CHAPTER I

INTRODUCTION


1.1 INTRODUCTION

The reliable performance of system is of immense interest in modern business, industrial and defence systems. In fact, increasing complexity of present day system has resulted in engineering problems involving high performance, reliability and maintainability. It is the importance of reliability in system analysis studies that has attracted attention of a large number of research workers from different disciplines to carry out their analysis from the reliability viewpoint. Investigation of reliability models with cost considerations is of utmost importance to system analysts.

We develop below basic definitions and terminology which are needed to understand a reliability model.

1.2 Definitions and Concepts

The terms/parameters which are used in reliability may be divided into two categories:

(A) System configuration parameters

(B) System effectiveness parameters.

(A) System Configuration Parameters:

1. **Series system**: is one in which a failure in any one of its units (components) leads to system failure.

2. **Redundant systems**: are those where more than one means (units) are available for performing a function, so that when one means (unit) fails, others are available to perform the task.
3. **Parallel or active redundant systems** :- are those where all redundant units and basic units operate simultaneously rather than being switched on when needed.

4. **Standby redundant systems** :- are those in which only one unit operates at a time, while the other redundant units (components) are switched on sequentially only when needed i.e., upon the failure of the operating units. Here, the system is said to have failed when all the units have failed.

The standby redundancy may be cold, warm or hot.

(a) **Cold standby redundant systems** :- are those in which redundant units do not fail when they are in standby.

(b) **Warm standby redundant systems** :- are those where redundant units can fail when they are in standby. Normally, in such systems failure probability of standby unit is less than that of an operating unit.

(c) **Hot standby redundant systems** :- are those in which redundant units can fail when they are in standby with failure probability same as that of an operating unit.

5. **k-out-of-n : F system** :- it consists of n units and the system fails if and only if k (< n) out of its n units are failed. Such systems are prevalent in practical systems, e.g., a communication system with 4 transmitters in which the average message load is such that at least 3 transmitters must be operational for successful message delivery, thus the system is 2-out-of-4 : F system.
6. **k-out-of-n : G system** :- it is a redundant system composed of n units and the system functions if and only if at least k (< n) out of n units are operable. It is clear that series and parallel systems are n-out-of-n : G and 1-out-of-n : G systems respectively.

7. **Complex systems** :- are those systems which do not fall into the category of either standby or parallel or series-parallel systems.

8. **Imperfect switchover** :- The phenomenon which occurs when a standby unit is switched to operate at the time of failure of an operative unit in a redundant system is referred to as 'switchover'. Some times the switchover is not perfect i.e., not 100% error free and/or instantaneous. This is called ‘imperfect switchover’.

9. **Maintainability** :- may be defined as a characteristic of design and installation which is expressed as the probability that a unit will be retained in or restored to a specified condition within a given period of time, when the maintenance is performed in accordance with certain rules. Maintenance includes all actions necessary for retaining a unit in or restoring it to a specified condition.

   It is further divided into following two categories :-

   (a) **Repair maintenance** :- whenever a unit fails, an activity is performed to restore the unit to a specified condition (operation).

   (b) **Preventive Maintenance (P.M.)** :- this is performed in an attempt to retain a unit in a specified condition by providing systematic, inspection,
detection and prevention of failure.

P.M. is required for costly systems like computers, satellites, radar, transmitters etc., where cost of failure or replacement is quite heavy and systems are required to operate without failure for a long time span. There are several types of P.M. policies.

10. **Service replacements** :- are those replacement which are made after the occurrence of failure.

11. **Planned replacements** :- are those replacement which are made after the occurrence of failure. Such replacement may be made at scheduled times.

12. **Delayed repair** :- the repair of a unit is some times delayed due to any physical or environmental reason. This process is called delayed repair.

13. **Fault detector** :- occurrence of a fault (or failure) in a system is not always detected automatically. As such some mechanism may be used to detect the fault, this is referred to as fault detector.

(B) **System effectiveness parameters** :

1. **Reliability** :- may be defined as the probability that a system will perform its function adequately for a definite period of time under specified conditions.

2. **Pointwise availability** :- probability that the system will be able to operate within certain tolerances at a given instant of time.
3. Interval reliability :- probability that a system continues to operate during a given interval of time.

4. Interval availability :- expected proportion of a given interval of time that the system will be able to operate within certain tolerances.

5. Steady-state availability :- expected proportion of time that a system is operable when it is allowed to operate for a very long period.

6. First passage time to system failure :- this is defined as the probability that starting in a given state (up) system enters a failed state for the first time in time $t$.

That is

$$\varphi_{OF}(t) = P[\text{system goes to state 'F' in time } \leq t \mid \text{system is in 'O' at } t=0].$$

7. Mean time to system failure :- if $T$ denotes time to first system failure starting from the beginning, then MTSF is given by $E(T)$ where $E$ stands for mathematical expectation.

This measure of system effectiveness appears to be more appropriate for the so called one-shot systems, e.g., missiles, rockets etc., which are used once for all.

Infact, first passage time distribution to system failure, $\varphi_0(t)$ and MTSF are related to the following manner.

$$\text{MTSF} = \left. -\frac{d}{ds} \varphi_0^*(s) \right|_{s=0}$$
where \( \varphi^*_0(s) \) denotes L.S. transform of \( \varphi_0(t) \).

8. **Expected profit** :- This is the total expected earnings per unit time of the system if the system is allowed to operate for sufficiently long period.

1.3 System Models in the Field of Reliability

Keeping in view the varied nature of practical problems, a large number of reliability models have been developed and analyzed in literature.

Most of the papers appeared are motivated to the following two directions:-

(1) **Analysis of special systems.**

(2) **Development of reliability, availability and cost optimization models.**

In the analysis of special systems one is interested in a specific system. The general approach for the analysis of such systems consists in defining and formulating the system model i.e., to state its assumptions, to enumerate different states and identify them as up and down ones, to develop the underlying equations governing the system behaviour and to apply an appropriate technique to obtain the solution.

For the second case viz., in the development of reliability, availability, cost optimization models, first of all one has to formulate the problem clearly i.e., to construct the objective function and constraints. Infact, in this area, formulation aspect is important in its own right, this involves clear understanding of effectiveness criterion, constraints visualisation, and their impact on objective functions. Once the problem is formulated successfully, next step is to search
and/or develop a suitable solution method to get optimal values for decision variables.

We now review the existing literature on reliability keeping in view the above classification.

(1) **Analysis of special system** :- In general following types of system models have been discussed under this category.

(i) **Cold standby redundant systems** :- Muth (1966) considered a cold standby system with constant hazard rate for the on line unit where the repair time is a random variable having a gamma distribution and obtained expressions for the Laplace transform of mean time-to-system-failure (MTSF) of the system. Subsequently, Linton and Baraswell (1973) have generalized the results of Muth (1966) for case in which life time of the unit and its repair time are generally distributed random variables. Nakagawa and Osaki (1974a) obtained for the same system, expressions for the Laplace transform of the reliability, steady-state availability and its MTSF. Nakagawa and Osaki (1974b) derived expressions for the expected number of visits to a state. Srinivasan and Gopalan (1973a) obtained expressions for the Laplace transform of reliability and pointwise availability. Kumar (1977a) obtained expression for steady-state profit for a 2-unit cold standby redundant system.

(ii) **Warm standby redundant systems** :- Branson and Shah (1971) considered a 2-unit system with exponential failure times and
general repair times and obtained the MTSF and steady-state availability. Srinivasan and Gopalan (1973b) obtained the Laplace transform of pointwise availability of the system comprising of dissimilar units by employing supplementary variable technique. Kumar (1977d) considered a 2-unit system with exponential failure times and general repair times and obtained the expression for expected profit of the system. Birolini (1975) analyzed his model when the failure time distribution of the on line unit follows general distribution and failure time of the standby unit follows exponential distribution, repair times are also of exponential nature by employing regenerative stochastic processes.

(iii) **Parallel redundant systems** :- Gaver (1963) studied a 2-unit parallel redundant system with constant state dependent hazard rate and arbitrary repair and obtained the Laplace transform of reliability of the system its MTSF and steady-state unavailability, using supplement variable technique. Kulshrestha (1968, 1970), Mine and Kawai (1974), Linton and Saw (1974) have studied some parallel redundant systems. Kodama and Deguchi (1974) obtained the MTSF of a parallel redundant system with Erlangian failure and general repair. Linton (1976) extended these results for the case when the failure and repair distributions of one of the units are Erlangian and those of the other are arbitrary by employing supplementary variable technique.

(iv) **2-Unit standby redundant system with imperfect switch over** :-
The importance of switch in standby redundant systems has attracted the attention of many research workers to incorporate switch behaviour in investigating the stochastic behaviour of standby redundant systems. Gopalan (1975), Osaki (1972a) Prakash (1973), Nakagawa and Osaki (1975), Nielsen and Runge (1974), Kumar (1977b) have studied such systems under different operational circumstances. Chow (1972, 1973) constructed mathematical models for the reliability of modularly redundant systems with unequal failure rates for the operating and standby units and different types of switch failures. Arnold (1973) introduced the concept of ‘coverage’ for repairable systems.

(v) **Redundant systems with delayed repair** :- When a unit or switch fails; repair facility may not be available because of its preoccupation with higher priority jobs and as such failed unit might have to wait for sometime. Several other reasons may also be attributed to delay in repair. The reliability models with delay in repair have been analyzed by a large number of workers. [Gopalan and Saxena (1977), Kapur and Kapoor (1975a), Khalil and Bogus (1975), Khalil (1977b), Kumar and Lal (1979) etc.].

(vi) **Redundant systems with two types of repair** :- Some electrical and mechanical devices can fail in two or more mutually exclusive failure modes. A vending machine or public telephone fails due to failure of units under benign environments or to the failure of human misuse. The failure (and thus equivalently repair) may
be categorized in 2 classes. Many papers with 2 types of repair have appeared in the literature, [Garg and Kumar (1977), Khalil (1977a,b), Kumar and Agarwal (1978), Osaki and Okumoto (1977), Proctor and Singh (1976) etc.].

(vii) **Complex systems** :- All redundant systems may not be either standby or parallel or series-parallel. Consider a complex system consisting of two classes of components denoted by \( L_1 \) and \( L_2 \). Class \( L_1 \) has \( n \) components in series and class \( L_2 \) has \( m \) identical components in parallel redundancy. Such systems have been investigated by Garg (1963a,b), Gupta (1973a,b), Sharma (1974), Agarwal (1975). Such systems have also been considered by Varma (1973) with repair priority and by Kapur and Kapoor (1975b) with demand pattern.

Different techniques were used by research workers in the evaluation study of reliability models to obtain parameters of interest. For example Gnedenko et al. (1965), Osaki (1970b) and Buzacott (1971) have used the renewal theoretic approach for the analysis of their models. The technique of Markov-renewal process has been successfully employed for the analysis of redundant systems by Srinivasan (1968), Osaki (1970a, 1972a,b), Osaki and Okumoto (1977). Further, Nakagawa and Osaki (1974a) have suggested a unique modification of MRP which is useful for the analysis of 2-unit redundant systems under general failure and repair time distributions. Later, this technique was employed in a large number of papers. [Nakagawa and Osaki (1974b, 1975), Kapur and Kapoor (1975a,b), Kapil and Sinha (1981)]. Gnedenko (1967),
Srinivasan et al. (1973) have used regeneration point technique for the analysis of redundant systems. The supplementary variable technique has been used in standby redundant system by Gaver (1963) and subsequently by others, Garg (1963a,b), Garg and Kumar (1977) to treat general distributions for repair times. Various other methods have also been applied by workers to analyze their models, e.g., discrete transforms Kulshrestha (1972), conditional transforms, Linton and Braswell (1973).

Branson and Shah (1971) applied the results from the theory of semi-Markov processes to reliability analysis for a 2-unit warm standby systems. Later on, this was applied to other systems [Kumar (1976, 1977d), Kumar and Lal (1979,1980)].

But a close look at the processes considered in these papers reveals the fact that the underlying process is not SMP in true sense. However, it looks so. The interesting fact is that the process satisfies certain conditions [Arndt (1977), Arndt and Franken (1977)] under which results from SMP theory are applicable. This has been pointed out in Kumar et al. (1981).

(2) Development of Reliability, Availability and Cost Optimization Models :- A large number of developments have appeared to optimize system effectiveness: reliability, availability, cost, profit etc. For problems of this nature, Tillman et al. (1980) may serve as a good reference. We review below some of the important optimization models.

(i) Optimization models in series and parallel system :- In these models there are several units (stages) say, connected in series and the units are statistically independent. If reliability of the
The $j^{th}$ unit is denoted by $R_j$, the system reliability $R$ is given by

$$ R = \prod_{j=1}^{n} R_j $$

Many workers [Ghare and Taylor (1969), Beraha and Misra (1974)] have made attempts to develop algorithms which maximize $R$ subject to $k$ resource constraints of the form

$$ \sum_{j=1}^{n} h_{ij}(R_j) \leq \alpha_i \quad i = 1, 2, \ldots, k $$

where $h_{ij}(R_j)$ denotes the requirement of the $i^{th}$ resource and $\alpha_i$ stands for availability of the $i^{th}$ resource. The functions $h_{ij}(R_j)$ may be linear or nonlinear depending upon situations under investigation.

Another useful area in series optimization models is redundancy allocation in an optimal manner. Banerjee and Rajamani (1973), Shetty and Sengupta (1975), Luus (1975) have considered problems to choose the optimum number of components, say $m_j$ in parallel at each of the $n$ stages in a series system so that reliability is maximized. Mathematically, the problem in general form is

$$ \text{Max } R ; \quad R = \prod_{j=1}^{n} (1 - Q_j^{m_j}), \quad Q_j = 1 - R_j $$

subject to the constraints given earlier.

Misra and Ljubojevic (1973), Tillman et al. (1977b) considered a simultaneous reliability and redundancy allocation problem viz.,

$$ \text{Max } R ; \quad R = \prod_{j=1}^{n} [1 - (1 - R_j)^{m_j}] $$
subject to

\[ \sum_{j=1}^{n} h_{ij}(R_j) = \sum_{j=1}^{n} h_{ij}(R_j, m_j) \leq \alpha_i \]

and choose \( R_j \) and \( m_j \).

They have used Lagrangian Multiplier technique to solve the problem. Shetty and Sengupta (1975) applied slacked sequential unconstraint minimization technique (SLUMT) which is superior to SUMT used by Tillman et al. (1970) for solving their problems.

Another formulation in series models has been due to Aggarwal (1977). He discussed the problem of minimizing the total cost under a lower bound on system reliability. In other words,

\[ \text{Min } C ; \quad C = \sum_{j=1}^{n} C_j(m_j) \]

subject to

\[ R = \prod_{j=1}^{n} R_j(m_j) \geq R_0 \]

\( C_j(m_j) \) being the cost of the \( j^{th} \) stage when \( m_j \) parallel components are used at this stage. He used heuristic approach for the solution.

Misra (1971a) gave a dynamic programming formulation of redundancy allocation at each stage under two nonlinear constraints.

The problem of choosing MTBF, MTTR and \( m_j \) by cost minimization subject to availability constraints has been formulated by Lambert et al. (1971). They have taken total cost \( C \) and availability \( A \) for a parallel series system.
as

\[ C = \sum_{j=1}^{n} (R_j, M_j, m_j) \]

\[ A = \prod_{j=1}^{n} A_j(R_j, M_j, m_j) \]

\[ A_j = \frac{R_j}{R_j + M_j} \]

R_j and M_j are MTBF and MTTR at the j\textsuperscript{th} stage respectively.

The problem is to minimize C by choosing R_j, M_j, m_j within permissible design requirements so as to meet a specified level of availability.

A typical design requirement for the j\textsuperscript{th} stage is

\[ R_j \geq 4m_j , \quad M_j \leq 5 \]

Dynamic programming technique was used to obtain the solution.

(ii) Reliability optimization models with repair schemes :- Pal and Bhattacharjee (1978) considered a system having n stages and j\textsuperscript{th} stage has m_j units. The objective is to maximize system reliability subject to nonlinear constraints depending upon maintenance. The system reliability is

\[ R(t) = \prod_{j=1}^{n} R_j(t) = \exp (-Zt) \]

where

\[ Z = \sum_{j=1}^{n} Z_j \]
and

\[ Z_j = \frac{[(1-\rho_j)^2 \rho_j^{m_j} - \lambda_j]}{1-\rho_j^{m_j} [1+m_j(1-\rho_j)]} \]

\[ \rho_j = \frac{\text{failure rate of stage } j}{\text{repair rate of stage } j} = \frac{\lambda_j}{\mu_j} \]

The steady-state probability \( P_{0j} \) of stage \( j \) with its units being idle is

\[ P_{0j} = \frac{1 - \rho_j}{1 - \rho_j^{m_j}} \]

The problem is

\[ \max_{m_j} R(t) \]

subject to

\[ m_j \geq 1 \quad (j = 1, 2, ..., n) \]

\[ \sum_{j=1}^{n} h_{ij} m_j \leq C_j \text{ (resource constraints)} \]

\[ t(1-P_{0j}) \leq T_j^* \text{ (busy period constraints)} \]

(iii) **Optimization in repair, replacement and inspection models** :-

Replacement is carried out when the failures cause serious damages and/or imply expensive repairs. Replacement can be made in accordance with a predefined policy.

Replacement theory is of great importance in reliability theory and has been investigated by many research workers. Barlow and Proschan (1965) discussed several interesting replacement problems and obtained the optimum
repair policies minimizing the expected cost per unit time. Glasser (1967) considered the age replacement problem of Barlow and Proschan (1965) and gave graphs of the optimum repair policies. Fox (1966) introduced discount rate for a replacement problem. Scheaffer (1971) investigated a replacement model with an increasing cost for an operating unit. Nakagawa and Osaki (1974c) obtained the optimum policies for a replacement model with delay. Nakagawa and Osaki (1974d) have taken a repair limit replacement policy which is defined as the policy in which the repair is stopped if it is not completed within a fixed time (called the repair limit time). The problem is to determine the optimum repair time limit τ, which minimizes the expected cost per unit time for an infinite time span. Kapil and Sinha (1981) have discussed repair limit suspension policy for a 2-unit redundant system with 2-phase repair. A failed unit is first repaired by a phase 1 repair.

If the repair is not completed in a fixed time then the unit goes for a phase 2 repair. They have obtained optimum repair limit suspension policies which maximize the availability using MRP results.

Muth (1977) also studied a similar problem in which a replacement takes place at the first failure time after a predetermined age τ.

Mine and Kawai (1975) discussed an optimal inspection and replacement policy for a unit which assumes any one of several Markov states. The policy evaluation function is expected cost per unit time over an infinite time span. The problem is formulated as a SMDP with a modified policy improvement routine.

Alam and Sarma (1974) considered maintenance policies for a machine
with degradation in performance with age and subject to failure.

Hastings (1969) developed a repair limit replacement method. In this, when an item requires repair it is first inspected and the repair cost is estimated. If the estimated cost exceeds a certain amount known as the repair limit, then the item is not repaired but replaced. Two types of problems, one in which condition of equipment is related to age and other in which condition is related to number of major repairs an item has had, have been analyzed. Dynamic Programming methods have been used to obtain optimum repair limits.

Diveroli (1974) has considered simultaneously the problems of preventive replacement and service replacement. Several procedures have been developed to determine optimal policies. He also proves the optimal preventive replacement policy is a function of the optimal repair limit.

(iv) **Profit optimization in redundant system models** :- Expected steady-state profit in redundant systems has been obtained in several papers [Kumar (1976), Kumar and Lal (1979)]. This measure of system performance was used to determine optimal P.M. in Kumar (1976). However, profit optimization problem was considered in Kumar and Lal (1980) to determine optimal maintenance policy for a redundant system. This method is based on Howard’s (1971) policy iteration. Srinivasan et al. (1971) have considered expected profit in a 2-unit redundant system by mixing of two renewal processes. Further, Kumar and Kapoor (1981a) extended the solution method to find optimal maintenance policy.
for a redundant system when the cost structure is discounted one.

(v) **Optimization models with fault detection**: The concept of fault detection in series system is important and needs attention. Gross (1970) had studied a 2-unit series system with fault detector, misclassification cost. He had minimized the cost of misclassification and determined the optimal cost spent on detector. Later, Kumar (1975) had extended the result to the case of n-unit series system. He obtained a sufficient condition under which a detection mechanism would be economically feasible i.e., expected cost of misclassification is minimized. Recently, Takami et al. (1978) have developed a Markov model for a class of series systems which have fault detectors to find component failures. The optimal allocation of fault detectors is determined by formulating the problem as 0-1 integer programming problem.

Recently, Kumar and Kapoor (1981b) have solved a problem concerning classification of failure in a 2-unit series system, the classification criterion depends upon the time to system failure.

### 1.4 Some Cost Structures in Reliability Models

The review of existing literature in previous sections reveals that authors who have contributed to the economic aspects of reliability had definite cost structures in their minds to superimpose on the system model. Generally speaking, reliability and cost are related parameters of a system. But a unique
expression of this relationship which is valid in reliability modelling is not feasible mathematically. Never the less, some properties originating from the conceptual frameworks which are desired of cost functions are :-

(I) Components with very low reliability cost very low.

(II) Similarly, components which are highly reliable cost very high.

(III) Cost is an increasing function of reliability.

(IV) Derivative of cost with respect to reliability is an increasing function of reliability.

We now discuss some important cost structures which have appeared in reliability modelling and serve a useful purpose to system analysts :-

(1) Beriphol (1961) has expressed the cost of a component as a function of its reliability as given below

\[ C_n = \frac{K_{1n}}{1-R_n(C_n)} \exp \left[-K_{2n}(1-R_n(C_n))\right] \]

where

- \( C_n \) : Cost of the \( n^{th} \) component
- \( R_n(C_n) \) : reliability of the \( n^{th} \) component
- \( K_{1n}, K_{2n} \) : constants depending upon components.

A modified version of the above cost structure has appeared in Aggarwal and Gupta (1975)

\[ C_n = K_n \left[\tan\left(\frac{\pi}{2}R_n\right)\right]^{(R_n)} \]
here \( f(R_n) \) is a function of \( R_n \). In many situations \( f(R_n) \) takes the form

\[
f(R_n) = 1 + R_n^{x_n}, \quad 0 < x_n < 1
\]

\[
f(R_n) = m_n, \quad 0 \leq m_n \leq 2
\]

(2) In the problem of maximizing \( R \) defined on page 12

\[
R = \prod_{j=1}^{n} [1 - (1 - R_j)^{m_j}]
\]

subject to

\[
\sum_{j=1}^{n} h_{ij}(R_j) \leq \alpha_i
\]

and to choose \( m_j \) and \( R_j \) simultaneously, Misra and Ljubojevic (1973) have taken the functions \( h_{ij}(R_j) \)'s as

\[
h_{ij}(R_j, m_j) = a_{ij} \exp[-b_{ij}/(1 - R_j)]
\]

Infact most of the models assume additivity of costs i.e., total cost \( C \) is additive in terms of the costs of the constituent units

\[
C = \sum_{j=1}^{n} C_j(m_j)
\]

where \( C_j(m_j) \) is the cost of the \( j^{th} \) stage assumed to be a known function of \( m_j \).

(3) Many developments are based on expected cost per unit time for an infinite time span. Nakagawa and Osaki (1974d), Kontoleon (1977), Muth (1977), Mine and Kawai (1974, 1975) have used this concept in their works. This is obtained as follows.
Find the expected cost from the instant a system starts operating up to the completion of the repair/replacement. (This constitutes one complete cycle. Divide this quantity by the mean time of one cycle.

For example, if \( F(t) \) is cdf of repair time of a system, \( C_0 \) is the replacement cost, \( C_r(t) \) is the expected repair cost incurred during \((0,T]\) then the expected total cost over one cycle is

\[
C = [C_0 + C_r(t)] \bar{F}(\tau) + \int_0^\tau C_r(t) \, dF(t)
\]

\[
= C_0 \bar{F}(\tau) + \int_0^\tau \bar{F}(t) \, dC_r(t)
\]

where \( \tau \) is repair limit time.

The mean time of the cycle is

\[
\lambda + \tau \bar{F}(t) + \int_0^\tau t \, dF(t) = \lambda + \int_0^\tau \bar{F}(t) \, dt
\]

Thus the expected cost per unit time over an infinite time span is

\[
L(\tau) = \frac{C_0 \bar{F}(\tau) + \int_0^\tau \bar{F}(t) \, dC_r(t)}{\lambda + \int_0^\tau \bar{F}(t) \, dt}
\]

For \( C_r(t) \) the two usual forms are:

\( C_r(t) = at^b, \quad a > 0, \quad 0 \leq b < 2 \)

i.e., repair cost is proportional to time and

\( C_r(t) = a(e^{bt} - 1), \quad a > 0, \quad b > 0 \)

i.e., exponential repair cost.
The problem is to obtain optimal $\tau$ which minimizes $L(\tau)$

(4) Shershin (1970) has discussed a problem of simultaneous apportionment of reliability and maintainability. He visualized the total cost function $C_N$ for a system with $N$ subsystems as the sum of the failure rate and maintainability costs. Assuming exponential failure rates, the failure rate $\lambda_s$ of the system and the failure rate for the $k^{th}$ subsystem are related by

$$\lambda_s = \sum_{k=1}^{N} \lambda_k$$

Further, let $d_k(\lambda_k)$ be the cost for achieving failure rate $\lambda_k$ for the $k^{th}$ subsystem. Then, the cost of achieving a system failure rate $\lambda_s$ is

$$C_N(\lambda_s) = \sum_{k=1}^{N} d_k(\lambda_k)$$

He has concentrated on maintenance time and has considered 3 major activities and their associated costs to be performed under maintenance time.

(i) Maintainability during design stage;

(ii) Performing maintenance to prevent failures termed as P.M.; and

(iii) Repairing a failed subsystem/subsystems termed as C.M. associated with each component.

(5) Nakagawa and Osaki (1976a) considered a problem of optimization for a one-unit reliability model which is provided with $n$ spares. They assumed in the model that whenever, the main unit fails, it undergoes repair and the spare unit starts functioning. After the repair of the main unit, it begins operation whereas the spare unit acts as standby. However, if the spare unit
fails before the repair of the main unit, it is not repaired and it is scrapped, the other spare unit starts operation. In this way, the first failure occurs when the main unit is under repair and the last spare unit fails. In the analysis of results to obtain total expected cost C(t) in (0,t], they used following costs :

\[ C_r : \text{cost of repairing the failed unit} \]

\[ C_s : \text{cost of replacing spare unit.} \]

Then, C, the expected cost per unit time for an infinite time span is

\[ C = \lim_{{t \to \infty}} \frac{C(t)}{t} = C_r \cdot [\text{expected number of failed units per unit time}] + C_s \cdot [\text{expected number of spares per unit time}]. \]

(6) Okumoto and Osaki (1977) discussed a cost structure in the context of obtaining preventive maintenance policies for a standby system. They assumed in their model that preventive maintenance is allowed for an operating unit only and when it is supported by a standby unit. But the failed unit is replaced as it cannot be repaired.

They have considered the following costs in their model.

\[ C : \text{replacement cost of a failed unit} \]

\[ a : \text{preventive maintenance cost per unit time.} \]

Then, the expected cost C(t) per unit time according to them is given by

\[ C(t) = a \cdot [\text{the limiting probability that a unit is under preventive maintenance}] + C \cdot [\text{expected number of replacements per unit time}]. \]
(7) Cleroux and Hanscom (1974) have taken a cost structure which takes into account adjustment costs, depreciation costs or interest charges which are paid at fixed equidistant intervals of time. They minimized the average cost per unit time over an infinite time span to determine optimal age replacement policy for a single unit system.

In notation let

\[ C_1 \] : cost for each replacement of a failed unit

\[ C_2 \] : cost for replacement of a non failed unit

\[ C_3(ik) \] : cost which is incurred at the age \( ik; \ i = 1, 2, \ldots \)

Above costs are known. Further, the sequence of costs \([C_3(ik)]\) helps in keeping the unit operative. The cost \([C_3(ik)]\) may include adjustment costs, depreciation costs, interest charges etc.

(8) In the problems related to detection of failures as and when they occur, some interesting cost structures have appeared in the literature Kumar (1975), Gross (1970), Kumar and Kapoor (1981b), Takami et al. (1978).

Kumar (1975) considered the problem of detection of failures in an \( n \)-component series system. He obtained a condition under which detection mechanism is economically feasible using the following cost structure

\[ p_j(c) \] : Prob. [detecting the \( j \)th component as failed whereas in fact \( i \)th component has failed]

where \( c \) is the amount of money spent on the detection mechanism that enhances the probability of correctly detecting the failed component.
\[ d_{ij} \] : extra incurred cost if \( i^{th} \) component has failed but failure is assigned to \( j^{th} \) component erroneously.

These \( p_{ij}(c) \) satisfy the following properties.

\[ \sum_{j=1}^{n} p_{ij}(c) = 1 \text{ for } i = 1, 2, ..., n \]

\[ 0 \leq p_{ij}(c) \leq 1 \text{ for } i, j = 1, 2, ..., n \]

\[ p_{ij}(0) = \frac{1}{n} \text{ for } i, j = 1, 2, ..., n \]

\[ p_{ij}(c) \geq \frac{1}{n} \text{ for } i = j = 1, 2, ..., n \text{ and } c \neq 0 \]

\[ p_{ij}(c) \leq \frac{1}{n} \text{ for } i \neq j = 1, 2, ..., n \text{ and } c \neq 0 \]

\[ p_{ii}(c) \uparrow c \text{ for } i = 1, 2, ..., n \]

\[ p_{ij}(c) \downarrow c \text{ for } i \neq j = 1, 2, ..., n \]

\[ \lim_{c \to \infty} p_{ij}(c) = \delta_{ij} \]

where \( \delta_{ij} \) is Kronecker's delta

\[ \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \]

Gross (1970) is a particular case of the above model when the number of components is 2.

(9) Kumar and Kapoor (1981b) have developed a classification rule for a 2-unit series system based on the life-time distribution of the system which minimizes the expected cost due to misclassification.
They have considered the following costs.

\[ c_{12} \]: cost of classifying the unit 2 as failed whereas unit 1 has failed.

\[ c_{21} \]: cost of classifying the unit 1 as failed whereas unit 2 has failed.

(10) Takami et al. (1978) have obtained the solution for the problem of optimal allocation of fault detectors in a series system. They have considered the following costs for their model.

\[ c_i \]: cost of the detector allocated to component \( i \)

\[ c_s \]: loss per unit time caused by the system failure.

(11) The following reward (cost) structure due to Howard (1971) has been extensively applied to obtain expected profit in redundant systems operating under different operational conditions, Kumar (1976, 1977c).

The cost structure is :-

\[ Y_i \]: earning rate of the system while it is in the state \( S_i \)

\[ r_{ij} \]: fixed transition cost incurred at the time of transition from \( S_i \) to \( S_j \).

It may be remarked that the above cost structure allows both types of costs viz., fixed and variable costs.

1.5 SUMMARY OF THE THESIS

Contribution and Distribution of Chapters :- The review given in the preceding pages reveals that for non-maintained systems problem of redundancy allocation is important whereas for efficient running of maintained systems it is worthwhile to determine optimal failure/repair rates. If a system is in the
working state (i.e., it is already designed) one will be interested in having a control over the repair process because failure properties might have already been prespecified in design stage. So, for maintained systems there is a genuine demand of research investigations in the direction of repair/maintenance which can help a maintenance engineer/analyst in making an optimal choice of repair rule (for a fixed failure rate) and/or in having better understanding of system's economics by studying parameters' effectiveness from cost considerations.

Keeping the above in view, the efforts in the thesis are motivated in the following directions.

(i) To provide a decision maker with readily available values of repair rates depending upon the earning rates and failure characteristics of the system which optimizes steady-state expected profit of the system and to provide basic cost equations for the better understanding of system's economic behaviour.

(ii) To suggest some guidelines to the management regarding installation of a new system/or replacing the existing one with a new one whose parameters like failure rate, repair rate, earnings in various states (viz., operable and failed) are known.

(iii) Further, in maintained systems, it is of great importance to analyze failures i.e., to locate/detect a failure, to classify a failure, to isolate the faulty component/components. In this direction, two problems of failure detection and classification have also been formulated and solved.

(iv) In the context of the non-maintained systems, a reliability
maximization problem in an n-component series system under constraint on cost has been discussed.

While formulating the problems at (i) and (ii) and discussing the solution procedures, the cost structure of Howard (1971) has been superimposed on the Markov process/semi-Markov process generated by a system model. The expected profit thus obtained has been maximized and optimal repair rates are determined. This cost structure allows different cost rates (variables) in each state and different transition costs incurred at the time when the system makes a transition from one state to another.

For the problem of failure detection in a series system, a cost structure involving probabilities of correct detection (depending upon cost spent on detection) has been mixed with Howard’s reward structure (1971) and is superimposed on a 2-unit series system. Optimal allocation of cost on detection mechanisms has been determined when the total cost for detection is fixed. Further, a problem of failure classification in a series system has been discussed after taking into account costs of misclassification whenever the system fails. A decision rule based on the life-time distribution of the system which minimizes the expected misclassification cost has been suggested. A procedure has also been suggested to examine the discriminatory power of the rule.

For the problem of cost allocation to different components in a series system under a constraint on the total cost, a useful and interesting reliability cost function is devised and a solution procedure is developed based on dynamic programming.
The thesis consists of four chapters:

Chapter I : Introduction

Chapter II : Optimal Repair Rate in Some Maintained Systems

Chapter III : Optimal Failure Analysis in Series Systems


Chapter II discusses the profit evaluation studies for some systems with repair maintenance. Three models describe single unit maintained systems under different operational conditions and one model deals with 2-unit system with parallel redundancy. The concept of minor repair has also been applied in one of the models. Optimal repair rates maximizing expected profit have been obtained analytically for almost all models. The four models on profit evaluation studies in this chapter are discussed in the following sections:

2.2- Profit evaluation in a single unit system

2.3- Profit evaluation in a single unit system with a less productive state

2.4- Profit considerations in a single unit system with minor repair

2.5- Profit evaluation in a 2-unit parallel redundant system.

Chapter III deals with failure analysis in series systems. Problems of failure classification and resource allocation on failure detection mechanisms have been considered in the following two sections:

3.2- Optimal failure classification in an n-unit series system.
3.3- Optimal allocation of cost to detectors in a 2-unit series system.

The last chapter discusses application of dynamic programming technique to solve problem in series systems. Problem of cost allocation has been formulated and solved through dynamic programming technique. This is discussed in the following section:

4.2- Optimum cost allocation in a series system under a constraint on the cost.