1, U^2) \). A function \( f : (X, T) \to (Y, S) \) is called an intuitionistic uniform* \( AB \)-border irresolute function if \( f^{-1}(IU^*Bd(U)) \) is intuitionistic \( AB \)-open symmetric member (resp. intuitionistic \( AB \)-closed symmetric member) in \((X, T)\) for every intuitionistic \( AB \)-open symmetric member (resp. intuitionistic \( AB \)-closed symmetric member) \( IU^*Bd(U) \) in \((Y, S)\).

Proposition 6.1.7. Let \((X, T)\) and \((Y, S)\) be any two intuitionistic uniform* topological spaces. Then \( f : (X, T) \to (Y, S) \) is an intuitionistic uniform* \( AB \)-border irresolute function if and only if \( f(IU^*AB^\#(IU^*Bd(U))) \subseteq IU^*AB^\#(f(IU^*Bd(U))) \), for every intuitionistic uniform* \( AB \)-border of intuitionistic symmetric member \( U \) of \((X, T)\).

Proof

Suppose that \( f \) is an intuitionistic uniform* \( AB \)-border irresolute function and \( IU^*Bd(U) \) be an intuitionistic uniform* border of intuitionistic symmetric member \( U = ((x_1, x_2), U^1, U^2) \) of \((X, T)\). Then \( IU^*AB^\#(f(IU^*Bd(U))) \) is
an intuitionistic $AB$-closed symmetric member in $(Y, S)$. By assumption, $f^{-1}(IU^*AB^\bigtriangleup(f(IU^*Bd(U))))$ is an intuitionistic $AB$-closed symmetric member of $(X, T)$. Now, $IU^*Bd(U) \subseteq f^{-1}(f(IU^*Bd(U))) \subseteq f^{-1}(IU^*AB^\bigtriangleup(f(IU^*Bd(U))))$.

Now,

$$IU^*Bd(U) \subseteq f^{-1}(IU^*AB^\bigtriangleup(f(IU^*Bd(U)))) \subseteq IU^*AB^\bigtriangleup(I$$

$$U^*Bd(U) \subseteq IU^*AB^\bigtriangleup(f^{-1}(IU^*AB^\bigtriangleup(f(IU^*Bd(U))))) \subseteq IU^*AB^\bigtriangleup(I$$

$$U^*Bd(U) \subseteq f^{-1}(IU^*AB^\bigtriangleup(IU^*AB^\bigtriangleup(f(IU^*Bd(U)))))$$

$$f(IU^*AB^\bigtriangleup(IU^*Bd(U))) \subseteq IU^*AB^\bigtriangleup(f(IU^*Bd(U)))$$

That is, $f(IU^*AB^\bigtriangleup(IU^*Bd(U))) \subseteq IU^*AB^\bigtriangleup(f(IU^*Bd(U)))$.

Conversely, suppose that $IU^*Bd(U)$ is an intuitionistic $AB$-closed symmetric member in $(Y, S)$. Then,

$$IU^*AB^\bigtriangleup(IU^*Bd(U)) = IU^*Bd(U) \quad (6.1.5)$$

Now by assumption,

$$f(IU^*AB^\bigtriangleup(f^{-1}(IU^*Bd(U)))) \subseteq IU^*AB^\bigtriangleup(f(f^{-1}(IU^*Bd(U))))$$

This implies that,

$$f(IU^*AB^\bigtriangleup(f^{-1}(IU^*Bd(U)))) \subseteq IU^*AB^\bigtriangleup(IU^*Bd(U))$$

By (6.1.5), $f(IU^*AB^\bigtriangleup(f^{-1}(IU^*Bd(U)))) \subseteq IU^*Bd(U)$

$$IU^*AB^\bigtriangleup(f^{-1}(IU^*Bd(U)) \subseteq f^{-1}(IU^*Bd(U)) \quad (6.1.6)$$

But, $IU^*AB^\bigtriangleup(f^{-1}(IU^*Bd(U)) \supseteq f^{-1}(IU^*Bd(U)) \quad (6.1.7)$
From (6.1.6) and (6.1.7), we have, $f^{-1}(IU^*Bd(U)) = IU^*ABf(f^{-1}(IU^*Bd(U)))$. That is, $f^{-1}(IU^*Bd(U))$ is an intuitionistic $AB$-closed symmetric member in $(X, T)$. Hence, $f$ is an intuitionistic uniform* $AB$-border irresolute function.

**Definition 6.1.14.** Let $(X, T)$ and $(Y, S)$ be any two intuitionistic uniform* topological spaces. A function $f : (X, T) \to (Y, S)$ is said to be intuitionistic uniform* $AB$-open if $f(A)$ is an intuitionistic $AB$-open symmetric member in $(Y, S)$ for each intuitionistic $AB$-open symmetric member $A$ in $(X, S)$.

**Proposition 6.1.8.** Let $(X, T)$ and $(Y, S)$ be any two intuitionistic uniform* topological spaces and $IU^*Bd(U)$ be an intuitionistic uniform* border of an intuitionistic symmetric member $U = ((x_1, x_2), U^1, U^2)$. Let $f : (X, T) \to (Y, S)$ be an intuitionistic uniform* $AB$-open function and surjective. Then $f^{-1}(IU^*ABf(IU^*Bd(U))) \subseteq IU^*ABf(f^{-1}(IU^*Bd(U)))$, for every $IU^*Bd(U)$ in $(Y, S)$. **Proof**

Let $IU^*Bd(U)$ be an intuitionistic uniform* border of $U$ in $(Y, S)$ and $IU^*Bd(V) = f^{-1}(IU^*Bd(U))$. Then, $IU^*ABf(f^{-1}(IU^*Bd(U))) = IU^*ABf(IU^*Bd(V))$ is an intuitionistic $AB$-open symmetric member in $(X, T)$. Now, $IU^*ABf(IU^*Bd(V)) \subseteq IU^*Bd(V)$. Hence, $f(IU^*ABf(IU^*Bd(V))) \subseteq f(IU^*Bd(V))$.

That is, $IU^*ABf(IU^*ABf(IU^*Bd(V))) \subseteq IU^*ABf(f(IU^*Bd(V)))$. This implies that, $IU^*ABf(f(IU^*ABf(IU^*Bd(V)))) \subseteq f(IU^*ABf(IU^*Bd(V))))$. Since $f$ is intuitionistic uniform* $AB$ open, $f(IU^*ABf(IU^*Bd(V)))$ is an intuitionistic $AB$-open symmetric member in $(Y, S)$. Therefore, $IU^*ABf(f(IU^*ABf(IU^*Bd(V))))$
\[ f(I^* A \tilde{B}(I^* B_d(V))). \] Now,

\[ I^* A \tilde{B}(f(I^* A \tilde{B}(I^* B_d(V))) \subseteq f(I^* A \tilde{B}(I^* B_d(V))) \]

\[ f(I^* A \tilde{B}(I^* A \tilde{B}(I^* B_d(V)))) \subseteq f(I^* A \tilde{B}(I^* B_d(V))) \]

\[ f(I^* A \tilde{B}(I^* B_d(V))) \subseteq I^* A \tilde{B}(f(I^* B_d(V))) \]

\[ f(I^* A \tilde{B}(f^{-1}(I^* B_d(U)))) \subseteq I^* A \tilde{B}(f^{-1}(I^* B_d(U))). \tag{6.1.8} \]

Now, (6.1.8) becomes,

\[ I^* A \tilde{B}(f^{-1}(I^* B_d(U))) \supseteq f^{-1}(I^* A \tilde{B}(I^* B_d(U))). \]

Hence,

\[ I^* A \tilde{B}(f^{-1}(I^* B_d(U))) \supseteq f^{-1}(I^* A \tilde{B}(I^* B_d(U))). \]

Therefore,

\[ f^{-1}(I^* A \tilde{B}(I^* B_d(U))) \subseteq I^* A \tilde{B}(f^{-1}(I^* B_d(U))). \]

### 6.2 ON INTUITIONISTIC UNIFORM* $AB$-BORDER SPACES

In this section, the concept of intuitionistic uniform* $AB$ border space is introduced and studied. In this connection, some interesting properties and characterizations are discussed.

**Definition 6.2.1.** An intuitionistic uniform* topological space \((X, T)\) is called an intuitionistic uniform* $AB$-border space if the intuitionistic uniform* $AB$ closure of intuitionistic uniform* $AB$ interior of intuitionistic uniform* border of each intuitionistic open symmetric member is an intuitionistic open symmetric member. That is, \(I^* A \tilde{B}(I^* A \tilde{B}(I^* B_d(U)))\) is an intuitionistic open symmetric member for each \(U = (x_1, x_2, U^1, U^2) \in T.\)
Example 6.2.1. Let \( X = \{a, b, c\} \) and \( Y = X \times X \). Let \( X = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\} \). Then the intuitionistic symmetric members \( M, N, O \) and \( P \) of \( X \) are defined by \( M = \{(x_1, x_2), \{(a, a), (b, c), (c, a)\}\}, N = \{(x_1, x_2), \{(b, c), (c, a)\}\}, O = \{(x_1, x_2), \{(a, a), (b, c), (c, a)\}, \varphi_\sim\} \) and \( P = \{(x_1, x_2), \varphi_\sim, \{(b, c), (c, a), (c, b)\}\} \). Let \( T = \{X_\sim, \varphi_\sim, M, N, O, P\} \) be an intuitionistic uniform* topology on \( X \). Clearly, \( (X, T) \) is an intuitionistic uniform* topological space. Hence the **intuitionistic uniform* topological space** \( (X, T) \) is an intuitionistic **uniform* \( AB \)-border space**.

Definition 6.2.2. Let \( (X, T) \) and \( (Y, S) \) be any two intuitionistic uniform* topological spaces. Let \( IU^*Bd(U) \) be an intuitionistic uniform* border of an intuitionistic symmetric member \( U = \{(x_1, x_2), U^1, U^2\} \). A function \( f : (X, T) \rightarrow (Y, S) \) is called an intuitionistic uniform* border continuous function if \( f^{-1}(IU^*Bd(U)) \) is intuitionistic open symmetric member in \( (X, T) \) for every intuitionistic open symmetric member \( IU^*Bd(U) \) in \( (Y, S) \).

Proposition 6.2.1. Let \( (X, T) \) and \( (Y, S) \) be any two intuitionistic uniform* topological spaces. Let \( f : (X, T) \rightarrow (Y, S) \) be an intuitionistic uniform* border continuous function, intuitionistic uniform* open function and surjective. If \( (X, T) \) is an intuitionistic uniform* \( AB \)-border space then \( (Y, S) \) is an intuitionistic uniform* \( AB \)-border space.

Proof

Let \( IU^*Bd(U) \) be an intuitionistic uniform* border of \( U = \{(x_1, x_2), U^1, U^2\} \),
Let \( U \in S \) be an intuitionistic open symmetric member in \((Y, S)\). Since \( f \) is an intuitionistic uniform* \( AB \)-border continuous function, \( f^{-1}(IU^*\text{Bd}(U)) \) is an intuitionistic open symmetric member in \((X, T)\).

Since \((X, T)\) is an intuitionistic uniform* \( AB \) border space, \( IU^*AB^r(IU^*\text{Bd}(U)) \) is an intuitionistic open symmetric member in \((X, T)\).

Since \( f \) is an intuitionistic open, \( f(IU^*AB^r(IU^*\text{Bd}(U))) \) is an intuitionistic open symmetric member in \((Y, S)\). By Proposition 6.1.8.,

\[
IU^*AB^r(f^{-1}(IU^*\text{Bd}(U))) \subseteq f^{-1}(f(IU^*AB^r(IU^*\text{Bd}(U)))) \subseteq IU^*AB^r(f^{-1}(IU^*\text{Bd}(U))))
\]

Now, \( f(IU^*AB^r(IU^*\text{Bd}(U))) \) is an intuitionistic open symmetric member in \((Y, S)\). Hence \((Y, S)\) is an intuitionistic uniform* \( AB \)-border space.

**Definition 6.2.3.** Let \((X, T)\) be an intuitionistic uniform* topological space. Let \( IU^*\text{Bd}(U) \) be an intuitionistic uniform* border of intuitionistic symmetric
member $U = ((x_1, x_2), U^1, U^2)$. Then the intuitionistic uniform* $AB$ closure of intuitionistic uniform* $AB$ interior of $IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U)))$ is denoted and defined by

$$IU^*AB^\delta(IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U)))) = \cap \{K = ((x_1, x_2), K^1, K^2) : K \text{ is intuitionistic } AB\text{-closed and } IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U))) \subseteq K \}.$$ 

**Remark 6.2.1.** Let $(X, T)$ be an intuitionistic uniform* topological space and $U = ((x_1, x_2), U^1, U^2)$ be an intuitionistic symmetric member of $X$. Let $IU^*Bd(U)$ be an intuitionistic uniform* border of $U$. Then the following conditions hold:

(i) $IU^*AB^\delta((IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U))))) = IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U)))$

if and only if $IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U)))$ is an $AB$-intuitionistic closed symmetric member.

(ii) $TU^*AB^\delta(TU^*AB^\delta(IU^*Bd(U))) = IU^*AB^\delta(IU^*AB^\delta(TU^*Bd(U)))$

(iii) $TU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U))) = IU^*AB^\delta(IU^*AB^\delta(TU^*Bd(U)))$

**Proposition 6.2.2.** Let $(X, T)$ be an intuitionistic uniform* topological space and $U = ((x_1, x_2), U^1, U^2)$ be an intuitionistic symmetric member of $X$. Let $IU^*Bd(U)$ be an intuitionistic uniform* border of intuitionistic symmetric member, $U \in T$. Then $TU^*Bd(U)$ is the complement of intuitionistic uniform* border of $U$. Then the following statements are equivalent:

(i) $(X, T)$ is an intuitionistic uniform* $AB$-border space.
(ii) For each intuitionistic closed symmetric member $\overline{IU^*Bd(U)}$,

$$IU^*AB^\hat{e}(IU^*AB^\hat{e}(\overline{IU^*Bd(U)}))$$

is an intuitionistic closed symmetric member.

(iii) For each intuitionistic open symmetric member $IU^*Bd(U)$,

$$IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U)))) = IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U))).$$

(iv) For every pair of intuitionistic open symmetric members $IU^*Bd(U)$ and $IU^*Bd(V)$, $V = (x_1, x_2, V^1, V^2)$ with $IU^*Bd(V) = IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U)))$,

we have $IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(V))) = IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U)))$.

**Proof**

(i) $\Rightarrow$ (ii)

Let $(X, T)$ be an intuitionistic uniform* $AB$-border space. Let $IU^*Bd(U)$ be an intuitionistic open symmetric member. Then $\overline{IU^*Bd(U)}$ is an intuitionistic closed symmetric member. Then by assumption, $IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U)))$ is an intuitionistic open symmetric member.

Then, $\overline{IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U)))}$ is an intuitionistic closed symmetric member. Which implies that, $IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U)))$ is an intuitionistic closed symmetric member. Hence (i) $\Rightarrow$ (ii).

(ii) $\Rightarrow$ (iii)

Let $IU^*Bd(U)$ be an intuitionistic open symmetric member. Then, $\overline{IU^*Bd(U)}$ is an intuitionistic closed symmetric member. Then by assumption, $IU^*AB^\hat{e}(IU^*AB^\hat{e}(IU^*Bd(U)))$ is an intuitionistic closed symmetric member.
closed symmetric member. Now, \( IU^*AB^c(IU^*AB^c(IU^*AB^c(IU^*Bd(U)))) \)
\[= IU^*AB^c(IU^*Bd(U)). \] Hence, (ii) \(\Rightarrow\) (iii).

(iii) \(\Rightarrow\) (iv)

Let \( IU^*Bd(U) \) and \( IU^*Bd(V) \) be any two intuitionistic open symmetric members such that \( IU^*Bd(V) = IU^*AB^c(IU^*Bd(U)) \). This implies that, \( IU^*Bd(V) = IU^*AB^c(IU^*Bd(U)) \). By (iii), \( IU^*AB^c(IU^*AB^c(IU^*Bd(U))) = IU^*AB^c(IU^*Bd(U)) \). This implies that, \( IU^*AB^c(IU^*AB^c(IU^*Bd(V))) = IU^*AB^c(IU^*Bd(U)) \). Hence, (iii) \(\Rightarrow\) (iv).

(iv) \(\Rightarrow\) (i)

Let \( IU^*Bd(U) \) and \( IU^*Bd(V) \) be any two intuitionistic open symmetric members such that \( IU^*Bd(V) = IU^*AB^c(IU^*Bd(U)) \). By (iv), it follows that, \( IU^*AB^c(IU^*Bd(V)) = IU^*AB^c(IU^*Bd(U)) \) is an intuitionistic closed symmetric member. That is, \( IU^*AB^c(IU^*Bd(U)) \) is an intuitionistic closed symmetric member.

This implies that, \( IU^*AB^c(IU^*Bd(U)) \) is an intuitionistic open symmetric member. Thus, \( (X, T) \) is an intuitionistic uniform* \( AB \) border space. Hence, (iv) \(\Rightarrow\) (i).

**Proposition 6.2.3.** Let \( (X, T) \) be an intuitionistic uniform* topological space. Then \( (X, T) \) is an intuitionistic uniform* \( AB \)-border space if and only if for each intuitionistic open symmetric member \( IU^*Bd(U) \) and intuitionistic closed
symmetric member \( IU^* \text{Bd}(V) \) such that \( IU^* \text{Bd}(U) \subseteq IU^* \text{Bd}(V) \),

\[
IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(U))) \subseteq IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))).
\]

**Proof**

Let \((X, T)\) be an intuitionistic uniform* \( AB \)-border space. Let \( IU^* \text{Bd}(U) \) be an intuitionistic open symmetric member and \( IU^* \text{Bd}(V) \) be an intuitionistic closed symmetric member with \( IU^* \text{Bd}(U) \subseteq IU^* \text{Bd}(V) \).

Then by (ii) of Proposition 6.2.2., \( IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))) \) is an intuitionistic closed symmetric member. Now, \( IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))) = IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))) \) is an intuitionistic closed symmetric member. Therefore,

\[
IU^* AB^\#(IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(U)))) = IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))).
\]

Since \( IU^* \text{Bd}(U) \) is an intuitionistic open symmetric member and \( IU^* \text{Bd}(U) \subseteq IU^* \text{Bd}(V) \), \( IU^* \text{Bd}(U) \subseteq IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))) \). Now, \( IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(U))) \subseteq IU^* AB^\#(IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V)))) \).

\[
\subseteq IU^* AB^\#(IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V)))).
\]

Hence, \( IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(U))) \subseteq IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))) \).

Conversely, let \( IU^* \text{Bd}(V) \) be an intuitionistic closed symmetric member. Then \( IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))) \) is an intuitionistic open symmetric symmetric member and \( IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))) \subseteq IU^* \text{Bd}(V) \). Therefore by assumption,

\[
IU^* AB^\#(IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V)))) \subseteq IU^* AB^\#(IU^* AB^\#(IU^* \text{Bd}(V))).
\]
Also,
\[ IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(V))) \subseteq IU^*AB^\delta(IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(V)))) \]

This implies that,
\[ IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(V))) = IU^*AB^\delta(IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(V)))) \]

Therefore, \( IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(V))) \) is an intuitionistic closed symmetric member. By (ii) of Proposition 6.2.2., \((X, T)\) is an intuitionistic uniform* \(AB\)-border space.

**Definition 6.2.4.** Let \((X, T)\) be an intuitionistic uniform* topological space. An intuitionistic symmetric member \(V = \langle x_1, x_2, V^1, V^2 \rangle\) is said to be intuitionistic clopen symmetric member if it is both intuitionistic open symmetric member and intuitionistic closed symmetric member.

**Remark 6.2.2.** Let \((X, T)\) be an intuitionistic uniform* \(AB\)-border space. Let \(\{IU^*Bd(U_i), IU^*Bd(V_j) | i \in N \}\) be a collection such that \(IU^*Bd(U_i)\)'s are intuitionistic open symmetric member and \(IU^*Bd(V_j)\)'s are intuitionistic closed symmetric member. Let \(IU^*Bd(U)\) and \(IU^*Bd(V)\) be any two intuitionistic clopen symmetric members. If \(IU^*Bd(U_i) \subseteq IU^*Bd(U) \subseteq IU^*Bd(V_j)\) and \(IU^*Bd(U_i) \subseteq IU^*Bd(V) \subseteq IU^*Bd(V_j)\) for all \(i, j \in N\), then there exists an intuitionistic clopen symmetric member \(IU^*Bd(W)\) such that \(IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(U_i))) \subseteq IU^*Bd(W) \subseteq IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(V_j)))\) for all \(i, j \in N\).
Proof

By Proposition 6.2.3.,

\[ IU^*AB^\#(IU^*AB^\#(IU^*Bd(U_i))) \subseteq IU^*AB^\#(IU^*AB^\#(IU^*Bd(U))) \cap IU^*AB^\#(IU^*AB^\#(IU^*Bd(V))) \]

\[ \subseteq IU^*AB^\#(IU^*AB^\#(IU^*Bd(V_j))) \] for all \( i, j \in \mathbb{N} \).

Therefore, \( IU^*Bd(W) = IU^*AB^\#(IU^*AB^\#(IU^*Bd(U))) \cap IU^*AB^\#(IU^*AB^\#(IU^*Bd(V))) \)

is an intuitionistic clopen symmetric member satisfying the required conditions.

Notation 6.2.1. \( ISM \) denotes the collection of all intuitionistic clopen symmetric members.

Proposition 6.2.4. Let \( (X, T) \) be an intuitionistic uniform* \( AB \)-border space. Let \( \{IU^*Bd(U_q)\}_{q \in Q} \) and \( \{IU^*Bd(V_q)\}_{q \in Q} \) be monotone increasing collections of intuitionistic open symmetric members and intuitionistic closed symmetric members of \( (X, T) \) respectively. Suppose that \( IU^*Bd(U_{q_1}) \subseteq IU^*Bd(V_{q_2}) \) whenever \( q_1 \leq q_2 \) (\( Q \) is the set of all rational numbers). Then there exists a monotone increasing collection \( \{IU^*Bd(P_q)\}_{q \in Q} \) of intuitionistic clopen symmetric members such that \( IU^*AB^\#(IU^*AB^\#(IU^*Bd(U_{q_1}))) \subseteq IU^*Bd(P_{q_2}) \) and \( IU^*Bd(P_{q_1}) \subseteq IU^*AB^\#(IU^*AB^\#(IU^*Bd(V_{q_2}))) \) whenever \( q_1 \leq q_2 \).

Proof

Let us arrange all rational numbers into a sequence \( \{q_n\} \) (without repetitions). For every \( n \geq 2 \), we shall define inductively a collection \( \{IU^*Bd(P_n)\} \) \( \subseteq \)
\(i \leq n\) \(\subseteq ISM\) of intuitionistic clopen symmetric members such that

\[I^*A^\delta(I^*A^\delta(I^*Bd(U_q))) \subseteq I^*Bd(P_q)\]

if \(q < q_i\), \(I^*Bd(P_q) \subseteq I^*A^\delta(I^*A^\delta(I^*Bd(V_q)))\) if \(q < q\) for all \(i < n\) \((S_n)\).

By Proposition 6.2.3., the countable collections \(\{I^*A^\delta(I^*A^\delta(I^*Bd(U_q)))\}\)
and

\(\{I^*A^\delta(I^*A^\delta(I^*Bd(V_q)))\}\) satisfy \(I^*A^\delta(I^*A^\delta(I^*Bd(U_q))) \subseteq I^*A^\delta(I^*A^\delta(I^*Bd(V_q)))\) if \(q_1 < q_2\). By Remark 6.2.2, there exists an intuitionistic clopen symmetric member \(I^*Bd(K_1)\) such that

\[I^*A^\delta(I^*A^\delta(I^*Bd(U_q))) \subseteq I^*Bd(K_1) \subseteq I^*A^\delta(I^*A^\delta(I^*Bd(V_q))).\]

Letting \(I^*Bd(P_{q_i}) = I^*Bd(K_1)\), we get \((S_2)\). Assume that \(I^*Bd(P_{q_i})\) are already defined for \(i < n\) and satisfy \((S_n)\).

Define \(M = \bigcup\{I^*Bd(P_{q_i})/i < n, q_i < q_n\} \cup I^*Bd(U_{q_n})\) and \(N = \bigcap\{I^*Bd(P_{q_j})/j < n, q_j > q_n\} \cap I^*Bd(V_{q_n})\). Then,

\[I^*A^\delta(I^*A^\delta(I^*Bd(P_{q_i}))) \subseteq I^*A^\delta(I^*A^\delta(M)) \subseteq I^*A^\delta(I^*A^\delta(I^*Bd(P_{q_j})))\]

and

\[I^*A^\delta(I^*A^\delta(I^*Bd(P_{q_i}))) \subseteq I^*A^\delta(I^*A^\delta(N)) \subseteq I^*A^\delta(I^*A^\delta(I^*Bd(P_{q_j})))\]

whenever, \(q_i < q_n < q_{(i,j < n)}\). Now, \(I^*Bd(U_q) \subseteq I^*A^\delta(I^*A^\delta(M)) \subseteq I^*Bd(V_q)\) and \(I^*Bd(U_q) \subseteq I^*A^\delta(I^*A^\delta(N)) \subseteq I^*Bd(V_q)\) whenever, \(q_i < q_n < q_{i^2}\).

This shows that the countable collections, \(\{I^*Bd(P_{q_i})/i < n, q_i < q_n\}\) \(\cup\)
\(\{I^*Bd(U_q)/q < q_n\}\) and \(\{I^*Bd(P_{q_j})/j < n, q_j > q_n\}\) \(\cup\) \(\{I^*Bd(V_q)/q > q_n\}\) together
with $M$ and $N$ fulfill all the conditions of Remark 6.2.2.

Hence there exists an intuitionistic clopen symmetric member $IU^*Bd(K_n)$ such that $IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(K_n))) \subseteq IU^*Bd(V_q)$ if $q_n < q$ and $IU^*Bd(U_q) \subseteq IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(K_n)))$ if $q < q_n$

$IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(P_{q_i}))) \subseteq IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(K_n)))$ if $q_i < q_n$ and

$IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(K_n))) \subseteq IU^*AB^\delta(IU^*AB^\delta(IU^*Bd(P_{q_j})))$ if $q_n < q_j$, where $1 \leq i, j \leq n - 1$.

Now, setting $IU^*Bd(P_{q_n}) = IU^*Bd(K_{q_n})$ we obtain an intuitionistic uniform* border of intuitionistic symmetric members $IU^*Bd(P_{q_1}), IU^*Bd(P_{q_2}),...,IU^*Bd(P_{q_n})$ that satisfy $(S_{m,1})$. Therefore the collection $\{IU^*Bd(P_{q_i})|i = 1, 2,...,j\}$ has the required property.

**CONCLUSION**

In this chapter, the concepts of intuitionistic uniform* topological spaces, intuitionistic uniform* $AB$-border of an intuitionistic symmetric member, intuitionistic uniform* $AB$-boundary of an intuitionistic symmetric member, intuitionistic uniform* $AB$-derived symmetric members, intuitionistic uniform* $AB$-border irresolute functions, intuitionistic uniform* $AB$-border open functions and intuitionistic uniform* $AB$-border spaces are studied. In this connection, some interesting properties are established.