5.1 Introduction

In the study of HIV infection and its consequences the seroconversion of the infected is a vital event. The estimation of the expected time to seroconversion of the HIV infected over the time interval \((a, t]\) is an important aspect which helps medical intervention. The concept of preventive strategy is gaining greater importance nowadays in view of both prevention and delaying the process of seroconversion. The effectiveness of such a preventive strategy is investigated using a stochastic model by appropriate conceptualization of the problem. In this stochastic model the expected time to
5.2 Motivation

The model prescribed in chapter 4 concerned with the estimation of expected time to seroconversion under the assumption that in every contact of a person with an unknown infected partner there is a possibility of acquiring HIV, due to this there is some role to the antigenic diversity. In the present model discussed in this chapter, the concept of preventive strategy is introduced. The electronic media is used to educate the people on the use of condom. It is assumed that the individual takes some precautionary measure with a view to avoid the HIV getting transmitted and this is called alertness. It is a well known fact that the use of condoms during the sexual contacts is the most useful preventive strategy against HIV infection. Specifically in our country the television and radio broadcast are used as media of information to propagate the use of condoms as a preventive strategy. The use of preventive strategies gives rise to the concept of alertness on the part of the individual who has sexual contacts with unknown partners. In this chapter the effectiveness of the adoption preventive strategy in the elongation of the expected time to seroconversion is investigated by introducing a stochastic model. A stochastic model for estimating the expected time to seroconversion and its variance under successive contact
with no preventive strategy has been previously discussed by [106] Sathiyamoorthi and Kannan (2001). In their study the concept of alertness and preventive strategy, a stochastic model for the estimation of expected time to seroconversion and its variance are derived under the assumption that the threshold level of antigenic diversity is a random variable which follows an exponential distribution. In developing such a stochastic model the concept of shock model and cumulative damage process developed by [43] Esary et al (1973) is used. This has motivated the investigator to develop a stochastic model for time to cross antigenic diversity threshold of HIV infection.

5.3 Model Description

This model portrays a situation which is more realistic and has concurrence with the medical findings. A person may be alert in any contact with probability ‘$p$’ and in-alert with probability ‘$q$’ so that $p + q = 1$. At the same time the transmission of HIV during a contact is not a sure event according to medical findings. So it is assumed that the probability of transmission on infection during a contact occurs with probability $\beta < 1$. Hence one can visualize the following situation.

(i) A person is alert with probability ‘$p$’ and during that contact invasion does not take place.

(ii) A person is in-alert with probability ‘$q$’ and transmission occurs with probability $\beta$. 
(iii) A person is in-alert with probability $q$ and transmission does not occur with probability $(1 - \beta)$.

5.3.1 Assumption

(i) Sexual contact is the only source of transmission of HIV.

(ii) During any contact in which a person is in-alert the transmission of HIV is a sure event.

(iii) If the total damage caused when a threshold level $y$ exceeds, which itself is a random variable, the seroconversion occurs and a person is recognized as an infected.

(iv) The cumulative damage is linear and additive.

(v) The damage occur if a partner is in-alert.

(vi) In any single contact a person is alert with probability $p$ and in-alert with probability $q$ so that $p + q = 1$.

5.3.2 Notations

$X_i$ - a random variable denoting the increase in the antigenic diversity arising due to the HIV transmitted during the $i^{th}$ contact $X_1, X_2 \ldots X_k$ are continuous i.i.d. random variable with p.d.f. $g(.)$ and c.d.f. $G(.)$. 
5.4 Analysis

\( Y \) - a random variable representing antigenic diversity threshold and follows exponential with parameter \( \theta \).

\( U_i \) - a continuous random variable denoting the inter-arrival times between successive contacts with p.d.f. \( . \) and c.d.f. \( F(\cdot) \).

\( M(z) \) - the random variable representing the time between damages \( g_k(\cdot) \) the p.d.f. of random variable \( \Sigma x_i \).

\( F_k(\cdot) \) - the convolution of \( F(\cdot) \).

\( T \) - is the continuous random variable denoting the time to seroconversion with p.d.f. \( l(\cdot) \) and c.d.f. \( L(\cdot) \).

\( l^*(s) \) is the Laplace transform of \( l(t) \).

\( f^*(s) \) is the Laplace transform of \( f(t) \).

\( V_k(t) \) is the probability of exactly \( k \) contacts in the time interval \( (o, t] \).

5.4 Analysis

\( S(t) = \text{Survival function} = P(T > t) \)

\[ = \sum_{k=0}^{\infty} P_r\{\text{there are exactly } k \text{ contact in } (o, t)\} \]

\[ \times P_r\{\text{the cumulative total of antigenic diversity } < y\} \]
The threshold variable $y$ has exponential distribution with parameter $\theta$, so that

$$P(Y < y) = 1 - e^{-\theta y}, \ y > \theta, \ \theta > 0$$

$$P[x_1 + x_2 + \cdots + x_k < y]$$

$$= \int_0^\infty g(x)e^{-\theta x}dx$$

$$= g^*(\theta)$$

$$= [g^*(\theta)]^k$$

Therefore, $S(t) = \sum_{k=1}^\infty [M_k(t) - M_{k+1}(t)][g^*(\theta)]^k$

$L(t) = 1 - S(t)$ is called prevalence function as mentioned in [61] Jewell and Shiboski (1990).

$$L(t) = P(T < t) = [1 - g^*(\theta)] \sum_{k=1}^\infty [g^*(\theta)]^{k-1}M_k(t)$$

The Laplace stieltjes transform of $L(t)$ is

$$L^*(S) = \frac{[1 - g^*(\theta)]M^*(S)}{[1 - g^*(\theta)M^*(S)]}$$

(5.4.1)

But the c.d.f. of $Z$ is

$$M(z) = q\beta \sum_{n=0}^\infty [p + q(1 - \beta)]^n F_{n+1}(z)$$

Taking Laplace stieltjes transform of $M(z)$, we get
\[ M^*(s) = \int_0^\infty e^{-st}dM(z) \]
\[ = q\beta g_f(S) \sum_{n=0}^{\infty} [(p + q(1 - \beta)]g_f(S)]^n \]
\[ M^*(S) = \frac{q\beta f^*(S)}{[1 - p + q(1 - \beta)]f^*(S)} \] (5.4.2)

Substituting (5.4.2) in (5.4.1) we get
\[ L^*(S) = 1 + \left[ \frac{1 - f^*(S)}{1 - g^*(\theta)q\beta f^*(S)} \right]^2. \]

The mean time to seroconversion is given by,
\[ \text{Mean} = \mu_t = E(T) = - \frac{dL^*(S)}{dS} \bigg|_{S=0} = \frac{\mu_u}{[1 - g^*(\theta)]q\beta}. \]
\[ E(T^2) = \frac{6u^2q\beta^2 + \mu_u^2(\beta^2 + q^2\beta^2g^*(\theta) + 1)}{(1 - g^*(\theta))^2q^2\beta^2} \]

Hence \[ V_t = E(T^2) - (E(T))^2 \]
\[ V_t = \frac{6u^2 + \mu_u^2/q + \mu_u^2 g^*(\theta)}{q [1 - g^*(\theta)]}. \]

Special Cases

With the assumptions that the random variable \( U \) is exponential with parameter \( \lambda \), it is seen that \( \mu_u = \frac{1}{\lambda} \).

This leads to the result \( \mu_{ua} = \frac{(\alpha + \theta)}{\lambda q\beta \theta} \).

If there is no alterness, then it follows that \( q = 1, \beta = 1 \) from which one obtains \( \mu_t = \frac{(\alpha + \theta)}{\lambda \theta} \).
Therefore, $\mu_{ta} > \mu_t$ and this implies that the mean time to seroconversion is largest in the case of alertness which is inversely proportional to the probability of non-alertness $q$, which is an interesting result. Further, it is seen that

$$
\sigma^2_t = \frac{(\alpha + \theta)^2}{\lambda^2 q^2 \beta^2 \theta^2}.
$$

When $q = 1$, $\beta = 1$, it follows that

$$
\sigma^2_t = \frac{(\alpha + \theta)^2}{\lambda^2 \theta^2}.
$$

## 5.5 Numerical Illustration

To illustrate the applications of the above result $\theta = 1$, $\alpha = 1$, $\beta = 0.5$, and $q = 0.1$, $\lambda = 1, 2, \ldots, 10$ then the values of $E(T)$ and $V(T)$ as follows.

### 5.5.1 Graphs of $E(T)$ and $V(T)$ for different values of $\lambda$

Using the formula for $E(T)$ and $V(T)$ taking the values $\theta = 1$, $\alpha = 1$, $\beta = 0.5$, and $q = 0.1$, $\lambda = 1, 2, \ldots, 10$ then $E(T)$ and $V(T)$ are found and the corresponding graphs are drawn.
Table 5.5.1: Table of $E(T)$ and $V(T)$ for different values of $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$E(T)$ Mean</th>
<th>$V(T)$ Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00</td>
<td>1600.00</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>400.00</td>
</tr>
<tr>
<td>3</td>
<td>1.30</td>
<td>177.80</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>100.00</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>64.00</td>
</tr>
<tr>
<td>6</td>
<td>0.70</td>
<td>44.40</td>
</tr>
<tr>
<td>7</td>
<td>0.60</td>
<td>32.65</td>
</tr>
<tr>
<td>8</td>
<td>0.50</td>
<td>25.00</td>
</tr>
<tr>
<td>9</td>
<td>0.44</td>
<td>19.75</td>
</tr>
<tr>
<td>10</td>
<td>0.40</td>
<td>16.00</td>
</tr>
</tbody>
</table>

Figure 5.5.1: Graph of $E(T)$ for table 5.5.1
5.5. Numerical Illustration

Figure 5.5.2: Graph of $V(T)$ for table 5.5.1

$\lambda$ - Parameter for inter arrival times

Observation

As the value of $\lambda$ increases the value of $E(T)$ and $V(T)$ decreases.

5.5.2 Graphs of $E(T)$ and $V(T)$ for different values of $\alpha$

Using the formula for $E(T)$ and $V(T)$ taking the values $\lambda = 1$, $\theta = 1$, $q = 0.1$, $\beta = 0.5$, $\alpha = 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5$ and the corresponding graphs are drawn.
Table 5.5.2: Table of $E(T)$ and $V(T)$ for different values of $\alpha$

<table>
<thead>
<tr>
<th>$\lambda = 1, \theta = 1, q = 0.1, \beta = 0.5$</th>
<th>$\alpha$</th>
<th>$E(T)$ Mean</th>
<th>$V(T)$ Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>40</td>
<td>1600</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>50</td>
<td>2500</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>60</td>
<td>3600</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>70</td>
<td>4900</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>80</td>
<td>6400</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>90</td>
<td>8100</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>100</td>
<td>10000</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>110</td>
<td>12100</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>120</td>
<td>14400</td>
</tr>
</tbody>
</table>

Figure 5.5.3: Graph of $E(T)$ for table 5.5.2
5.5. Numerical Illustration

5.5.4 Graph of $V(T)$ for table 5.5.2

$\alpha$ - Threshold parameter

Observation

As the value of $\alpha$ increases the value of $E(T)$ and $V(T)$ increases

5.5.3 Graphs of $E(T)$ and $V(T)$ for different values of $q$

Using the formula for $E(T)$ and $V(T)$, taking the values $\lambda = 1$, $\theta = 1$, $\alpha = 1$, $\beta = 0.5$, and $q = 0.1, 0.2 \ldots 0.9$ then the corresponding graphs are drawn.
Table 5.5.3: Table of $E(T)$ and $V(T)$ for different values of $q$

<table>
<thead>
<tr>
<th>$q$</th>
<th>$E(T)$ Mean</th>
<th>$V(T)$ Variance</th>
<th>$E(T)$ Mean</th>
<th>$V(T)$ Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>40.00</td>
<td>1600.00</td>
<td>28.6000</td>
<td>816.000</td>
</tr>
<tr>
<td>0.2</td>
<td>20.00</td>
<td>400.00</td>
<td>14.3000</td>
<td>204.000</td>
</tr>
<tr>
<td>0.3</td>
<td>13.33</td>
<td>177.78</td>
<td>9.5000</td>
<td>90.670</td>
</tr>
<tr>
<td>0.4</td>
<td>10.00</td>
<td>100.00</td>
<td>7.1500</td>
<td>51.000</td>
</tr>
<tr>
<td>0.5</td>
<td>8.00</td>
<td>64.00</td>
<td>5.7200</td>
<td>32.640</td>
</tr>
<tr>
<td>0.6</td>
<td>6.67</td>
<td>44.40</td>
<td>4.7670</td>
<td>22.670</td>
</tr>
<tr>
<td>0.7</td>
<td>5.77</td>
<td>32.65</td>
<td>4.0857</td>
<td>16.650</td>
</tr>
<tr>
<td>0.8</td>
<td>5.00</td>
<td>25.00</td>
<td>3.5750</td>
<td>12.750</td>
</tr>
<tr>
<td>0.9</td>
<td>4.44</td>
<td>19.80</td>
<td>3.1780</td>
<td>10.074</td>
</tr>
</tbody>
</table>

Figure 5.5.5: Graph of $E(T)$ and $V(T)$ for table 5.5.3
5.5. Numerical Illustration

Figure 5.5.6: Graph of $E(T)$ and $V(T)$ for table 5.5.3

$q$ - Inalert probability

Observation

As the value of $q$ increases the value of $E(T)$ and $V(T)$ decreases

5.5.4 Graphs of $E(T)$ and $V(T)$ for different values of $\theta$

Using the formula for $E(T)$ and $V(T)$ taking the values $\theta = 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5$, $\alpha = 1$, $\beta = 0.5$, and $q = 1$, $\lambda = 1$ then $E(T)$ and $V(T)$ are found and the corresponding graphs are drawn.
Table 5.5.4: Table of \( E(T) \) and \( V(T) \) for different values of \( \theta \)

<table>
<thead>
<tr>
<th>( \lambda = 1, \theta = 1, q = 0.1, \beta = 0.5 )</th>
<th>( \theta )</th>
<th>( E(T) ) Mean</th>
<th>( V(T) ) Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00</td>
<td>16.000</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>3.33</td>
<td>11.110</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
<td>9.000</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>2.80</td>
<td>7.840</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.67</td>
<td>7.110</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>2.57</td>
<td>6.610</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.50</td>
<td>6.250</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>2.44</td>
<td>5.975</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.40</td>
<td>4.800</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.5.7: Graph of \( E(T) \) and \( V(T) \) for table 5.5.4

\( \theta \) - Threshold probability

Observation

As the value of \( \theta \) increases the value of \( E(T) \) and \( V(T) \) decreases
5.6 Conclusion

1. $\lambda$ the parameter of the distribution of random variable denoting the inter contact
time increases then expected time to seroconversion decreases and variance of it also
decreases.

2. $\alpha$ is the parameter of the random variable depicting the amount of antigenic
diversity contribution of antigenic diversity increases, then both $E(T)$ and $V(T)$
also increases.

3. It is observed from the table that as the probability of in-alertness increases, the mean
time to seroconversion decreases. It is also quite reasonable as regard to the variance
as it could be seen as the value of $q$ increases the variance decreases. It is observed
that the value of $\beta$ increases, then there is a decreases in $E(T)$ and $V(T)$.

4. It is observed from the table and also from the graph as the value of $\theta$ which is
the parameter of the exponential distribution depicting the threshold increases then
the mean time to seroconversion decreases. It is also quite reasonable as regard to
variance. It could be seen that if the value of $\alpha$ increases then the variance decreases.
The above model shows that the adoption of preventive strategies leads to the elongation of the expected time to seroconversion. Based on the concept of alertness and preventive strategy the expected time to seroconversion and its variance are derived under the assumption that the threshold level of antigenic diversity is a random variable which follows an exponential distribution.