CHAPTER 3
ANALYSIS OF NON-ISOTHERMAL CONTINUOUS STIRRED TANK REACTOR IN THE PRESENCE OF MULTIPLICATIVE NOISE
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3.1 INTRODUCTION AND PROBLEM FORMULATION

In the previous chapter the role of additive noise has been considered and the results have brought out the behavioural features of a CSTR in its presence. The present chapter considers the role of another type of noise referred to as the multiplicative noise. This type of noise can enter the system, for example, by considering the random variations in the reaction rate. The reaction rate as measured experimentally represents an average value because local variations can be caused due to fluctuations in the temperature, concentration or even a parameter such as activation energy. It is this latter parameter which is assumed to bring about variations in rate in the present analysis. The governing equations in dimensionless forms for this case can be written as:

\[
\frac{dx_1}{dt} = -x_1 + Da \exp\left(\frac{x_2}{1 + x_2/\beta}\right) \exp\left(\frac{\gamma \xi(t)}{1 + x_2/\gamma}\right) (1 - x_1) \tag{3.1}
\]

\[
\frac{dx_2}{dt} = -x_2 + Da B (1 - x_1) \exp\left(\frac{x_2}{1 + x_2/\beta}\right) \exp\left(\frac{\gamma \xi(t)}{1 + x_2/\gamma}\right) - (x_2 - x_c) \beta \tag{3.2}
\]

\[
\frac{d\xi}{dt} = -\epsilon^{-2} + \epsilon^{-1} D \frac{dw_1}{dt} \tag{3.3}
\]
It may be noted from these governing equations that the random noise now interacts with the state variable of the system. This type of interaction can therefore be expected to have more drastic effects than those noted in the earlier case of additive noise where no such interactions were present. The details and results of region-wise calculations showing the effects of varying noise intensity and time-correlation will now be discussed. The numerical method employed to solve these equations is already discussed in Chapter 2.

3.2 RESULTS AND DISCUSSION

The results obtained in presence of multiplicative noise indicate that for a system operating in region I of the macroscopic model, noise does not alter the system behaviour significantly. Further, the stochastic solution never stabilises and keeps oscillating around the macroscopic solution. As before increasing the noise parameters bring about greater deviations. In region II, in the entire parameter space, the results show behaviour similar to that in region I. In other words, the bistable feature of the reacting system, permissible from a macroscopic viewpoint, seems to be lost in presence of noise.

Region IIIa

As stated earlier, the macroscopic system operating in this region exhibits A, C, H and E types of solutions depending on the range of parameter values, e.g. $D_c$. For lower values of the parameter $D_a$ the system invariably operates with a single stable node solution typical of A type of behaviour. With progressive increase in the value of the parameter $D_a$ the system
Figure III-a-1: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise (R = 0.4, D = 0.000062, x = 0.00681).
successively exhibits three solutions (two of which are unstable and a stable mode) that characterize the C type of behaviour. Further increase in the value of Da allows four stationary solutions of the system with two stable and two unstable solutions. This is type H behaviour of the macroscopic model. Finally a further increase in the value of Da leads to a bistable situation with two stable solutions and a middle unstable solution. The presence of additive type noise as seen in the previous chapter, alters some of the characteristics of this macroscopic bifurcation pattern. However, it does not generate any new solution. The effects of multiplicative type of noise are however considerably different in this respect. For each one of these possible types of solutions (viz. A, C, H and E) we shall now present the behaviour in presence of multiplicative noise.

Figure IIIa-1 shows the stochastic behaviour for A type of solution. The parameter values are $B = 15$, $\beta = 0.4$ and $Da = 0.000632$. The corresponding macroscopic solution is also indicated in the figure. As in the case of additive noise, even here, the stochastic profiles considerably deviate from the macroscopic solution although they move around and close to it. The effect of increase in the intensity of the noise and its time-correlation are clearly evident in the figure which shows increased deviation with increase in the noise parameters.

Figure IIIa-2 shows a situation for a system exhibiting C type of behaviour. The parameter values for this case are $B = 15$, $\beta = 0.4$ and $Da = 0.00064$. The corresponding macroscopic model for this set of parameter values possesses three stationary states, with one stable and two unstable solutions $(x_{2s} = 0.00689, 9.51, 9.63)$. The solution that can be realized in practice
FIGURE IIIa-2: DIMENSIONLESS TEMPERATURE - TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

$B = 15$, $\beta = 0.4$, $D_n = 0.00064$, Initial condition $x_{-\infty} = 0.00689$
are underlined for the purpose of identification. The stochastic solution for the situation beginning with the stable stationary state as the initial condition is sketched in Fig. IIIa-2 and the nature of the profiles is similar to those observed in the case of A type solution.

Figure IIIa-3 illustrates the profiles obtained by beginning with one of the unstable solutions. The figure shows the influence of the intensity and time-correlation effects of the noise. An interesting feature to note in this figure is the fact that for certain values of the noise intensity and time-correlation the macroscopic unstable solution stabilizes to a new solution which is not permissible in the macroscopic model. This solution represents a noise induced solution and is clearly a result of the presence of multiplicative noise in the system. It may be noted that the earlier case of additive noise also does not permit the existence of this solution. Beginning with the third unstable type C solution, the same feature recurs. It therefore appears that the two unstable states in the macroscopic model (i.e., type C) are transferred into a single stable solution. The transition is therefore purely noise induced.

Figures IIIa-4-IIIa-6 show stochastic behaviour of the system in the H type region. The parameter values for this case are $B = 15, \beta = 0.4$ and $Da = 00066$. The stationary solutions of the macroscopic model for this set of parameter values are $(x_{28} = 0.005, 9.25, 9.51, 9.61)$

The stable solutions realizable from the practical viewpoint are as before underlined. Figure IIIa-4 shows the stochastic behaviour for the case beginning with the stable node. The stochastic profiles describe a behaviour that is familiar and similar to those encountered for a single node.
Figure IIIa-3: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise

\( B = 15, \beta = 0.4, D_n = 0.00064, \text{initial condition } x_0 = 9.51 \)
FIGURE IIIa-4: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT
VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
($\alpha = 15, \beta = 0.4, D_0 = 0.00066$, initial condition $x_0 = 0.005$)
in the A or C type of solutions.

Beginning with the unstable limit cycle, Fig.IIIa-5 shows the corresponding stochastic behaviour. Clearly as in the case of C type of solutions, the unstable limit cycle of the macroscopic model, in the presence of fluctuations, transforms itself into a stable solution. This occurs for certain sets of parameter values of the noise. Starting with the saddle point in H type of solutions, a similar feature of noise induced stationary state is realized (Fig.IIIa-6). The interesting feature of the system operating in this region is that it permits the existence of either a stationary state that corresponds to a macroscopic stationary state or gets driven to a totally new state that has been generated due to presence of noise.

Region IIIb

The system operating in region IIIb also shows a plethora of behaviour in a manner analogous to the system in region IIIa. The types of solutions possible in this region are A, C, F and B. A systematic analysis in presence of multiplicative noise in region IIIb leads to results that are summarised below.

For low values of Da, the system operating in this region exhibits A type of solutions. As is known, the system should then evolve to a single node. The stochastic profiles for different intensities and time-correlation effects of the noise are indicated in Fig. IIIb-1. Like in the previous cases, the effects of the noise parameter is to bring about deviations from the macroscopic solutions. The deviations increase with larger intensity of the noise.
Figure III.5: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise (B = 15, \( \beta = 0.4, D_g = 0.00063 \), initial condition \( x_{25} = 9.25 \))
Figure III-6: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise

(B = 15, β = 0.4, D = 0.00066, initial condition x₂₀ = 9.51)
For higher values of Da (= 0.05), the macroscopic model shows C type of behaviour with one stable and two unstable solutions \((x_{28} = 0.37, 3.91, 4.38)\). The corresponding stochastic profiles beginning with each one of these macroscopic solutions as the initial condition are depicted in Figs. IIIb-2, IIIb-3 and IIIb-4. In each one of these cases it becomes evident from the figures that the only permissible stochastic solution is the macroscopically stable solution. The effects of the noise parameters eventually die down and the system approximates the macroscopic solution. The results also indicate that unlike in the case of region IIIa, where for the C type of solutions the presence of multiplicative noise generates an additional noise induced solution, in the present case no such additional solution is created.

For systems operating with somewhat higher values of Da (= 0.084) the macroscopic system possesses F type of solutions. As is known, for this situation, the system has one stable node, one stable limit cycle and two unstable solutions that can be identified as unstable focus and a saddle point. Beginning with the macroscopic solution corresponding to the stable node, Fig. IIIb-5 sketches the stochastic profiles in the presence of noise of different intensities and time correlation effects. For lower values of the noise intensity \((D = 0.01)\) and lower values of time correlation effects \((\tau = 0.01)\) the macroscopic stable solution under the perturbation of noise is seen to break into an oscillatory pattern. For constant values of time correlation effects, the effect of increasing the noise intensity is to bring about a faster transition. Thus, for instance, for \(D = 0.015\) the oscillations appear around the value of time \(t = 5.3\), while for \(D = 0.02\), they now appear at a value of time \(t = 3.5\). The effect of time correlation is also analogous.
FIGURE III b-1: DIMENSIONLESS TEMPERATURE - TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

\( B = 22, \beta = 3, D_0 = 0.049, \) initial condition \( x_{25} = 0.36 \)
FIGURE IIIb-2: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

(B = 22, \( \beta = 3 \), \( D_q = 0.05 \), initial condition \( x_{25} = 0.37 \))
FIGURE III.b-3: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
(B = 22, \( \beta = 3 \), \( D_0 = 0.05 \), initial condition \( x_{25} = 3.91 \))
FIGURE III b-4: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

(B = 22, β = 3, Da = 0.05, initial condition x = 4.38)
FIGURE IIIb-5: DIMENSIONLESS TEMPERATURE TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
(B = 22, β = 3, Da = 0.084, initial condition \( x_{25} = 1.17 \))
For a given value of $D$ its effect is to bring about an early transition to an oscillatory pattern.

Beginning with the unstable limit cycle and the saddle point solutions, the stochastic profiles ultimately end up in an oscillating solution which is the same as the one found when beginning with the stable node. In fact in the entire region of parameter space with $F$ type solutions it appears that the only feasible solution in presence of multiplicative noise is the stable limit cycle. All other solutions are destroyed or transformed to this limit cycle behaviour. Figures IIIb-6 and IIIb-7 sketch some of the stochastic solutions beginning with different initial conditions in this region.

Further increase in the value of parameter $D_a$ pushes the system into region $B$. The system here has either a stable limit cycle or an unstable solution as the possible stationary states. The numerical computations beginning with these solutions as the initial conditions suggest that even here the macroscopic solutions are transformed into a stable limit cycle. This typical feature is sketched in Fig. IIIb-8, where the effects of noise parameters are also clearly seen.

Region IVa

Similar to regions IIIa and IIIb, this region also shows a variety of possibilities of the system behaviour. Depending on the value of the parameter $D_a$, one could realize either $A$, $C$, $J$, $G$ or $B$ type of solutions. The stochastic solutions in each one of these cases have been generated for varying intensities and time correlation effects of the noise and the results are presented below.
Figure III.6: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise.

\( B = 22, \beta = 3, D_0 = 0.084 \), initial condition \( x_{25} = 1.47 \).
FIGURE IIIb-8: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

\( B = 22, \beta = 3, D_a = 0.085, \) initial condition \( x_{25} = 5.13 \)
Figure IVa.1 shows the behaviour of the system with A type of solution. The macroscopic solution indicates that the system is possessing a stable solution while the induction of noise of varying strength and correlation effects makes the system behave in a somewhat haphazard way and eventually breaks into a smooth oscillating profile. Thus, for instance, for values of $D = 0.05$ and $\tau = 0.03$ or 0.05 the system shows a smooth oscillatory pattern. The effect of time correlation is to bring about a slight shift on the time scale where the first peak appears. Increase in the value of the parameter $D_a$ pushes the macroscopic system operating in this region to exhibit C type of solutions. As is known, for this case the macroscopic behaviour of the system indicates the existence of one stable and two unstable solutions. Beginning with each one of these solutions as the initial condition, Figs. IVa.2-4 show the corresponding stochastic behaviour in presence of varying strengths of noise and correlation effects. As is evident from Fig IVa-2 for lower values of intensity of noise and lower correlation effects, the deterministic and stochastic solutions approach each other. An increase in the correlation effect of the noise parameter brings about a transition to oscillatory behaviour. Further increase in the correlation effect of the noise results in bringing about an early transition to the oscillatory mode. Increase in the intensity of noise also has a similar effect. Figures IVa-3 and IVa-4 respectively indicate the transient behaviour with initial conditions corresponding to the two unstable solutions of the macroscopic model. As is evident from these figures the system eventually returns to an oscillating pattern. It thus appears that the only permissible solution for the system operating in this region is the oscillatory one.
FIGURE IV-a-1: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

(B = 19, β = 3, D = 0.084, initial condition x_{25} = 0.66)
Figure IV.a-2: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise (β=1.0, α=3.0, D_0 = 0.10, initial condition x_{0,0} = 0).

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FIGURE IVa-3: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
\( B=19, \beta=3, D_0=0.101, \) initial condition \( x_{2s}=1.81 \)
FIGURE IV.4: DIMENSIONLESS TEMPERATURE TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

(B = 95, $\beta = 13$, $D_a = 0.101$, initial condition $x_0 = 4.02$)
Progressive increase in the value of the parameter $Da$ leads to $J$, $G$ or $B$ type of solutions. With $J$ type of solutions, the macroscopic system has one stable node and one stable limit cycle that are practically realizable. The other three stationary solutions that are permissible for this type of behaviour are unstable and cannot be realized practically. For $G$ type of solutions, the only permissible solution from practical viewpoint is again a stable limit cycle and the remaining three solutions are unstable. Similarly, in region $B$ the only permissible solution is stable limit cycle. Beginning with each one of these macroscopic stationary solutions the corresponding stochastic solutions have been generated. In the entire parameter range studied (where $J$, $G$ or $B$ type solutions are possible) the numerical results indicate the existence of only the stable limit cycle solution. One can therefore conclude that in the entire region $IVa$ the presence of noise destroys all other solutions and leaves the stable limit cycle as the only possible solution.

Region $IVb$

The system operating in this region is known to be capable of showing $A$, $B$, $D$, $C$, $B$ and $A$ types of solutions, on progressive increase in the value of parameter $Da$. In this region the corresponding stationary solutions permissible from the stochastic viewpoint for progressive increase in the value of parameter $Da$ have been generated for each one of these types of solutions. The results are described below.

For lower values of parameter $Da$, the system possesses $A$ type of solution which like in the previous case is transformed into a oscillatory solution. For higher values of $Da$ the system starts exhibiting $D$ type of
**FIGURE IVa-5: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE**

\[ B = 19, \beta = 3, D_n = 0.1028, \text{initial condition } x_0 = 4.02 \]
solution with one stable node, one unstable and one stable limit cycle. The stable node solution under the influence of noise in general transforms into oscillations. The details of the stochastic profiles are sketched in Figs. IVb-1,2. It may be noticed from these figures that the original macroscopic stable solution is realizable from the stochastic viewpoint only for infinitesimally small extents of noise. For higher values of noise parameter the system invariably shows oscillatory pattern. The effects of the noise intensity and time correlation are also clearly brought out in this figure. As seen from the figure, the effects of time correlation for a fixed value of the noise intensity is to shift the profiles to lower values of time. Increasing the value of the noise intensity has a similar effect of shifting the profiles on time axis. The intensity and time correlation of noise while does not seem to affect the amplitude of oscillations their time period seems to get affected considerably. For systems with higher noise intensities or with higher correlation time the number of cycles on the time axis therefore increases.

Figure IVb-2 shows the situation when one begins with the unstable limit cycle solution of the macroscopic model as the initial condition. The presence of noise transforms this solution into a stable limit cycle. This corresponds to the permissible macroscopic limit cycle that exists in this region. It appears from Fig. IVb-1 and IVb-2 that for D type of solutions, the stochastic analysis indicates the existence of only one solution viz. the stable limit cycle solution. Incidentally, this solution corresponds to the stable limit cycle of the macroscopic model.
Figure IVb-1: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise
(B = 16.2, β = 3, Dq = 0.12823, initial condition x_{25} = 1.28)
FIGURE IVb-2: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
(\(B=16\cdot2, \beta=3, D_0=0.12823\), initial condition \(x_{25}=2.025\))
A further increase in the value of Da transforms the system behaviour from D type of solution to B type of solution. As is known for B type of solutions the system can possess either the stable or unstable limit cycle. In presence of noise both these solutions are transformed into a stable limit cycle solution. The effects of the time correlation for a fixed value of noise intensity is to shift the profiles to lower values on time axis. Similar effect of noise intensity, for a constant value of its time correlation, is also evident in these figures (Figs. IVb-3 and IVb-4).

A progressive increase in the parameter Da transforms system with B type of solutions to one showing G type of solutions. Here the system possesses two unstable focii (a saddle point and a stable limit cycle) as the possible stationary solutions. Beginning with each one of these solutions as the initial condition Figs. IVb-5 to IVb-8 show the corresponding stochastic profiles with different extents of noise. In each one of these cases, as is evident from these figures, the only permissible solution from the stochastic viewpoint is a stable limit cycle. Like in the previous case, the extent of noise and its time correlation does not affect the amplitude of oscillations but the time period of oscillation is altered and the system, in general, exhibits larger number of cycles on the time scale.

For higher values of parameter Da, the system again exhibits type B behaviour. This behaviour also subsequently vanishes in favour of the A type of solution for further increase in the parameter value Da. Corresponding stochastic solutions in these regions are also indicated in Fig. IVb-9. As clearly evident from the figure, the system has an oscillatory solution as the only possible solution. In the entire IVb region it thus appears that the macroscopic solution comprising of types A, D, C, G and B and A types
FIGURE IV 4: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

\( B = 15.2, \beta = 3, D_0 = 0.131, \) initial condition \( x_{2S} = 2.78 \)
FIGURE IVb-5: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
\( B = 16.2, \beta = 3, D_0 = 0.132, \) initial condition \( x_{25} = 1.66 \)
FIGURE IVb-6: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

(β = 16.2, β = 3, D₂ = 0.132, initial condition x₀ = 1.78)
Figure IVb-7: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise

(B = 16.2, β = 3, D_q = 0.132, initial condition x_{2s} = 2.43)
FIGURE IV.b-8: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
(B=16.2, β=3, D_0 = 0.132, initial condition x_{2s} = 2.91)
Figure IXb-9: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise ($\beta = 16.2, \gamma = 3, D_0 = 0.2$, initial condition $x^* = 3.46$).
are radically transformed and the only permissible solution under the circumstances, seems to be a stable limit cycle.

Region Va

Systems operating in this region can possess either A, D, B or A type of solutions that are successively realized on increasing the parameter value Da. Figure Va-1 shows the system behaviour for A type of solutions where it is seen that the macroscopic stable solution is transformed into a limit cycle behaviour. Figures Va-2 to 4 show the situation for D type of solutions. The results clearly indicate that the entire D region of macroscopic model is transformed into a region containing only one type of solution that is a stable limit cycle. In fact even the B region of the macroscopic model is transformed into a region containing only a stable limit cycle as the possible type of solution (Figs. Va-5 and Va-6).

It thus appears from these results that like in the case of systems operating in region IVb even here the entire parameter space is transformed to possess only a stable limit cycle as the possible type of solution.

Region Vb

For systems operating in this region with progressive increase in the parameter value Da one realizes, from the macroscopic viewpoint, either A, or B type of solutions. Figures Vb-1 to 4 sketch the stochastic profiles for each one of these types of solutions in this region. As is evident from Fig. Vb.1 the stationary macroscopic solution is now totally distorted and the system behaves in a haphazard manner with no inclination to converge to any stationary state. The same feature seems to continue even for B
FIGURE Vα-1: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

(β = 14, β = 3, D₀ = 0.161, initial condition x₂₅ = 1.29)
FIGURE V.a-2: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

($B = 14, \beta = 3, D_0 = 0.162$, initial condition $x_{29} = 1.4$)
FIGURE Vα-3: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
(B=14, β=3, D_q = 0.162, initial condition x_{25} = 2.1)
Figure 10.4: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise.

(B = 14, \( \beta = 3 \), \( D_y = 0.162 \), initial condition \( x_{20} = 2.56 \))
FIGURE 7α-5: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

\( B=14, \beta=3, D_0=0.166, \text{initial condition } x_{20}=1.50 \)
FIGURE V a-6: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
(B = 14, β = 3, D₀ = 0.166, initial condition x₂₅ = 2.68)
Figure 5b-1: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise
($B = 11, \beta = 3, D_0 = 0.37$, initial condition $x_{2S} = 1.51$)
Figure 1b-2: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise (B = 1, $\beta = 3$, $D_{o} = 0.32$, initial condition $x_{20} = 1.82$).
FIGURE Vb-3: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

\[ B = 11, \beta = 3, D_0 = 0.32, \text{ initial condition } x_{28} = 2.14 \]
Figure Vb-4: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise
\( (B = 11, \beta = 3, D_0 = 0.44, \text{initial condition } x_{29} = 2.25) \)
Figure VI - 1: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise

(B = 0.06, β = 0.74, D = 0.1318, initial condition x₂ = 1.6)
FIGURE VI - 2: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE
(B = 0.06, β = 0.74, D_a = 0.1319, initial condition x_s = 1.62)
Figure 6.3: Dimensionless temperature-time curves for different values of the intensity and correlation time of noise (B = 0.06, β = 0.04, Da = 0.1319, initial condition $x_{25} = 2.15$).

- Deterministic
- $D = 0.01, \tau = 0.01$
- $D = 0.01, \tau = 0.03$
- $D = 0.01, \tau = 0.05$
- $D = 0.03, \tau = 0.01$
- $D = 0.03, \tau = 0.03$
- $D = 0.03, \tau = 0.05$
- $D = 0.05, \tau = 0.01$
- $D = 0.05, \tau = 0.03$
- $D = 0.05, \tau = 0.05$
FIGURE VI-4: DIMENSIONLESS TEMPERATURE-TIME CURVES FOR DIFFERENT VALUES OF THE INTENSITY AND CORRELATION TIME OF NOISE

\( B = 7.06, \beta = 0.74, D_q = 0.1319 \), initial condition \( x_{28} = 2.29 \)
type of solution in this region and a state of chaotic behaviour persists all through the region totally.

It thus appears that the system operating in this region from a stochastic viewpoint exhibits totally distorted and chaotic behaviour.

Region VI

For systems operating in this region, the progressive increase in the value of parameter Da leads to A, E, F, B or A type of solutions. Representative figures for some of these type of solutions are presented in Fig. VI.1 to VI.4. As in the case of the region Va, one can notice that even in this region the system does not seem to follow any order. It therefore appears that the entire region behaves chaotically and the system may not possess any stationary state from a stochastic viewpoint.

3.3 CONSTRUCTION OF STOCHASTIC BIFURCATION DIAGRAM

The large number of computations presented in the last section can be brought together in a single diagram. This generalised diagram in presence of multiplicative noise is indicated in Fig. 3.1. Much similar to the effects of additive noise (see Fig. 2.2) even here, we notice that the number of permissible solutions are few.

A close comparison of the Figs. 2.3 and 3.1 reveal that for lower values of Da, in presence of noise, the system reaches the lower stable solution permissible in the macroscopic model. Thus for instance, in each of the regions I, II, IIIa, IIIb and IVa, for smaller values of Da, the only solution even from stochastic viewpoint is the same as that given by the macroscopic
FIGURE 3.1—PERMISSIBLE STATIONARY STATES USING A STOCHASTIC MODEL (multiplicative noise)
The stochastic solution of course does not entirely coincide with the macroscopic one but keeps oscillating around it depending on the extent of noise. For even slightly higher values of Da (such as for C type of solutions in region IIIa) the behaviour changes drastically. The upper stable solution, wherever it exists, is seen to be approached more smoothly using the stochastic model. Very drastic variations are realised for higher values of Da, and especially in regions IVa onwards the noise entirely transforms the stationary states of the macroscopic model into an oscillatory mode. In the entire region IVb, Va and for most parts of region IVa, the only type of solution feasible is the stationary limit cycle. In regions Vb, the lower macroscopic stable solution is additionally possible in the stochastic model, but even here the solution is approached only in the mean. In region VI, it is difficult to ascertain whether a stationary state has been reached. The deviations from the macroscopic solutions are always found to be too large.

The comparison of the effects of an additive and multiplicative noise also reveals certain features. Thus we note that while the presence of noise, in general, destroys a large number of the macroscopically possible solutions their intrinsic role can be quite different. For instance, the presence of additive noise does not alter the stability of the macroscopic solutions, but on the other hand, multiplicative noise can change the stability. In fact new solutions can be generated and these originate because of the noise. Such noise induced solutions are the main finding of this work. The qualitative and quantitative deviations of the results from a stochastic analysis, when compared to those from the macroscopic model, suggest the important role played by the random noise in deciding the evolution of the system.
The above analysis has shown the important role of noise in reacting systems and its effects on system behaviour. In view of the existence of disturbances in all practical systems it is appropriate that the dynamics of these systems be determined in the way indicated here.