RELIABILITY MODELING AND COST BENEFIT ANALYSIS OF TWO NON IDENTICAL COLD STAND BY UNIT WITH TWO STAGES OF REPAIR

5.1 INTRODUCTION

Two-unit redundant systems have been extensively studied by several Authors including Goel. et. al. [25-30, 32-33, 35, 38, 40], Singh and Singh [121, 122], Gupta [46-47], Chaudhary et. al. [16] and Gupta et. al.[48, 50-52, 56-57] in the field of reliability theory. They have analyzed the two-unit cold standby system models by considering the various concepts like different operating modes of units, two types of repair, patience time of repairman, imperfect and slow switching devices, allowed down time and the priority unit etc.

The present paper is devoted to discuss the concept of two-stage repair process for a two-unit cold standby system in which operative and standby units are interchanged with constant rates after a random amount of time, this process of interchanging is automatic for which no switching device is required. The repair process is divided into two stages i.e. the repairing process of failed unit is started in the first stage but it doesn’t gets completed instead it is completed in the second stage of the repairing process. All the failure time distributions and rates of interchanging from operative to standby mode and standby to operative mode are taken to be negative exponential and all the repair time distributions are assumed to be general. Various reliability characteristics of interest have been studied along with graphical behaviour.

5.2 ASSUMPTIONS

The system model is analyzed under following practical assumptions:

1. The units are similar and functions independently.
2. Failures and repairs are stochastically independent.
3. The repair process is divided into two stages
4. In the first stage the repair doesn’t completed and the repairing process is completed in the second stage.

5. A repaired unit is as good as new and is immediately reconnected to the system.

6. All the failure time distributions are taken to be negative exponential.

7. All the repair time distributions are taken as arbitrary.

5.3 NOTATIONS AND STATES OF THE SYSTEM

\( \alpha_1 \): Failure rate of the first unit i.e. A.

\( \alpha_2 \): Failure rate of the second unit i.e. B.

\( \alpha_3 \): Rate of transition of first unit from operative to standby.

\( \alpha_4 \): Rate of transition of second unit from operative to standby.

\( F_1(.) \): First phase repair rate of first unit i.e. A.

\( F_2(.) \): Second phase repair rate of first unit i.e. A.

\( G_1(.) \): First phase repair rate of second unit i.e. B.

\( G_2(.) \): Second phase repair rate of second unit i.e. B.

Symbols for the states of the system

\( A_0 \): First unit is in operative mode.

\( A_s \): First unit is in standby mode.

\( B_0 \): Second unit is in operative mode.

\( B_s \): Second unit is in standby mode.

\( A_{r1} \): First unit is under first phase of repair.
\[ A_{r2} : \text{First unit is under second phase of repair.} \]
\[ B_{r1} : \text{Second unit is under first phase of repair.} \]
\[ B_{r2} : \text{Second unit is under second phase of repair.} \]
\[ A_{wr1} : \text{First unit is waiting for repair.} \]
\[ B_{wr1} : \text{Second unit is waiting for repair.} \]

With the help of the above symbols the possible states of the system are:

\[ S_0 = [A_o, B_o] \]
\[ S_1 = [A_s, B_o] \]
\[ S_2 = [A_{r1}, B_o] \]
\[ S_3 = [A_{r2}, B_o] \]
\[ S_4 = [A_{r1}, B_{wr1}] \]
\[ S_5 = [A_{r2}, B_{wr1}] \]
\[ S_6 = [A_o, B_{r1}] \]
\[ S_7 = [A_o, B_{r2}] \]
\[ S_8 = [A_{wr1}, B_{r1}] \]
\[ S_9 = [A_{wr1}, B_{r2}] \]

The transition diagram along with all the transitions is shown in fig 5.1
Fig. 5.1
5.4 TRANSITION PROBABILITIES AND SOJOURN TIMES

Let \( T_0 (\equiv 0), T_1, T_2, \ldots \) denotes the regenerative epochs and \( X_n \) denotes the state visited at epoch \( T_n^+ \), i.e. just after the transition at \( T_n \) then \( \{ X_n, T_n \} \) constitute a Markov-Renewal process with state space \( E \), set of regenerative states and

\[
Q_{ij}(t) = \mathbb{P}[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i].
\]

is the semi Markov kernel over \( E \).

Then the transition probability matrix of the embedded Markov chain is

\[
p = p_{ij} = [Q_{ij}(\infty)] = [Q(\infty)]
\]

The various transitions probabilities may be obtained as follows:

\[
Q_{01}(t) = \alpha_3 \int_0^t \exp[-(\alpha_1 + \alpha_3)u] \, du
\]

\[
Q_{02}(t) = \alpha_1 \int_0^t \exp[-(\alpha_1 + \alpha_3)u] \, du
\]

\[
Q_{10}(t) = \alpha_4 \int_0^t \exp[-(\alpha_2 + \alpha_4)u] \, du
\]

\[
Q_{16}(t) = \alpha_2 \int_0^t \exp[-(\alpha_2 + \alpha_4)u] \, du
\]

\[
Q_{23}(t) = \int_0^t \exp[-(\alpha_2)u] \, dG_1(u)
\]

\[
Q_{25}^{(4)}(t) = \alpha_2 \int_0^t \exp[-(\alpha_2)u] \, du \int_u^t dG_1(v)
\]

\[
Q_{30}(t) = \int_0^t \exp[-(\alpha_2)u] \, dG_2(u)
\]

\[
Q_{36}^{(5)}(t) = \alpha_2 \int_0^t \exp[-(\alpha_2)u] \, du \int_u^t dG_2(v)
\]
Steady state transition probabilities

By taking the limit as $t$ tends to $\infty$ in equation (1), we obtain the following steady state transition probabilities:

$$P_{01} = \lim_{t \to \infty} Q_{01}(t) = \alpha_1 \int_0^\infty \exp[-(\alpha_1 + \beta_1)t] \, dt = \frac{\alpha_3}{\alpha_1 + \alpha_3}$$

$$P_{02} = \frac{\alpha_1}{\alpha_1 + \alpha_3}$$

$$P_{16} = \frac{\alpha_2}{\alpha_2 + \alpha_4}$$

$$P_{25}^{(4)} = 1 - g_1^*(\alpha_2)$$

$$P_{36}^{(5)} = 1 - g_2^*(\alpha_2)$$

$$P_{45} = 1$$
From the obtained steady state probabilities, it can be easily seen that the following results hold good:

\[ p_{56} = 1 \quad p_{67} = f_1^*(\alpha_1) \]
\[ p_{69}^{(9)} = 1 - f_1^*(\alpha_1) \quad p_{71} = f_2^*(\alpha_1) \]
\[ p_{72}^{(9)} = 1 - f_2^*(\alpha_1) \quad p_{89} = 1 \]
\[ p_{92} = 1 \quad (2) \]

Mean sojourn time

The mean sojourn time in state \( S_i \) denoted by \( \mu_i \) is defined as the expected time taken by the system in state \( S_i \) before transiting to any other state. To obtain mean sojourn time \( \mu_{i'} \) in state \( S_{i'} \), we observe that as long as the system is in state \( S_i \), there is no transition from \( S_i \) to any other state. If \( T_i \) denotes the sojourn time in state \( S_i \) then mean sojourn time in state \( S_i \) is

\[
\mu_i = E[T_i] = \int_0^\infty P[T_i > t] dt
\]

Thus
\[
\mu_0 = \int_0^\infty \exp[-(\alpha_1 + \alpha_3)t] \, dt = \frac{1}{\alpha_1 + \alpha_3}
\]
\[
\mu_1 = \int_0^\infty \exp[-(\alpha_2 + \alpha_4)t] \, dt = \frac{1}{\alpha_2 + \alpha_4}
\]
\[
\mu_2 = \int_0^\infty \exp[-(\alpha_2)t] \, \bar{g}_1(t) \, dt = \frac{1}{\alpha_2} [1 - g_1^*(\alpha_2)]
\]
\[
\mu_3 = \int_0^\infty \exp[-(\alpha_1)t] \, \bar{g}_2(t) \, dt = \frac{1}{\alpha_2} [1 - g_2^*(\alpha_2)]
\]
\[
\mu_5 = \int_0^\infty \bar{g}_2(t) \, dt
\]
\[
\mu_6 = \int_0^\infty \exp[-(\alpha_1)t] \, \bar{F}_1(t) \, dt = \frac{1}{\alpha_1} [1 - f^*_1(\alpha_1)]
\]
\[
\mu_7 = \int_0^\infty \exp[-(\alpha_1)t] \, \bar{F}_2(t) \, dt = \frac{1}{\alpha_1} [1 - f^*_2(\alpha_1)]
\]
\[
\mu_9 = \int_0^\infty \bar{F}_2(t) \, dt
\]

5.5 MEAN TIME TO SYSTEM FAILURE

Let the random variable \( T_i \) denotes the time to system failure when \( E_0 = E_1 \in E \) and \( \phi_1(t) \) is the c.d.f. of the time to system failure for the first time when the system starts operation from state \( S_i \). On the basis of arguments used for regenerative processes we obtain following relations for \( \phi_1(t) \).

\[
\phi_0(t) = Q_{01}(t) \phi_1(t) + Q_{02}(t) \phi_2(t)
\]
\[
\phi_1(t) = Q_{10}(t) \phi_0(t) + Q_{16}(t) \phi_6(t)
\]
\[
\phi_2(t) = Q_{23}(t) \phi_3(t) + Q_{24}(t)
\]
\[
\phi_3(t) = Q_{30}(t) \phi_0(t) + Q_{35}(t)
\]
\[
\phi_6(t) = Q_{67}(t) \phi_7(t) + Q_{68}(t)
\]
\[
\phi_7(t) = Q_{71}(t) \phi_1(t) + Q_{79}(t)
\]

Taking the Laplace Transform of above equations we get:

\[
\bar{\phi}_0(s) = \bar{Q}_{01}(s) \, \bar{\phi}_1(s) + \bar{Q}_{02}(s) \, \bar{\phi}_2(s)
\]
\[\tilde{\phi}_1(s) = \tilde{Q}_{10}(s)\tilde{\phi}_0(s) + \tilde{Q}_{16}(s)\tilde{\phi}_6(s)\]
\[\tilde{\phi}_2(s) = \tilde{Q}_{23}(s)\tilde{\phi}_3(s) + \tilde{Q}_{24}(s)\]
\[\tilde{\phi}_3(s) = \tilde{Q}_{30}(s)\tilde{\phi}_0(s) + \tilde{Q}_{35}(s)\]
\[\tilde{\phi}_6(s) = \tilde{Q}_{67}(s)\tilde{\phi}_7(s) + \tilde{Q}_{68}(s)\]
\[\tilde{\phi}_7(s) = \tilde{Q}_{71}(s)\tilde{\phi}_1(s) + \tilde{Q}_{79}(s)\]

on solving above equations for \(\tilde{\phi}_0(s)\), we have

\[\tilde{\phi}_0(s) = \frac{N_1(s)}{D_1(s)}\] (5)

where

\[N_1(s) = \tilde{Q}_{01}(s)\tilde{Q}_{16}(s)\tilde{Q}_{67}(s)\tilde{Q}_{79}(s) + \tilde{Q}_{01}(s)\tilde{Q}_{16}(s)\tilde{Q}_{68}(s) + \tilde{Q}_{02}(s)\tilde{Q}_{24}(s) + \tilde{Q}_{23}(s)\tilde{Q}_{35}(s)[1 - \tilde{Q}_{16}(s)\tilde{Q}_{67}(s)\tilde{Q}_{71}(s)]\] (6)

and

\[D_1(s) = [1 - \tilde{Q}_{16}(s)\tilde{Q}_{67}(s)\tilde{Q}_{71}(s)][1 - \tilde{Q}_{02}(s)\tilde{Q}_{23}(s)\tilde{Q}_{30}(s)] - \tilde{Q}_{01}(s)\tilde{Q}_{10}(s)\] (7)

on taking \(s \to 0\) in, (6) and (7) and using the relation \(\tilde{Q}_{ij}(s) \to p_{ij}\), we get

\[N_1(0) = p_{01}p_{16}[p_{67}p_{79} + p_{68}] + p_{02}[p_{24} + p_{23}p_{35}][1 - p_{16}p_{67}p_{71}]\]

\[= 1 - p_{01}p_{10} - p_{01}p_{76}p_{71}p_{16} - p_{02}p_{67}p_{71}p_{16} + p_{02}p_{23}p_{30}[1 - p_{16}p_{67}p_{71}]\]

\[= [1 - p_{16}p_{67}p_{71}][1 - p_{02}p_{23}p_{30}] - p_{01}p_{10}\]

and

\[D_1(0) = [1 - p_{16}p_{67}p_{71}][1 - p_{02}p_{23}p_{30}] - p_{01}p_{10}\] (8)

which is equal to \(N_1(0)\)
\[ \Phi_0(s) = \frac{N_i(s)}{D_i(s)} = 1 \]

This shows that \( \Phi_0(s) \) is a proper cdf. Therefore, mean time to system failure will be

\[ \text{MTSF} = E(T) = -\frac{d}{ds} \Phi_0(s) \bigg|_{s=0} = \frac{D_i'(0) - N_i'(0)}{D_i(0)} \quad (9) \]

To obtain numerator of (9), we collect the coefficients of the relevant \( m_{ij} \)'s, where \( m_{ij} \)'s is the mean elapsed time of the system in state \( S_i \) before transiting to state \( S_j \).

In notation

\[ m_{ij} = \tilde{Q}_{ij}(s) \bigg|_{s=0} = \frac{d}{ds} \int_0^\infty \exp[-(st)] \partial Q_{ij}(t) \bigg|_{s=0} \]

also we know that \( \sum_j m_{ij} = \mu_i \)

Thus the coefficient of various \( m_{ij} \)'s in \( D_i'(0) - N_i'(0) \)

The coefficient of \( m_{01} = p_{10} + p_{16}p_{67}p_{79} + p_{16}p_{68} = 1 - p_{16}p_{67}p_{71} \)

The coefficient of \( m_{02} = 1 - p_{16}p_{67}p_{71} \)

The coefficient of \( m_{10} = p_{01} \)

The coefficient of \( m_{16} = p_{01} \)

The coefficient of \( m_{23} = p_{02}[1 - p_{16}p_{67}p_{71}] \)

The coefficient of \( m_{30} = p_{02}p_{23}[1 - p_{16}p_{67}p_{71}] \)

The coefficient of \( m_{67} = p_{01}p_{16} \)

The coefficient of \( m_{71} = p_{01}p_{16}p_{67} \)

Therefore, on using the relations (3) and above coefficients, the numerator of (9) becomes

\[ D_i'(0) - N_i'(0) = [\mu_0 + \mu_2p_{02} + \mu_3p_{03}p_{23}][1 - p_{16}p_{67}p_{71}] + p_{01}[\mu_1 + \mu_6p_{16} + \mu_7p_{16}p_{67}] \]

\[ \text{MTSF} = \frac{D_i'(0) - N_i'(0)}{D_i(0)} \quad (10) \]
where $D_1(0)$ is given by (8)

Now by substituting the values of $\mu_i$'s and $p_{ij}$'s in (9), we get the expressions for numerator and denominator of MTSF as

$$D'_1(0) - N'_1(0) = \alpha_1 [\alpha_2 + \alpha_1 \{1 - g_1^*(\alpha_2)\}] + \alpha_2 \{1 - g_2^*(\alpha_2)\} g_1^*(\alpha_2) \{[\alpha_2 + \alpha_4] - \alpha_2 f_1^*(\alpha_1) f_2^*(\alpha_1)\} + \alpha_2 \alpha_3 [\alpha_1 + \alpha_2 \{1 - f_1^*(\alpha_1)\}] + \{1 - f_2^*(\alpha_1)\}$$

and

$$D_1(0) = \alpha_1 \alpha_2 [(\alpha_1 + \alpha_3) - \alpha_1 g_1^*(\alpha_2) g_2^*(\alpha_2)] \{[\alpha_2 + \alpha_4] - \alpha_2 f_1^*(\alpha_1) f_2^*(\alpha_1)\} - \alpha_3 \alpha_4$$

### 5.6 Availability Analysis

Let $Z(t)$ be the probability that the system is up at epoch $t$ when it initially started operation from regenerative state $S_1$ and remains up continuously till time $t$ without passing through any other regenerative state. Therefore we have

$$Z_0(t) = \exp[-(\alpha_1 + \alpha_3)t] \quad Z_1(t) = \exp[-(\alpha_2 + \alpha_4)t]$$

$$Z_2(t) = \exp[-(\alpha_1 + \alpha_2 + \gamma)t] \quad Z_3(t) = \int_0^\infty \exp[-(\alpha_1)t] \bar{G}_2(t)$$

$$Z_5(t) = \bar{G}_2(t)\quad Z_6(t) = \exp[-(\alpha_1)t] \bar{F}_1(t)$$

$$Z_7(t) = \exp[-(\alpha_1)t] \bar{F}_2(t) \quad Z_0(t) = \bar{F}_2(t)$$

Also we define $A_1(t)$ as the probability that the system is in up-state at epoch ‘$t$’ when it initially started from regenerative state $S_1$. To obtain recurrence relations among different point wise availabilities we use the simple probabilistic arguments.

As an illustration $A_0(t)$ is the sum of the following probabilities:

1. The system remains up in state $S_0$ without making any transition to any other regenerative state up to time ‘$t$’, the probability of this event equals

$$Z_0(t) = \exp[-(\alpha_1 + \alpha_3)t]$$
2. The system transits from state $S_0$ to state $S_1$ during $(u, u + du)$, $u \leq t$ and then starting at epoch $u$ from $S_1$ it is available for remaining time $(t - u)$ the probability of this event is:

$$\int_0^t q_{01}(u) A_1(t - u) du = q_{01}(t) \odot A_1(t).$$

3. The system transits from state $S_0$ to state $S_2$ during $(u, u + du)$, $u \leq t$ and then starting at epoch $u$ from $S_3$ it is available for remaining time $(t - u)$ the probability of this event is:

$$\int_0^t q_{02}(u) A_2(t - u) du = q_{02}(t) \odot A_2(t).$$

Therefore $A_0(t)$ becomes

$$A_0(t) = Z_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t)$$

By similar arguments, we have

$$A_1(t) = Z_1(t) + q_{10}(t) \odot A_0(t) + q_{16}(t) \odot A_6(t)$$

$$A_2(t) = Z_2(t) + q_{23}(t) \odot A_3(t) + q_{25}^{(4)}(t) \odot A_5(t)$$

$$A_3(t) = Z_3(t) + q_{30}(t) \odot A_0(t) + q_{36}^{(5)}(t) \odot A_6(t)$$

$$A_5(t) = q_{56}(t) \odot A_6(t)$$

$$A_6(t) = Z_6(t) + q_{67}(t) \odot A_7(t) + q_{69}^{(8)}(t) \odot A_9(t)$$

$$A_7(t) = Z_7(t) + q_{71}(t) \odot A_1(t) + q_{72}^{(9)}(t) \odot A_2(t)$$

$$A_9(t) = q_{92}(t) \odot A_2(t)$$

Taking the Laplace transform of above equations, we get a set of linear equations in $A_i^*(s)$ as

$$A_0^*(s) = Z_0^*(s) + q_{01}(s) A_1^*(s) + q_{02}(s) A_2^*(s)$$

$$A_1^*(s) = Z_1^*(s) + q_{10}(s) A_0^*(s) + q_{16}(s) A_6^*(s)$$

$$A_2^*(s) = Z_2^*(s) + q_{23}(s) A_3^*(s) + q_{25}^{(4)}(s) A_5^*(s)$$
On solving above equations, the Laplace-transformation of the point wise availability is

\[ A_0^*(s) = \frac{N_2(s)}{D_2(s)} \]  \hspace{1cm} (11)

where

\[ N_2(s) = [q_{23}^* q_{36}^{* (5)} + q_{25}^* q_{56}^*] [Z_7^* q_{02}^* q_{67}^* - Z_6^* q_{02}^* - Z_1^* q_{01}^* q_{92} q_{69}^* - q_{02}^* q_{67} q_{71}^*] + Z_0^* (1 - q_{16}^* q_{67} q_{71}^*) + Z_1^* q_{01}^* + Z_2^* [q_{01}^* q_{16}^* q_{69}^* + q_{02}^* q_{67} q_{71}^*] + Z_2^* [q_{02}^* q_{23}^* + q_{16}^* q_{67} q_{72}^* - q_{01}^* q_{72}^* - q_{01}^* q_{16}^* q_{69}^* + q_{92}^* q_{23}^*] + Z_6^* q_{01}^* q_{16}^* + Z_7^* q_{01}^* q_{16}^* q_{67}^* \]  \hspace{1cm} (12)

and

\[ D_2(s) = 1 - (q_{23}^* q_{36}^{* (5)} + q_{25}^* q_{56}^*) [q_{92}^* q_{69}^* + q_{67}^* q_{72}^* - q_{01}^* q_{10}^* q_{67} q_{72}^* + q_{01}^* q_{02}^* q_{67} q_{71}^* - q_{01}^* q_{10}^* q_{92} q_{69}^* - q_{02}^* q_{23}^* q_{30}^* - q_{16}^* q_{67} q_{23}^* q_{30}^* (q_{01}^* q_{72}^*) - q_{16}^* q_{67} q_{71}^* q_{02}^*) - q_{01}^* q_{16}^* q_{69}^* q_{92} q_{23}^* q_{30}^* - q_{16}^* q_{67} q_{71}^* q_{01}^* q_{10}^*] \]  \hspace{1cm} (13)

The steady state availability of the system will be given by

\[ A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} s A_0^*(s) = N_2(0)/D_2(0) \]

As we know that, \( q_{ij}(t) \) is the pdf of the time of transition from state \( S_i \) to \( S_j \) and \( q_{ij}(t) \) is the probability of transition from state \( S_i \) to \( S_j \) during the interval \( (t, t + dt) \), thus

\[ q_{ij}(s)|_{s=0} = q_{ij}(0) = p_{ij} \]

and

\[ -q_{ij}(s)|_{s=0} = m_{ij} \]

Also we know that
\[ \lim_{s \to 0} Z_i^*(s) = \int_0^\infty Z_i(t) \, dt = \mu_i \]

Therefore, we have

\[
\begin{align*}
Z_0^*(s) &= \frac{1}{a_1 + a_3} = \mu_0 \\
Z_1^*(s) &= \frac{1}{a_2 + a_4} = \mu_1 \\
Z_2^*(s) &= \frac{1}{a_2} \left[ 1 - g_1^*(a_2) \right] = \mu_2 \\
Z_3^*(s) &= \frac{1}{a_4} \left[ 1 - g_2^*(a_4) \right] = \mu_3 \\
Z_4^*(s) &= \int_0^c \bar{G}_2(t) \, dt = \mu_5 \\
Z_5^*(s) &= \frac{1}{a_5} \left[ 1 - f_1^*(a_5) \right] = \mu_6 \\
Z_6^*(s) &= \frac{1}{a_6} \left[ 1 - f_2^*(a_6) \right] = \mu_7 \\
Z_7^*(s) &= \int_0^\infty \bar{F}_2(t) \, dt = \mu_9
\end{align*}
\]

Using relations in (3) and (14) in (11), the expressions of \( N_2(0) \) and \( D_2(0) \) becomes

\[
N_2(0) = (1 - p_{23} p_{30}) \left[ \mu_7 p_{02} p_{67} - \mu_6 p_{02} - \mu_1 (p_{01} - p_{67} p_{71}) \right] - \mu_0 (1 - p_{16} p_{67} p_{71}) + \mu_1 p_{01} + \mu_2 (p_{02} + p_{01} p_{16} - p_{16} p_{67} p_{71}) + \mu_3 [1 - p_{01} p_{10} - p_{16} p_{67} p_{71}] + \mu_6 p_{01} p_{16} + \mu_7 p_{01} p_{16} p_{67}
\]

and

\[
D_2(0) = 1 - (p_{23} p_{30}^{(5)} + p_{25}^{(4)}) [p_{69}^{(8)} + p_{72}^{(9)} p_{67} - p_{01} p_{10} p_{72}^{(9)} p_{67} + p_{10} p_{02} p_{67} p_{71} - p_{01} p_{10} p_{69}^{(8)} - p_{02} p_{23} p_{30} - p_{23} p_{30} p_{16} p_{67} (p_{01} p_{72}^{(9)} - p_{71} p_{02}) - p_{01} p_{23} p_{30} p_{69}^{(8)} - p_{16} p_{67} p_{71} - p_{01} p_{10} p_{23} p_{30} - p_{01} p_{67} p_{23} p_{30} - p_{23} p_{30} p_{01} p_{10} + p_{23} p_{30} p_{01} p_{10} p_{67} - p_{02} p_{23} p_{30}]
\]

\[
= 0, \text{ as it should be}
\]

Thus the steady state availability of the system starting from state \( S_0 \) is obtained as follows:

\[
A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} s A_0^*(s) = \frac{N_2(0)}{D_2(0)}
\]

Where \( N_2(0) \) is given by equation (15)

To obtain \( D_2(0) \), we first collect the coefficients of \( m_i \)'s as

The coefficient of \( m_{01} \) = \[ p_{23} p_{30} p_{16} p_{67} p_{72}^{(9)} + p_{10} + p_{23} p_{30} p_{16} p_{92} p_{69}^{(8)} - (p_{23} p_{30}^{(5)} + p_{25}^{(4)}) (p_{10} p_{67} p_{72}^{(9)} + p_{10} p_{92} p_{69}^{(8)}) \]
\[ = p_{10}p_{67}p_{71} + p_{23}p_{30} - p_{23}p_{30}p_{67}p_{71} \]

The coefficient of \( m_{02} \) = \( p_{10}p_{67}p_{71} + p_{23}p_{30} - p_{23}p_{30}p_{67}p_{71} \)

The coefficient of \( m_{10} \) = \( (p_{23}p_{36}^{(5)} + p_{25}^{(4)})(p_{02}p_{67}p_{71} - p_{01}p_{92}p_{69}^{(8)} - p_{01}p_{67}p_{72}^{(9)}) + p_{01} \)

\[ = (1 - p_{23}p_{30})p_{67}p_{71} + p_{23}p_{30}p_{01} \]

The coefficient of \( m_{16} \) = \( (1 - p_{23}p_{30})p_{67}p_{71} + p_{23}p_{30}p_{01} \)

The coefficient of \( m_{23} \) = \( 1 - p_{16}p_{67}p_{71} - p_{10}p_{01} \)

The coefficient of \( m_{25}^{(4)} \) = \( 1 - p_{16}p_{67}p_{71} - p_{10}p_{01} \)

The coefficient of \( m_{30} \) = \( p_{23}[1 - p_{16}p_{67}p_{71} - p_{10}p_{01}] \)

The coefficient of \( m_{36}^{(5)} \) = \( p_{23}[1 - p_{16}p_{67}p_{71} - p_{10}p_{01}] \)

The coefficient of \( m_{56}^{(4)} \) = \( p_{25}^{(4)}[1 - p_{16}p_{67}p_{71} - p_{10}p_{01}] \)

The coefficient of \( m_{67} \) = \( [1 - p_{10}p_{01}][1 - p_{23}p_{30}] + p_{23}p_{30}p_{16}p_{01} \)

The coefficient of \( m_{69}^{(8)} \) = \( [1 - p_{10}p_{01}][1 - p_{23}p_{30}] + p_{23}p_{30}p_{16}p_{01} \)

The coefficient of \( m_{71} \) = \( p_{16}p_{67}[1 - p_{02}p_{23}p_{30}] + [1 - p_{23}p_{30}]p_{10}p_{02}p_{67} \)

The coefficient of \( m_{72}^{(9)} \) = \( p_{16}p_{67}[1 - p_{02}p_{23}p_{30}] + [1 - p_{23}p_{30}]p_{10}p_{02}p_{67} \)

The coefficient of \( m_{92} \) = \( p_{16}^{(8)}[(1 - p_{10}p_{01})(1 - p_{23}p_{30}) + p_{01}p_{16}p_{23}p_{30}] \)

Therefore, on collecting the above coefficients and using the relations (3), denominator of (17) becomes

\[ \]

\[ \]

Therefore \( D_2'(0) = \mu_0[p_{23}p_{30}(1 - p_{16}p_{67}p_{71}) + p_{10}p_{67}p_{71}] + \mu_1[p_{67}p_{71}(1 - p_{23}p_{30}) + p_{01}p_{23}p_{30}] + \mu_2 + \mu_3 + \mu_5^{(4)}[1 - p_{16}p_{67}p_{71} - p_{10}p_{01}] + \mu_6 + \mu_9^{(8)}[(1 - p_{10}p_{01})(1 - p_{23}p_{30}) + p_{01}p_{16}p_{23}p_{30}] + \mu_7[p_{16}p_{67} \]

\[ (1 - p_{02}p_{23}p_{30}) + (1 - p_{23}p_{30})p_{10}p_{02}p_{67} \]

Now by substituting the values of \( \mu_i \)'s and \( p_{ij} \)'s in (17), we get the expressions for numerator and denominator of steady state availability as
BUSY PERIOD ANALYSIS

Let $B_i(t)$ be the probability that the system is under repair at epoch $t$ given that the system entered regenerative state $S_i$ at $t = 0$. Now we will determine these probabilities. To illustrate the calculations we consider $B_0(t)$ and similar arguments may be employed for other probabilities.

$B_0(t)$ consists of the sum of the following independent contingencies and their respective probabilities:

1. The system transits to state $S_1$ from $S_0$ during $(u, u + du)$, $u \leq t$ and then repairman may be found busy at epoch $(t - u)$, starting from $S_1$, the probability of this event is

$$\int_0^t q_{01}(t)B_1(t - u)du = q_{01}(t)\odot B_1(t)$$
2. The system transits to state $S_2$ from $S_0$ during $(u, u + du)$, $u \leq t$ and then repairman may be found busy at epoch $(t - u)$, starting from $S_2$, the probability of this event is

$$\int_0^t q_{02}(t)B_2(t - u)du = q_{02}(t)\circ B_2(t)$$

the expression for $B_0(t)$ becomes

$$B_0(t) = q_{01}(t)\circ B_1(t) + q_{02}(t)\circ B_2(t)$$

By similar arguments we have

$$B_1(t) = q_{10}(t)\circ B_0(t) + q_{16}(t)\circ B_6(t)$$

$$B_2(t) = Z_2(t) + q_{23}(t)\circ B_3(t) + q_{25}(t)\circ B_5(t)$$

$$B_3(t) = Z_3(t) + q_{30}(t)\circ B_0(t) + q_{36}(t)\circ B_6(t)$$

$$B_5(t) = Z_5(t) + q_{56}(t)\circ B_6(t)$$

$$B_6(t) = Z_6(t) + q_{67}(t)\circ B_7(t) + q_{69}(t)\circ B_9(t)$$

$$B_7(t) = Z_7(t) + q_{71}(t)\circ B_1(t) + q_{72}(t)\circ B_2(t)$$

$$B_9(t) = Z_9(t) + q_{92}(t)\circ B_2(t)$$

Taking Laplace transform of these relations, we get

$$B_0^*(s) = q_{01}^*(s)B_1^*(s) + q_{02}^*(s)B_2^*(s)$$

$$B_1^*(s) = q_{10}^*(s)B_0^*(s) + q_{16}^*(s)B_6^*(s)$$

$$B_2^*(s) = \frac{Z_2^*(s)}{s} + q_{23}^*(s)B_3^*(s) + q_{25}^*(s)B_5^*(s)$$

$$B_3^*(s) = \frac{Z_3^*(s)}{s} + q_{30}^*(s)B_0^*(s) + q_{36}^*(s)B_6^*(s)$$

$$B_5^*(s) = \frac{Z_5^*(s)}{s} + q_{56}^*(s)B_6^*(s)$$

$$B_6^*(s) = \frac{Z_6^*(s)}{s} + q_{67}^*(s)B_7^*(s) + q_{69}^*(s)B_9^*(s)$$

$$B_7^*(s) = \frac{Z_7^*(s)}{s} + q_{71}^*(s)B_1^*(s) + q_{72}^*(s)B_2^*(s)$$
\[ B'_0(s) = Z'_0(s) + q_{92}^* B'_2(s) \]

On solving above equations for \( B'_0(s) \) we have

\[ B'_0(s) = \frac{N_3(s)}{D_3(s)} \]  \hspace{1cm} (20)

where

\[ N_3(s) = \left[ q_{01}^* q_{16}^* (q_{92}^* q_{69}^{* (8)} + q_{67}^* q_{72}^{* (9)}) + q_{02}^* (1 - q_{16}^* q_{67}^* q_{71}^*) [Z_2^* + Z_5^* q_{25}^{* (4)}] + \right. \]

\[ \left. [q_{01}^* q_{16}^* - q_{02}^* (q_{23}^* q_{36}^{* (5)} + q_{25}^{* (4)} q_{56}^*)] [Z_6^* + Z_7^* q_{67}^* + Z_9^* q_{69}^{* (8)}] + Z_5^* [q_{02}^* \right. \]

\[ q_{23}^* + q_{16}^* q_{67}^* q_{23}^* (q_{01}^* q_{72}^{* (9)} - q_{71}^* q_{02}^*) - q_{01}^* q_{16}^* q_{69}^{* (8)} q_{92}^* q_{23}^* \]

and

\[ D_3(s) \] is same as \( D_2(s) \) as obtained in availability is given by (13)

Thus in the long run, the fraction of time for which system is under repair is given by

\[ B_0 = \lim_{t \to \infty} B_0(t) = \lim_{s \to 0} sB'_0(s) = \frac{N_2(0)}{D'_2(0)} \]  \hspace{1cm} (21)

where

\[ N_3(0) = [p_{01}^* (1 - p_{67} p_{71}) + p_{02}^* (1 - p_{16} p_{67} p_{71})] [\mu_2 + \mu_5 p_{25}^{(4)}] + [p_{01} p_{16} - \]

\[ p_{02}^* (1 - p_{23} p_{30}) [\mu_6 + \mu_7 p_{67} + \mu_9 p_{69}^{(8)}] + \mu_3 [p_{23} (1 - p_{01} p_{10} - p_{16} \]

\[ p_{67} p_{71})] \]  \hspace{1cm} (22)

and

\[ D'_2(0) = D'_2(0) \] is same as obtained in availability analysis which is given by (18)

Now by substituting the values of \( \mu_i \)'s and \( p_{ij} \)'s in (21), we get the expressions for numerator and denominator of \( B_0 \) as
N₃(0) = α₁[α₂α₃(1 − f₁*(α₁))f₂*(α₁)] + α₄((α₂ + α₄) − α₂f₁*(α₁)f₂*(α₁))[1 − g₁*(α₁)]

\[\text{and}\]

D₃'(0) = α₁α₂[g₁*(α₂)g₂*(α₂)((α₂ + α₄) − α₂f₁*(α₁)f₂*(α₁)] + α₁α₂[(α₁ + α₃)f₁*(α₁)]

\[\int \tilde{G}_2(t)dt\] + α₂[α₂α₃ − α₁(α₂ + α₄){1 − g₁*(α₂)]} + (1 − g₁*(α₂)]f₁*(α₁) + α₁ \int \tilde{F}_2(t)dt

\{1 − f₁*(α₁]) + α₄(1 − g₂*(α₂)]g₁*(α₂)((α₂ + α₄)(α₄ + α₃) − α₃α₄ −

(α₁ + α₃)α₂f₁*(α₁)f₂*(α₁)]

\[\text{and}\]

\[\text{5.8 EXPECTED NUMBER OF VISITS BY THE REPAIR FACILITIES}\]

Let us define \(V₁(t)\) as the expected no. of repairs of the unit during the time interval \((0, t]\) when the system initially starts from regenerative state \(S₁\). Using the definition of \(V₁(t)\), the recursive relation among \(V₁(t)\)'s can be easily developed

\[V₀(t) = Q_{01}(t)\otimes V₁(t) + Q_{02}(t)\otimes [1 + V₂(t)]\]

\[V₁(t) = Q_{10}(t)\otimes V₀(t) + Q_{16}(t)\otimes [1 + V₆(t)]\]

\[V₂(t) = Q_{23}(t)\otimes V₃(t) + Q_{25}^{(4)}(t)\otimes V₅(t)\]

\[V₃(t) = Q_{30}(t)\otimes V₀(t) + Q_{36}^{(5)}(t)\otimes V₆(t)\]

\[V₅(t) = Q_{56}(t)\otimes V₆(t)\]

\[V₆(t) = Q_{67}(t)\otimes V₇(t) + Q_{69}^{(8)}(t)\otimes V₉(t)\]

\[V₇(t) = Q_{71}(t)\otimes V₁(t) + Q_{72}^{(9)}(t)\otimes V₂(t)\]
\[ V_0(t) = Q_{92}(t) \circ V_2(t) \]  

(23)

Taking Laplace Stieltjes transformation of these relations, we get

\[
\begin{align*}
\bar{V}_0(t) &= \bar{Q}_{01}(t)(s)\bar{V}_1(t) + \bar{Q}_{02}(t)(s)[1 + \bar{V}_2(t)] \\
\bar{V}_1(t) &= \bar{Q}_{10}(t)(s)\bar{V}_0(t) + \bar{Q}_{16}(t)(s)[1 + \bar{V}_6(t)] \\
\bar{V}_2(t) &= \bar{Q}_{23}(t)(s)\bar{V}_3(t) + \bar{Q}_{25}^{(4)}(t)(s)\bar{V}_5(t) \\
\bar{V}_3(t) &= \bar{Q}_{30}(t)(s)\bar{V}_0(t) + \bar{Q}_{36}^{(5)}(t)(s)\bar{V}_6(t) \\
\bar{V}_5(t) &= \bar{Q}_{56}(t)(s)\bar{V}_6(t) \\
\bar{V}_6(t) &= \bar{Q}_{67}(t)(s)\bar{V}_7(t) + \bar{Q}_{69}^{(8)}(t)(s)\bar{V}_9(t) \\
\bar{V}_7(t) &= \bar{Q}_{71}(t)(s)\bar{V}_1(t) + \bar{Q}_{72}^{(9)}(t)(s)\bar{V}_2(t) \\
\bar{V}_9(t) &= \bar{Q}_{92}(t)(s)\bar{V}_2(t)
\end{align*}
\]

On solving the above equations \( \bar{V}_0(s) \) the Laplace Stieltjes transformation of the expected number of visits is given by we get

\[
\bar{V}_0(s) = \frac{N_4(s)}{D_4(s)}
\]

(24)

Where

\[
N_4(s) = \bar{Q}_{01}[1 - (\bar{Q}_{69}^{(8)}\bar{Q}_{92} + \bar{Q}_{72}^{(9)}\bar{Q}_{67})(\bar{Q}_{36}^{(5)}\bar{Q}_{23} + \bar{Q}_{25}^{(4)}\bar{Q}_{56}) - \bar{Q}_{16}\bar{Q}_{67}\bar{Q}_{71}] + \bar{Q}_{01}
\]

\[
[\bar{Q}_{01} - (\bar{Q}_{36}^{(5)}\bar{Q}_{23} + \bar{Q}_{25}^{(4)}\bar{Q}_{56})(\bar{Q}_{67}\bar{Q}_{72}\bar{Q}_{01} - \bar{Q}_{71}\bar{Q}_{202}\bar{Q}_{67}) + \bar{Q}_{01}\bar{Q}_{92}\bar{Q}_{69}]
\]

and \( D_4(s) \) can be obtained by replacing \( q_{ij}^{*} \)'s by \( \bar{Q}_{ij} \)'s in relation (13)

In steady state the number of visits per unit time is given by

\[ V_0 = \lim_{t \to \infty} V_0(t) = \lim_{s \to 0} s\bar{V}_0(s) = \frac{N_4(0)}{D_4'(0)} \]

(25)

where

\[
N_4(0) = p_{02}[1 - p_{16}p_{67}p_{71} - (1 - p_{23}p_{30})(1 - p_{67}p_{71})] + p_{16}[p_{01} - (1 - p_{23}p_{30})(p_{01} - p_{67}p_{71})]
\]
and $D'_4(0) = D'_2(0)$ is already specified in equation (18)

Now by substituting the values of $\mu_i$'s and $p_{ij}$'s in (25), we get the expressions for numerator and denominator of $V_0$ as

\[
N_4(0) = \alpha_1 \alpha_2 \{\alpha_2 + \alpha_4\} \alpha_1 \{1 - \alpha_2 f'_1(\alpha_1) f'_2(\alpha_1)\} - (\alpha_2 + \alpha_4) \alpha_1 \{1 - f'_1(\alpha_1) f'_2(\alpha_1)\} \]
\[
\{1 - g'_1(\alpha_2) g'_2(\alpha_2)\} + \alpha_1 \alpha_2 \alpha_3 - \alpha_2 \{1 - g'_1(\alpha_2) g'_2(\alpha_2)\} \alpha_3 - (\alpha_1 + \alpha_3) f'_1(\alpha_1) f'_2(\alpha_1)\}
\]

and

\[
D'_4(0) = \alpha_1 \alpha_2 \{g'_1(\alpha_2) g'_2(\alpha_2)\} \{\alpha_2 + \alpha_4\} - \alpha_2 f'_1(\alpha_1) f'_2(\alpha_1)\} + \alpha_1 \alpha_2 \{\alpha_1 + \alpha_3\} f'_1(\alpha_1) f'_2(\alpha_1)\}
\]
\[
f'_2(\alpha_1) \{1 - g'_1(\alpha_2) g'_2(\alpha_2)\} + g'_1(\alpha_2) g'_2(\alpha_2) \alpha_3 + \alpha_1 \{1 - g'_1(\alpha_2)\} + \{1 - g'_1(\alpha_2)\}^2 + \alpha_2 \int \bar{G}_2(t) dt \{\alpha_1 + \alpha_3\} \alpha_2 + \alpha_4 - \{\alpha_2 + \alpha_4\} - \alpha_2 f'_1(\alpha_1) \]
\[
f'_2(\alpha_1) \{\alpha_1 + \alpha_3\} - \alpha_3 \alpha_4 + \alpha_2 \{1 - f'_1(\alpha_1)\} + \alpha_1 \int \bar{F}_2(t) dt \{1 - f'_1(\alpha_1)\} \}
\]
\[
\{\alpha_1 + \alpha_3\} \alpha_2 + \alpha_4 - \alpha_3 \alpha_4 \{1 - g'_1(\alpha_2) g'_2(\alpha_2)\} + \alpha_2 \alpha_3 g'_1(\alpha_2) g'_2(\alpha_2) + \alpha_2 \{1 - f'_2(\alpha_1)\} \alpha_2 f'_1(\alpha_1) \{1 - g'_1(\alpha_2) g'_2(\alpha_2)\} + \alpha_1 \alpha_4 f'_1(\alpha_1) \{1 - g'_1(\alpha_2) g'_2(\alpha_2)\} + \alpha_1 \alpha_4 f'_1(\alpha_1) \{1 - g'_1(\alpha_2) g'_2(\alpha_2)\}
\]

\section{5.9 Cost Benefit Analysis}

The expected uptime and down time of the system and busy period of the repair man in $(0, t]$ are

\[
\mu_{up}(t) = \int_0^t A_0(u) du
\]
\[
\mu_{dn}(t) = t - \mu_{up}(t)
\]

and

\[
\mu_b = \int_0^t B_0(u) du
\]

So that

\[
\mu^*_{up}(s) = A'_0(s)/s
\]
\[
\mu^*_{dn}(s) = 1/s^2 - \mu^*_{up}(s)
\]
and

\[ \mu_b^*(s) = B_0^*(s)/s \]

The expected profits incurred in \((0, t] = \) expected total revenue in \((0, t] - \) expected total repair in \((0, t] - \) expected cost of visits by repairman in \((0, t].\)

\[ P = K_1 \mu_{up}(t) - K_2 \mu_b(t) - K_3 V_0(t) \quad (26) \]

where

\( K_1 = \text{Revenue per unit up time of the system.} \)

\( K_2 = \text{Cost per unit time for which the repair is busy.} \)

\( K_3 = \text{Cost per unit visits by the repairman.} \)
5.10 GRAPHICAL STUDY OF SYSTEM BEHAVIOR

The behavior of MTSF and availability of the system is studied graphically in this section and to plot their graphs, the replacement and repair time distributions are also assumed to be distributed exponentially. The graphs of MTSF and that of availability are depicted with respect to the different parameters. It is observed that the MTSF decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF increases with increasing repair rates. Thus, we can conclude that the expected life of the system can be increased by increasing repair rate of the unit. Further, it is observed that the availability of the system gradually decreases with increasing failure rates irrespective of type of failure and increases with increasing repair rate of the unit.

For fixed values of the parameters $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4$ and changing $\alpha_2$, TABLE-1 is obtained.

**TABLE-1: Effect of $\alpha_2$ and fixed parameters $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$ and $\beta_4$ on MTSF for three different values of $\beta_2$.**

<table>
<thead>
<tr>
<th>$\alpha_2$</th>
<th>MTSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 0.053, \quad \alpha_3 = 0.043,\quad \alpha_4 = 0.062, \quad \beta_1 = 0.10, \quad \beta_2 = 0.90, \quad \beta_3 = 0.70, \quad \beta_4 = 0.20$</td>
<td>$\alpha_1 = 0.053, \quad \alpha_3 = 0.043, \quad \alpha_4 = 0.062, \quad \beta_1 = 0.10, \quad \beta_2 = 0.50, \quad \beta_3 = 0.70, \quad \beta_4 = 0.20$</td>
</tr>
<tr>
<td>0.1</td>
<td>48.3617</td>
</tr>
<tr>
<td>0.2</td>
<td>33.4173</td>
</tr>
<tr>
<td>0.3</td>
<td>28.4369</td>
</tr>
<tr>
<td>0.4</td>
<td>25.9545</td>
</tr>
<tr>
<td>0.5</td>
<td>24.4723</td>
</tr>
<tr>
<td>0.6</td>
<td>23.4898</td>
</tr>
<tr>
<td>0.7</td>
<td>22.7924</td>
</tr>
<tr>
<td>0.8</td>
<td>22.2726</td>
</tr>
<tr>
<td>0.9</td>
<td>21.8709</td>
</tr>
<tr>
<td>1.0</td>
<td>21.5516</td>
</tr>
</tbody>
</table>

In fig. 5.2, we plot MTSF w.r.t. $\alpha_2$ and fixed values of parameters $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$ and $\beta_4$ for three different values of $\beta_2$. It is observed that MTSF of the system decreases w.r.t. $\alpha_2$ irrespective of the other parameters so that we conclude that expected life of the system increases with decreasing failure rate.
Behaviour of MTSF w.r.t. $\alpha_2$ for three different value of $\beta_2$

Fig. 5.2
For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3, \beta_4$ and changing $\beta_2$, TABLE-2 is obtained

TABLE- 2: Effect of $\beta_2$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3, \beta_4$ on MTSF for three different values of $\alpha_2$.

<table>
<thead>
<tr>
<th>$\beta_2$</th>
<th>MTSF</th>
<th>MTSF</th>
<th>MTSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 0.053, \alpha_2 = 0.20,$ $\alpha_3 = 0.048, \alpha_4 = 0.062,$ $\beta_1 = 0.10, \beta_3 = 0.70,$ $\beta_4 = 0.20$</td>
<td>$\alpha_1 = 0.053, \alpha_2 = 0.40,$ $\alpha_3 = 0.048, \alpha_4 = 0.062,$ $\beta_1 = 0.10, \beta_3 = 0.70,$ $\beta_4 = 0.20$</td>
<td>$\alpha_1 = 0.053, \alpha_2 = 0.30,$ $\alpha_3 = 0.048, \alpha_4 = 0.062,$ $\beta_1 = 0.10, \beta_3 = 0.70,$ $\beta_4 = 0.20$</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>29.9018</td>
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</tr>
<tr>
<td>0.3</td>
<td>31.7198</td>
<td>25.1219</td>
<td>27.3032</td>
</tr>
<tr>
<td>0.4</td>
<td>32.2166</td>
<td>25.3415</td>
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</tr>
<tr>
<td>0.5</td>
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<td>28.2016</td>
</tr>
<tr>
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<tr>
<td>0.9</td>
<td>33.4173</td>
<td>25.9545</td>
<td>28.4369</td>
</tr>
<tr>
<td>1.0</td>
<td>33.5433</td>
<td>26.0267</td>
<td>28.5285</td>
</tr>
</tbody>
</table>

In fig. 5.3, we plot MTSF w.r.t. $\beta_2$ and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3$ and $\beta_4$ for three different values of $\alpha_2$. It is quiet clear that MTSF of the system increases w.r.t. $\beta_2$ irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate.
Behaviour of MTSF w. r. t. $\beta_2$ for three different values of $\alpha_2$

- $\alpha_2 = 0.2$
- $\alpha_2 = 0.4$
- $\alpha_2 = 0.3$

**Fig. 5.3**
For fixed values of the parameters $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4$ and changing $\alpha_2$, TABLE-3 is obtained.

TABLE-3: Effect of $\alpha_2$ and fixed parameters $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$ and $\beta_4$ on Availability for three values of $\beta_2$.

<table>
<thead>
<tr>
<th>$\alpha_2$</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1 = 0.053$, $\alpha_3 = 0.048$, $\alpha_4 = 0.062$, $\beta_1 = 0.10$, $\beta_2 = 0.10$, $\beta_3 = 0.70$, $\beta_4 = 0.20$</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.6620</td>
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<td>0.6330</td>
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<td>0.6108</td>
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<td>0.5793</td>
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<tr>
<td>0.9</td>
<td>0.5677</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5580</td>
</tr>
</tbody>
</table>

In fig. 5.4, we plot Availability w.r.t. $\alpha_2$ and fixed values of parameter $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$ and $\beta_4$. For three values of $\beta_2$. It is observed that Availability of the system decreases w.r.t. $\alpha_2$ irrespective of the other parameters. Therefore, we conclude that expected life of the system increases with decreasing failure rate.
Behaviour of Availability w.r.t. $\alpha_2$ for three different value of $\beta_2$

Fig. 5.4
For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3, \beta_4$ and changing $\beta_2$, TABLE-4 is obtained.

TABLE- 4: Effect of $\beta_2$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3$ and $\beta_4$ on MTSF with three different values of $\alpha_2$.

<table>
<thead>
<tr>
<th>$\beta_2$</th>
<th>$\alpha_1 = 0.053$, $\alpha_2 = 0.70,$</th>
<th>$\alpha_1 = 0.053$, $\alpha_2 = 0.40,$</th>
<th>$\alpha_1 = 0.053$, $\alpha_2 = 0.20,$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_3 = 0.048$, $\alpha_4 = 0.062,$</td>
<td>$\alpha_3 = 0.048$, $\alpha_4 = 0.062,$</td>
<td>$\alpha_3 = 0.048$, $\alpha_4 = 0.062,$</td>
</tr>
<tr>
<td></td>
<td>$\beta_1 = 0.10,$ $\beta_3 = 0.70,$</td>
<td>$\beta_1 = 0.10,$ $\beta_3 = 0.70,$</td>
<td>$\beta_1 = 0.10,$ $\beta_3 = 0.70,$</td>
</tr>
<tr>
<td></td>
<td>$\beta_4 = 0.20$</td>
<td>$\beta_4 = 0.20$</td>
<td>$\beta_4 = 0.20$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5933</td>
<td>0.6620</td>
<td>0.7552</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6794</td>
<td>0.7384</td>
<td>0.8075</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7127</td>
<td>0.7656</td>
<td>0.8226</td>
</tr>
<tr>
<td>0.4</td>
<td>0.7299</td>
<td>0.7788</td>
<td>0.8288</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7402</td>
<td>0.7863</td>
<td>0.8318</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7470</td>
<td>0.7910</td>
<td>0.8334</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7517</td>
<td>0.7941</td>
<td>0.8343</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7551</td>
<td>0.7963</td>
<td>0.8348</td>
</tr>
<tr>
<td>0.9</td>
<td>0.7577</td>
<td>0.7980</td>
<td>0.8352</td>
</tr>
<tr>
<td>1.0</td>
<td>0.7597</td>
<td>0.7990</td>
<td>0.8353</td>
</tr>
</tbody>
</table>

In fig. 5.5, we plot Availability w.r.t. $\beta_2$ and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3$ and $\beta_4$. It is quiet clear that Availability of the system increases w.r.t. $\beta_2$ irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate.
Behaviour of Availability w. r. t. $\beta_2$ for three different values of $\alpha_2$

Fig. 5.5
For fixed values of the parameters $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, K_0, K_1, K_2$ and changing $\alpha_2$, Table-5 is obtained.

**TABLE-5**: Effect of $\alpha_1$ and fixed parameters $\alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, K_0, K_1$ and $K_2$ on Profit function.

<table>
<thead>
<tr>
<th>$\alpha_2$</th>
<th>Profit Function</th>
<th>Profit Function</th>
<th>Profit Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 0.053, \alpha_3 = 0.048, \alpha_4 = 0.062, \beta_1 = 0.10, \beta_2 = 0.12, \beta_3 = 0.70, \beta_4 = 0.20, K_0 = 1000, K_1 = 500, K_2 = 300$</td>
<td>544.950</td>
<td>536.42</td>
<td>531.619</td>
</tr>
<tr>
<td>0.2</td>
<td>499.195</td>
<td>491.426</td>
<td>485.431</td>
</tr>
<tr>
<td>0.3</td>
<td>469.426</td>
<td>457.662</td>
<td>446.811</td>
</tr>
<tr>
<td>0.4</td>
<td>446.609</td>
<td>429.798</td>
<td>413.893</td>
</tr>
<tr>
<td>0.5</td>
<td>427.636</td>
<td>405.801</td>
<td>385.385</td>
</tr>
<tr>
<td>0.6</td>
<td>411.129</td>
<td>384.637</td>
<td>360.368</td>
</tr>
<tr>
<td>0.7</td>
<td>396.372</td>
<td>365.683</td>
<td>338.173</td>
</tr>
<tr>
<td>0.8</td>
<td>382.946</td>
<td>348.522</td>
<td>318.298</td>
</tr>
<tr>
<td>0.9</td>
<td>370.586</td>
<td>332.854</td>
<td>300.356</td>
</tr>
<tr>
<td>1.0</td>
<td>359.109</td>
<td>318.457</td>
<td>284.049</td>
</tr>
</tbody>
</table>

In fig. 5.6, we plot Profit Function w.r.t. $\alpha_2$ and fixed values of parameter $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, K_0, K_1$ and $K_2$ for three different values of $\beta_2$. It is observed that Profit Function of the system decreases w.r.t. $\alpha_2$ irrespective of the other parameters. Therefore, we conclude Profit increases with decreasing failure rate.
Fig. 5.6
For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3, \beta_4, K_0, K_1, K_2$ and changing $\beta_2$, Table-3 is obtained

TABLE-6: Effect of $\beta_2$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3, \beta_4, K_0, K_1$ and $K_2$ on Profit Function for three different values of $\alpha_2$.

<table>
<thead>
<tr>
<th>$\beta_2$</th>
<th>Profit Function</th>
<th>Profit Function</th>
<th>Profit Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1 = 0.053, \alpha_2 = 0.40, \alpha_3 = 0.048, \alpha_4 = 0.062, \beta_1 = 0.10, \beta_3 = 0.70, \beta_4 = 0.20, K_0 = 1000, K_1 = 500, K_2 = 300$</td>
<td>$\alpha_1 = 0.053, \alpha_2 = 0.30, \alpha_3 = 0.048, \alpha_4 = 0.062, \beta_1 = 0.10, \beta_3 = 0.70, \beta_4 = 0.20, K_0 = 1000, K_1 = 500, K_2 = 300$</td>
<td>$\alpha_1 = 0.053, \alpha_2 = 0.20, \alpha_3 = 0.048, \alpha_4 = 0.062, \beta_1 = 0.10, \beta_3 = 0.70, \beta_4 = 0.20, K_0 = 1000, K_1 = 500, K_2 = 300$</td>
</tr>
<tr>
<td>0.1</td>
<td>520.291</td>
<td>563.614</td>
<td>621.848</td>
</tr>
<tr>
<td>0.2</td>
<td>638.969</td>
<td>668.999</td>
<td>704.534</td>
</tr>
<tr>
<td>0.3</td>
<td>681.584</td>
<td>704.901</td>
<td>730.252</td>
</tr>
<tr>
<td>0.4</td>
<td>702.460</td>
<td>721.866</td>
<td>741.688</td>
</tr>
<tr>
<td>0.5</td>
<td>714.430</td>
<td>731.328</td>
<td>747.779</td>
</tr>
<tr>
<td>0.6</td>
<td>721.997</td>
<td>737.178</td>
<td>751.406</td>
</tr>
<tr>
<td>0.7</td>
<td>727.112</td>
<td>741.060</td>
<td>753.738</td>
</tr>
<tr>
<td>0.8</td>
<td>730.746</td>
<td>743.775</td>
<td>755.325</td>
</tr>
<tr>
<td>0.9</td>
<td>733.427</td>
<td>745.751</td>
<td>756.451</td>
</tr>
<tr>
<td>1.0</td>
<td>735.467</td>
<td>747.236</td>
<td>757.278</td>
</tr>
</tbody>
</table>

In fig 5.7, we plot Profit Function w.r.t. $\beta_2$ and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3, \beta_4, K_0, K_1$ and $K_2$. It is observed that Profit Function of the system increases w.r.t. $\beta_2$ irrespective of the other parameters. Therefore, we conclude Profit increases with increasing repair rate.
Behaviour of Profit w.r.t. $\beta_2$ for three different value of $\alpha_2$

![Graph showing the behaviour of profit with respect to $\beta_2$ for different values of $\alpha_2$.]

Fig. 5.7