4.1 INTRODUCTION

Although a lot of work has been done in the field of reliability but most of it is concerned with hypothetical models which are not of much practical utility because they may not even exists in real life. Very few authors Singh, Pandey and Kumar [119], Singh and Singh [123], Singh [124] and Kumar et. al. [78], Kumar and Bharti [81-82], Arora and Kumar [5] and Gupta et. al. [55] have studied the industrial system models with real existing situations. For the purpose of analyzing real existing system and keeping in view the practical applicability of the subject a real existing C.F.L. manufacturing system, located at J&K S.I.D.C.O. (J&K State Industrial Development Corporation) Jammu, Jammu and Kashmir is selected for study and a model has been proposed to study its reliability analysis. J&K S.I.D.C.O. was established in 1967 and it act as a catalyst to inspire and accelerate the Industrial development in the J&K State. The proposed system consists of three units of varying in nature.

The production of fluorescent lamps is done in following three Machines:

Mount Making Machine (MM) :- This machine works in three steps , in the first step, a specially designed part with twelve heads located around the edge of the turn table flanges tubes of the glass into flares. The flare is then separated from the glass by a flame which polishes the surface of the glass as it cuts it. In the second step, the stems are automatically formed by feeding exhaust tubes and lead-in wires along with flares into the stem making part. In the third step, stems are forwarded to the last portion of mount making machine, where oxidized, tungsten wire filaments are attached, thus completing the mount making process.

Glass and Base Preparation Machine (GB) :- This machine also works in various stages, glass tubes of appropriate length are placed in the washing and coating part of the machine, this part uses hot water and hot air to wash and dry the tubes before the inner walls of the tubes are coated with fluorescent powder.

After being coated with fluorescent powder the tubes are automatically loaded into roller conveyer which transport them through an oven and then through a cooling chamber. As the
tubes pass through the oven the fluorescent coating is baked onto the tubes. A two end cleaning part of the machine automatically brushes the fluorescent coating from the ends of the tubes. The conveyer then transports the tubes to the sealing part of the machine for the assembly. For base preparation the sealing compound is made by mixing the required portions of the required ingredients in a cement-mixing machine. The sealing compound is then dispensed into the bases of the lamps by an automatic filling part of the machine. The bases are then forwarded to the basing part of the machine for the final assembly.

Assembly Machine (A) :- The mounts, glass tubes and the bases are then finally assembled into finished goods by feeding these into assembly machine

Using regenerative point technique the following important reliability characteristics of interest are obtained:
1. Transition probabilities and mean sojourn times.
2. Mean time to system failure (MTSF).
3. Point wise and steady-state availabilities of the system.
4. Expected up-time of the system.
5. Expected busy period of the repairman during \((0, t]\) and in the steady state.
6. Expected number of repairs during \((0, t]\) and in the steady state.
7. Net expected profit incurred by the system during \((0, t]\) and in steady state.

4.2 ASSUMPTIONS
1. Failures and repairs are stochastically independent.
2. A single repair facility is always available with the system to repair a failed unit.
3. A repaired unit is as good as new and is immediately reconnected to the system.
4. All the failure time and repair time distributions are taken to be negative exponential with different parameters.
5. Stock-out time is a random variable and follows negative exponential distribution.
6. Rate of production of Mount Making Machine and Glass Tube and Base Preparation Machine is different.
7. Priority in repair is given to Mount Making Machine over the other units and the preference is given to Glass Tube and base Preparation machine over the Assembler.
8. For the System functioning, working of atleast Mount Making Machine or Glass Tube and Base Preparation Machine is required.

### 4.3 NOTATIONS AND POSSIBLE STATES OF THE SYSTEM

\[ \alpha_i \] : Failure rate of the unit MM/GB/A respectively, for \( i = 1, 2, 3 \).

\[ \gamma_i \] : Repair rate of the unit MM/GB/A respectively, for \( i = 1, 2, 3 \).

\[ B \] : Rate of transition from idle to operative state.

Symbols for the states of the system

\[ \text{MM}_0 \] : Mount Making Machine is operative.

\[ \text{MM}_r \] : Mount Making Machine is under repair.

\[ \text{GB}_0 \] : Glass Tube and Base Preparation Machine is operative.

\[ \text{GB}_r \] : Glass Tube and Base Preparation Machine is under repair.

\[ \text{GB}_{wr}/\text{GB}_{wr_1} \] : Glass Tube and Base Preparation Machine is waiting for repair.

\[ A_0 \] : Assembler is operative.

\[ A_i \] : Assembler is under idle condition.

\[ A_r \] : Assembler is under repair.

\[ A_{wr}/A_{wr_2} \] : Assembler is waiting for repair.

With the help of the above symbols the possible states of the system are:

\[ S_0 = [\text{MM}_0, \text{GB}_0, A_0] \quad S_1 = [\text{MM}_r, \text{GB}_0, A_0] \]

\[ S_2 = [\text{MM}_r, \text{GB}_0, A_i] \quad S_3 = [\text{MM}_0, \text{GB}_r, A_0] \]

\[ S_4 = [\text{MM}_0, \text{GB}_r, A_i] \quad S_5 = [\text{MM}_r, \text{GB}_{wr}, A_i] \]

\[ S_6 = [\text{MM}_0, \text{GB}_r, A_{wr}] \quad S_7 = [\text{MM}_0, \text{GB}_0, A_r] \]

\[ S_8 = [\text{MM}_r, \text{GB}_0, A_{wr}] \quad S_9 = [\text{MM}_r, \text{GB}_{wr_1}, A_{wr_2}] \]

The transition diagram along with all the transitions is shown in fig. 2.1
TRANSITION DIAGRAM

Fig. 4.1

- **Up State**

- **Failed State**
4.4 TRANSITION PROBABILITIES AND SOJOURN TIMES

Let $T_0(= 0)$, $T_1$, $T_2$, \ldots, denotes the regenerative epochs and $X_n$ denotes the state visited at epoch $T_{n+}$, i.e. just after the transition at $T_n$ then $\{X_n, T_n\}$ constitute a Markov-Renewal process with state space $E$, set of regenerative states and

$$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t \mid X_n = i].$$

is the semi Markov kernel over $E$.

Then the transition probability matrix of the embedded Markov chain is

$$p = [Q_{ij}(\infty)] = [Q(\infty)].$$

The various transitions probabilities may be obtained as follows:

$Q_{01}(t) = \alpha_1 \int_0^t \exp[-(\alpha_1 + \alpha_2 + \alpha_3)u] \, du$

$Q_{02}(t) = \alpha_2 \int_0^t \exp[-(\alpha_1 + \alpha_2 + \alpha_3)u] \, du$

$Q_{03}(t) = \alpha_3 \int_0^t \exp[-(\alpha_1 + \alpha_2 + \alpha_3)u] \, du$

$Q_{11}(t) = \gamma_1 \int_0^t \exp[-(\gamma_1 + \beta + \alpha_2 + \alpha_3)u] \, du$

$Q_{12}(t) = \beta \int_0^t \exp[-(\gamma_1 + \beta + \alpha_2 + \alpha_3)u] \, du$

$Q_{13}(t) = \alpha_2 \int_0^t \exp[-(\gamma_1 + \beta + \alpha_2 + \alpha_3)u] \, du$

$Q_{14}(t) = \beta \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{15}(t) = \beta \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{16}(t) = \beta \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{20}(t) = \gamma_1 \int_0^t \exp[-(\gamma_1 + \alpha_2)u] \, du$

$Q_{21}(t) = \alpha_2 \int_0^t \exp[-(\gamma_1 + \alpha_2)u] \, du$

$Q_{22}(t) = \alpha_1 \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{23}(t) = \beta \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{30}(t) = \gamma_2 \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{31}(t) = \beta \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{32}(t) = \beta \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$

$Q_{33}(t) = \alpha_3 \int_0^t \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)u] \, du$
Steady state transition probabilities

By taking the limit as $t$ tends to $\infty$ in equation (1), we obtain the following steady state transition probabilities:

\begin{align*}
Q_{40}(t) &= \gamma_2 \int_0^t \exp[-(\gamma_2 + \alpha_1)u] \, du \\
Q_{45}(t) &= \alpha_1 \int_0^t \exp[-(\gamma_2 + \alpha_1)u] \, du \\
Q_{53}(t) &= \gamma_1 \int_0^t \exp[-(\gamma_1)u] \, du \\
Q_{67}(t) &= \gamma_2 \int_0^t \exp[-(\gamma_2 + \alpha_1)u] \, du \\
Q_{69}(t) &= \alpha_1 \int_0^t \exp[-(\gamma_2 + \alpha_1)u] \, du \\
Q_{70}(t) &= \gamma_3 \int_0^t \exp[-(\gamma_3 + \alpha_1 + \alpha_2)u] \, du \\
Q_{76}(t) &= \alpha_2 \int_0^t \exp[-(\gamma_3 + \alpha_1 + \alpha_2)u] \, du \\
Q_{78}(t) &= \alpha_1 \int_0^t \exp[-(\gamma_3 + \alpha_1 + \alpha_2)u] \, du \\
Q_{87}(t) &= \gamma_1 \int_0^t \exp[-(\gamma_1 + \alpha_2)u] \, du \\
Q_{89}(t) &= \alpha_2 \int_0^t \exp[-(\gamma_1 + \alpha_2)u] \, du \\
Q_{96}(t) &= \gamma_1 \int_0^t \exp[-(\gamma_1)u] \, du
\end{align*}

(1)

**Steady state transition probabilities**

By taking the limit as $t$ tends to $\infty$ in equation (1), we obtain the following steady state transition probabilities:

\[ p_{01} = \lim_{t \to \infty} Q_{01}(t) = \alpha_1 \int_0^\infty \exp[-(\alpha_1 + \alpha_2 + \alpha_3)t] \, dt = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3} \]

\[ p_{03} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \alpha_3} \]

\[ p_{10} = \frac{\beta}{\beta + \gamma_1 + \alpha_2 + \alpha_3} \]

\[ p_{15} = \frac{\alpha_2}{\beta + \gamma_1 + \alpha_2 + \alpha_3} \]

\[ p_{20} = \frac{\gamma_1}{\gamma_1 + \alpha_2} \]

\[ p_{25} = \frac{\alpha_2}{\gamma_1 + \alpha_2} \]

\[ p_{30} = \frac{\beta}{\beta + \gamma_2 + \alpha_1 + \alpha_3} \]

\[ p_{34} = \frac{\beta}{\beta + \gamma_2 + \alpha_1 + \alpha_3} \]

\[ p_{35} = \frac{\alpha_1}{\beta + \gamma_2 + \alpha_1 + \alpha_3} \]

\[ p_{36} = \frac{\alpha_3}{\beta + \gamma_2 + \alpha_1 + \alpha_3} \]
From the obtained steady state probabilities, it can be easily seen that the following results hold good:

\[ p_{01} + p_{03} + p_{07} = 1 \]
\[ p_{20} + p_{25} = 1 \]
\[ p_{40} + p_{45} = 1 \]
\[ p_{67} + p_{69} = 1 \]
\[ p_{87} + p_{89} = 1 \]

\[ p_{01} + p_{12} + p_{15} + p_{18} = 1 \]
\[ p_{30} + p_{34} + p_{35} + p_{36} = 1 \]
\[ p_{53} = 1 \]
\[ p_{70} + p_{76} + p_{78} = 1 \]
\[ p_{96} = 1 \]

\[ (1) \]

Mean sojourn time

The mean sojourn time in state \( S_i \) denoted by \( \mu_i \) is defined as the expected time taken by the system in state \( S_i \) before transiting to any other state. To obtain mean sojourn time \( \mu_i \) in state \( S_i \), we observe that as long as the system is in state \( S_i \), there is no transition from \( S_i \) to any other state. If \( T_i \) denotes the sojourn time in state \( S_i \) then mean sojourn time in state \( S_i \) is

\[ \mu_i = E[T_i] = \int_0^\infty P[T_i > t] dt , \]

Thus

\[ \mu_0 = \int_0^\infty \exp[-(\alpha_1 + \alpha_2 + \alpha_3)t] dt = \frac{1}{\alpha_1 + \alpha_2 + \alpha_3} \]

\[ \mu_1 = \int_0^\infty \exp[-(\gamma_1 + \beta + \alpha_2 + \alpha_3)t] dt = \frac{1}{\gamma_1 + \beta + \alpha_2 + \alpha_3} \]
\[ \mu_2 = \frac{1}{\gamma_1 + a_2} \quad \mu_3 = \frac{1}{\gamma_2 + \beta + a_1 + a_3} \]
\[ \mu_4 = \frac{1}{\gamma_2 + a_1} \quad \mu_5 = \frac{1}{\gamma_1} \]
\[ \mu_6 = \frac{1}{\gamma_2 + a_1} \quad \mu_7 = \frac{1}{\gamma_3 + a_1 + a_2} \]
\[ \mu_8 = \frac{1}{\gamma_1 + a_2} \quad \mu_9 = \frac{1}{\gamma_1} \] (4)

### 4.5 MEAN TIME SYSTEM FAILURE

Let the random variable \( T_i \) denotes the time to system failure when \( E_0 = E_1 \in E \) and \( \phi_i(t) \) is the c.d.f. of the time to system failure for the first time when the system starts operation from state \( S_i \). On the basis of arguments used for regenerative processes we obtain following relations for \( \phi_i(t) \).

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t)\phi_1(t) + Q_{03}(t)\phi_3(t) + Q_{07}(t)\phi_7(t) \\
\phi_1(t) &= Q_{10}(t)\phi_0(t) + Q_{12}(t)\phi_2(t) + Q_{15}(t) + Q_{18}(t)\phi_8(t) \\
\phi_2(t) &= Q_{20}(t)\phi_0(t) + Q_{25}(t) \\
\phi_3(t) &= Q_{30}(t)\phi_0(t) + Q_{34}(t)\phi_4(t) + Q_{35}(t) + Q_{36}(t)\phi_6(t) \\
\phi_4(t) &= Q_{40}(t)\phi_0(t) + Q_{45}(t) \\
\phi_6(t) &= Q_{67}(t)\phi_7(t) + Q_{69}(t) \\
\phi_7(t) &= Q_{71}(t)\phi_1(t) + Q_{76}(t)\phi_6(t) + Q_{78}(t)\phi_8(t) \\
\phi_8(t) &= Q_{87}(t)\phi_7(t) + Q_{89}(t) \\
\end{align*}
\] (5)

Taking the Laplace Transform of above equations we get:

\[
\begin{align*}
\tilde{\phi}_0(s) &= \tilde{Q}_{01}(s)\tilde{\phi}_1(s) + \tilde{Q}_{03}(s)\tilde{\phi}_3(s) + \tilde{Q}_{07}(s)\tilde{\phi}_7(s) \\
\tilde{\phi}_1(s) &= \tilde{Q}_{10}(s)\tilde{\phi}_0(s) + \tilde{Q}_{12}(s)\tilde{\phi}_2(s) + \tilde{Q}_{15}(s) + \tilde{Q}_{18}(s)\tilde{\phi}_8(s) \\
\end{align*}
\]
\[ \Phi_2(s) = \tilde{Q}_{20}(s)\Phi_0(s) + \tilde{Q}_{25}(s) \]
\[ \Phi_3(s) = \tilde{Q}_{30}(s)\Phi_0(s) + \tilde{Q}_{34}(s)\Phi_4(s) + \tilde{Q}_{35}(s) + \tilde{Q}_{36}(s)\Phi_6(s) \]
\[ \Phi_4(s) = \tilde{Q}_{40}(s)\Phi_0(s) + \tilde{Q}_{45}(s) \]
\[ \Phi_6(s) = \tilde{Q}_{67}(s)\Phi_7(s) + \tilde{Q}_{69}(s) \]
\[ \Phi_7(s) = \tilde{Q}_{71}(s)\Phi_1(s) + \tilde{Q}_{76}(s)\Phi_6(s) + \tilde{Q}_{78}(s)\Phi_8(s) \]
\[ \Phi_8(s) = \tilde{Q}_{87}(s)\Phi_7(s) + \tilde{Q}_{89}(s) \]

(6)

On solving above equations for \( \Phi_0(s) \), we have

\[ \Phi_0(s) = \frac{N_1(s)}{D_1(s)} \]  

(7)

where

\[ N_1(s) = \left[ 1 - \tilde{Q}_{67}(s)\tilde{Q}_{76}(s) - \tilde{Q}_{78}(s)\tilde{Q}_{87}(s) \right] \left[ \tilde{Q}_{01}(s)\tilde{Q}_{15}(s) + \tilde{Q}_{01}(s)\tilde{Q}_{12}(s)\tilde{Q}_{25}(s) + \tilde{Q}_{03}(s)\tilde{Q}_{35}(s) + \tilde{Q}_{03}(s)\tilde{Q}_{34}(s)\tilde{Q}_{45}(s) \right] + \tilde{Q}_{01}(s)\tilde{Q}_{18}(s)\tilde{Q}_{87}(s)\tilde{Q}_{76}(s)\tilde{Q}_{69}(s) + \tilde{Q}_{03}(s)\tilde{Q}_{36}(s)\tilde{Q}_{67}(s)\tilde{Q}_{76}(s)\tilde{Q}_{89}(s) + \tilde{Q}_{03}(s)\tilde{Q}_{36}(s)\tilde{Q}_{69}(s)[1 - \tilde{Q}_{78}(s)\tilde{Q}_{87}(s)] + \tilde{Q}_{07}(s)\tilde{Q}_{76}(s)\tilde{Q}_{69}(s) + \tilde{Q}_{07}(s)\tilde{Q}_{78}(s)\tilde{Q}_{89}(s) + [1 - \tilde{Q}_{67}(s)\tilde{Q}_{76}(s)] \]
\[ \tilde{Q}_{01}(s)\tilde{Q}_{18}(s)\tilde{Q}_{89}(s) \]  

(8)

and

\[ D_1(s) = \left[ 1 - \tilde{Q}_{67}(s)\tilde{Q}_{76}(s) - \tilde{Q}_{78}(s)\tilde{Q}_{87}(s) \right] \left[ 1 - \tilde{Q}_{01}(s)\tilde{Q}_{10}(s) + \tilde{Q}_{01}(s)\tilde{Q}_{12}(s)\tilde{Q}_{20}(s) + \tilde{Q}_{03}(s)\tilde{Q}_{30}(s) + \tilde{Q}_{03}(s)\tilde{Q}_{34}(s)\tilde{Q}_{40}(s) \right] - \tilde{Q}_{01}(s)\tilde{Q}_{18}(s)\tilde{Q}_{87}(s)\tilde{Q}_{70}(s) - \tilde{Q}_{07}(s)\tilde{Q}_{70}(s) - \tilde{Q}_{03}(s)\tilde{Q}_{36}(s)\tilde{Q}_{67}(s)\tilde{Q}_{70}(s) \]  

(9)

on taking \( s \to 0 \) in (8) and (9) and using the relation \( \tilde{Q}_{ij}(s) \to p_{ij} \), we have

\[ N_1(0) = (1 - p_{67}p_{76} - p_{78}p_{87})(p_{01}p_{15} + p_{01}p_{12}p_{25} + p_{03}p_{35} + p_{03}p_{34}p_{45}) + p_{07}p_{78}p_{89} + p_{03}p_{36}p_{69}(1 - p_{78}p_{87}) + p_{01}p_{18}p_{89}(1 - p_{67}p_{76}) + p_{01}p_{18}p_{87}p_{76}p_{69} + p_{07}p_{76}p_{69} + p_{03}p_{36}p_{67}p_{76}p_{89} + p_{07}p_{78}p_{89} = (1 - p_{67}p_{76} - p_{78}p_{87})(p_{01} - p_{01}p_{18} - p_{01}p_{12} - p_{01}p_{10} + p_{01}p_{12} - p_{01}p_{12}p_{20} + p_{03} - p_{03}p_{36} - p_{03}p_{34} - p_{03}p_{30} + p_{03}p_{34} - p_{03}p_{34}p_{40}) + p_{03}p_{36}p_{69} - p_{03}p_{36}p_{69}p_{78}p_{87} \]
\[ \begin{align*}
+ & p_{01} p_{18} p_{87} p_{76} p_{69} + p_{07} p_{76} p_{69} + p_{01} p_{18} p_{89} - p_{01} p_{18} p_{89} p_{67} p_{67} + p_{07} p_{78} p_{89} + p_{03} p_{36} p_{67} p_{78} p_{89} \\
= & (1 - p_{67} p_{76} - p_{78} p_{87})(1 - p_{01} p_{10} - p_{01} p_{12} p_{20} - p_{03} p_{30} - p_{03} p_{34} p_{40}) - p_{03} p_{36} \\
+ & p_{07} p_{78} p_{87} + p_{07} p_{76} p_{67} - p_{01} p_{18} + p_{01} p_{18} p_{87} p_{76} + p_{01} p_{18} p_{67} p_{76} + p_{03} p_{36} p_{78} p_{87} \\
+ & p_{03} p_{36} p_{67} p_{76} + p_{03} p_{36} p_{69} - p_{03} p_{36} p_{69} p_{78} p_{87} + p_{01} p_{18} p_{87} p_{76} p_{69} + p_{07} p_{76} p_{69} + p_{01} p_{18} p_{89} - p_{01} p_{18} p_{89} p_{67} p_{76} + p_{07} p_{78} p_{89} + p_{03} p_{36} p_{67} p_{78} p_{89} - p_{07} \\
= & (1 - p_{67} p_{76} - p_{78} p_{87})(1 - p_{01} p_{10} - p_{01} p_{12} p_{20} - p_{03} p_{30} - p_{03} p_{34} p_{40}) - p_{07} + \\
+ & p_{07} p_{78} p_{87} + p_{07} p_{76} (p_{67} + p_{69}) - p_{01} p_{18} + p_{01} p_{18} p_{87} p_{76} + p_{01} p_{18} p_{67} p_{76} - \\
& p_{03} p_{36} p_{67} + p_{03} p_{36} p_{78} p_{87} + p_{03} p_{36} p_{67} p_{76} - p_{03} p_{36} p_{69} p_{78} p_{87} + p_{01} p_{18} p_{87} p_{76} p_{69} \\
- & p_{01} p_{18} p_{89} p_{67} p_{76} + p_{03} p_{36} p_{67} p_{78} p_{89} + p_{07} p_{78} p_{89} \\
= & (1 - p_{67} p_{76} - p_{78} p_{87})(1 - p_{01} p_{10} - p_{01} p_{12} p_{20} - p_{03} p_{30} - p_{03} p_{34} p_{40}) + p_{07} p_{76} \\
+ & p_{07} p_{78} p_{87} - p_{01} p_{18} p_{87} p_{70} - p_{01} p_{18} p_{87} p_{76} p_{67} + p_{01} p_{18} p_{76} p_{67} - p_{03} p_{36} p_{67} + \\
& p_{03} p_{36} p_{67} p_{78} p_{87} + p_{03} p_{36} p_{67} p_{76} - p_{01} p_{18} p_{89} p_{76} p_{67} + p_{03} p_{36} p_{67} p_{78} p_{89} + p_{07} p_{78} p_{89} \\
- & p_{07} \\
= & (1 - p_{67} p_{76} - p_{78} p_{87})(1 - p_{01} p_{10} - p_{01} p_{12} p_{20} - p_{03} p_{30} - p_{03} p_{34} p_{40}) - p_{07} + \\
& p_{07} p_{76} - p_{01} p_{18} p_{87} p_{70} - p_{03} p_{36} p_{67} p_{70} + p_{07} p_{78} p_{87} + p_{03} p_{36} p_{67} p_{76} \\
- & p_{03} p_{36} p_{67} p_{76} + p_{07} p_{78} p_{89} \\
= & (1 - p_{67} p_{76} - p_{78} p_{87})(1 - p_{01} p_{10} - p_{01} p_{12} p_{20} - p_{03} p_{30} - p_{03} p_{34} p_{40}) - p_{07} - \\
& - p_{01} p_{18} p_{87} p_{70} - p_{03} p_{36} p_{67} p_{70} + p_{07} p_{78} + p_{07} p_{76} \\
= & (1 - p_{67} p_{76} - p_{78} p_{87})(1 - p_{01} p_{10} - p_{01} p_{12} p_{20} - p_{03} p_{30} - p_{03} p_{34} p_{40}) - p_{07} p_{78} \\
- & - p_{01} p_{18} p_{87} p_{70} - p_{03} p_{36} p_{67} p_{70}
\end{align*}\]

and

\[ D_1(0) = (1 - p_{67} p_{76} - p_{78} p_{87})(1 - p_{01} p_{10} - p_{01} p_{12} p_{20} - p_{03} p_{30} - p_{03} p_{34} p_{40}) - p_{07} p_{78} \]

\[ - p_{01} p_{18} p_{87} p_{70} - p_{03} p_{36} p_{67} p_{70} \]

which is equal to \( N_4(0) \)

\[ \tilde{\phi}_0(s) = \frac{N_4(s)}{D_1(s)} = 1 \]

This shows that \( \tilde{\phi}_0(s) \) is a proper cdf. Therefore, mean time to system failure will be

\[ \text{MTSF} = E(T) = -\frac{\partial}{\partial s} \tilde{\phi}_0(s) \bigg|_{s=0} = \frac{D_1'(0) - N_4'(0)}{D_1(0)} \]
To obtain numerator of (11), we collect the coefficients of the relevant $m_{ij}$’s, where $m_{ij}$’s is the mean elapsed time of the system in state $S_i$ before transiting to state $S_j$.

In notation

$$m_{ij} = \tilde{Q}_{ij}(s) \bigg|_{s=0} = \frac{\partial}{\partial s} \int_0^\infty \exp[-(st)] \partial Q_{ij}(t) \bigg|_{s=0}$$

Also we know that $\sum_j m_{ij} = \mu_i$

Thus the coefficient of various $m_{ij}$’s in $D_1'(0) - N_1'(0)$

The coefficient of $m_{01} = (1 - p_{67}p_{76} - p_{78}p_{87})(p_{10} + p_{12}p_{20}) + p_{15}(1 - p_{78}p_{87}) + p_{18}p_{87}p_{70}

- p_{15}p_{67}p_{76} + p_{12}p_{25}(1 - p_{78}p_{87}) - p_{12}p_{25}p_{67}p_{76} + p_{18}p_{87}p_{76}p_{69} + p_{18}p_{89}(1 - p_{67}p_{76})

= (1 - p_{67}p_{76} - p_{78}p_{87})(p_{10} + p_{12}p_{20} + p_{15} + p_{12}p_{25}) + p_{18}p_{87}p_{70} + p_{18}p_{87}p_{76}p_{69} + p_{18}p_{89}(1 - p_{67}p_{76})

= (1 - p_{67}p_{76} - p_{78}p_{87})(1 - p_{18}) + p_{18}p_{87}p_{70} + p_{18}p_{87}p_{76}p_{69} + p_{18}p_{89}(1 - p_{67}p_{76})

= (1 - p_{67}p_{76} - p_{78}p_{87}) - p_{18} + p_{18}p_{87} - p_{18}p_{87}p_{67}p_{76} + p_{18}p_{89}

- p_{18}p_{89}p_{67}p_{76} + p_{18}p_{67}p_{76}

= 1 - p_{67}p_{76} - p_{78}p_{87}$

The coefficient of $m_{03} = (1 - p_{67}p_{76} - p_{78}p_{87})(p_{30} + p_{34}p_{40} + p_{35} + p_{34}p_{45}) + p_{36}p_{67}p_{70}

+ p_{36}p_{69} - p_{36}p_{69}p_{78}p_{87} + p_{36}p_{67}p_{78}p_{89}

= (1 - p_{67}p_{76} - p_{78}p_{87})(p_{30} + p_{34} + p_{35} + p_{34}) + p_{36}p_{67}p_{70} + p_{36}p_{69}

- p_{36}p_{69}p_{78}p_{87} + p_{36}p_{67}p_{78}p_{89}

= (1 - p_{67}p_{76} - p_{78}p_{87}) + p_{36} + p_{36}p_{78}p_{87} - p_{36}p_{67}p_{78}(1 - p_{89})

- p_{36}p_{69}p_{78}p_{87} - p_{36}

= 1 - p_{67}p_{76} - p_{78}p_{87}$

The coefficient of $m_{07} = p_{70} + p_{76}p_{69} + p_{78}p_{89}$

= 1 - p_{67}p_{76} - p_{78}p_{87}$

The coefficient of $m_{10} = p_{01}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{12} = p_{20}p_{01}(1 - p_{67}p_{76} - p_{78}p_{87}) + p_{01}p_{25}(1 - p_{67}p_{76} - p_{78}p_{87})$

= $p_{01}(1 - p_{67}p_{76} - p_{78}p_{87})$

125
The coefficient of $m_{15} = p_{01}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{18} = p_{01}p_{87}p_{70} + p_{01}p_{87}p_{76}p_{69} + p_{01}p_{89}(1 - p_{67}p_{76})$

$= p_{01}(p_{70} - p_{89}p_{70} + p_{87}p_{76} - p_{87}p_{76}p_{67} + p_{89} - p_{89}p_{76}p_{67})$

$= p_{01}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{20} = p_{01}p_{12}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{25} = p_{03}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{30} = p_{03}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{34} = p_{03}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{35} = p_{03}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{36} = p_{03}(p_{70}p_{67} + p_{69} - p_{69}p_{78}p_{87} + p_{67}p_{78}p_{89})$

$= p_{03}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{40} = p_{03}p_{34}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{45} = p_{03}p_{34}(1 - p_{67}p_{76} - p_{78}p_{87})$

The coefficient of $m_{67} = p_{76}(1 - p_{01}p_{10} - p_{01}p_{12}p_{20} - p_{03}p_{30} - p_{03}p_{34}p_{40}) + p_{03}p_{36}p_{70}$

$- p_{76}(p_{01}p_{15} + p_{01}p_{12}p_{25} + p_{03}p_{35} + p_{03}p_{34}p_{45}) - p_{01}p_{18}p_{89}p_{76}$

$+ p_{03}p_{36}p_{78}p_{89}$

$= p_{76}(1 - p_{01}p_{10} - p_{01}p_{12}p_{20} - p_{03}p_{30} - p_{03}p_{34}p_{40}) + p_{03}p_{36}p_{70}$

$- p_{76}1 - p_{01}p_{10} - p_{01}p_{12}p_{20} - p_{03}p_{30} - p_{03}p_{34}p_{40} + p_{07}p_{76} +$ $p_{01}p_{18}p_{76} + p_{03}p_{36}p_{76} - p_{01}p_{18}p_{89}p_{76} + p_{03}p_{36}p_{78}p_{89}$

$= p_{03}p_{36} - p_{03}p_{36}p_{78}p_{87} + p_{07}p_{76} + p_{01}p_{18}p_{87}p_{76}$

$= p_{03}p_{36}(1 - p_{78}p_{87}) + p_{07}p_{76} + p_{01}p_{18}p_{87}p_{76}$

The coefficient of $m_{69} = p_{03}p_{36}(1 - p_{78}p_{87}) + p_{07}p_{76} + p_{01}p_{18}p_{87}p_{76}$

The coefficient of $m_{70} = p_{03}p_{36}p_{67} + p_{01}p_{18}p_{87} + p_{07}$
The coefficient of $m_{76} = p_{67}(1-p_{01}p_{10}-p_{01}p_{12}p_{20}-p_{03}p_{30}-p_{03}p_{34}p_{40})+p_{07}p_{69}-$ 
p_{67}(p_{01}p_{15} + p_{01}p_{12}p_{25}+p_{03}p_{35} + p_{03}p_{34}p_{45})+p_{01}p_{18}p_{87}p_{69} - 
p_{01}p_{18}p_{89}p_{67} = p_{03}p_{36}p_{67}+p_{01}p_{18}p_{87}+p_{07} \[4pt]
The coefficient of $m_{78} = p_{87}(1-p_{01}p_{10}-p_{01}p_{12}p_{20}-p_{03}p_{30}-p_{03}p_{34}p_{40})+p_{07}p_{89}-$ 
p_{87}(p_{01}p_{15} + p_{01}p_{12}p_{25}+p_{03}p_{35} + p_{03}p_{34}p_{45})+p_{03}p_{36}p_{67}p_{89} = p_{03}p_{36}p_{67}+p_{01}p_{18}p_{87}+p_{07}+p_{03}p_{36}p_{87} - p_{03}p_{36}p_{87} = p_{03}p_{36}p_{67}+p_{01}p_{18}p_{87}+p_{07} \[4pt]
The coefficient of $m_{87} = p_{78}(1-p_{01}p_{10} - p_{01}p_{12}p_{20} - p_{03}p_{30} - p_{03}p_{34}p_{40})+p_{01}p_{18}p_{70} - 
p_{78}(p_{01}p_{15} + p_{01}p_{12}p_{25}+p_{03}p_{35} + p_{03}p_{34}p_{45})+p_{03}p_{36}p_{67}p_{78} + p_{01}p_{18}p_{67}p_{76} = p_{07}p_{78}+p_{01}p_{18}+p_{03}p_{36}p_{67}p_{78} - p_{01}p_{18}p_{67}p_{76} \[4pt]
The coefficient of $m_{89} = p_{07}p_{78}+p_{01}p_{18}+p_{03}p_{36}p_{67}p_{78} - p_{01}p_{18}p_{67}p_{76}$ \[4pt]
Therefore, on using the relations (3) and above coefficients, the numerator of (11) becomes

$$D_1'(0) - N_1'(0) = (1 - p_{67}p_{76} - p_{78}p_{87})(m_{01} + m_{03} + m_{07}) + [p_{01}(1 - p_{67}p_{76} - p_{78}p_{87})]$$ 
$(m_{10} + m_{12} + m_{15} + m_{18}) + [p_{01}p_{12}(1 - p_{67}p_{76} - p_{78}p_{87})](m_{20} + m_{25}) + [p_{03}(1 - p_{67}p_{76} - p_{78}p_{87})](m_{30} + m_{34} + m_{35} + m_{36}) + (m_{40} + m_{45})$ 
$[p_{03}p_{34}(1 - p_{67}p_{76} - p_{78}p_{87})] + [p_{03}p_{36}(1 - p_{78}p_{87}) + p_{07}p_{76} + p_{01}p_{18}p_{87}p_{76}](m_{67} + m_{69}) + [p_{03}p_{36}p_{67} + p_{01}p_{18}p_{87} + p_{07}](m_{70} + m_{76} + m_{78}) + (m_{87} + m_{89})[p_{07}p_{78} + p_{01}p_{18} + p_{03}p_{36}p_{67}p_{78} - p_{01}p_{18}p_{67}p_{76}]$
\[ (1 - p_67p_{76} - p_{78}p_{87})(m_{01} + m_{03} + m_{07}) + [p_{01}(1 - p_67p_{76} - p_{78}p_{87})]
\[ (m_{10} + m_{12} + m_{15} + m_{18}) + [p_{01}p_{12}(1 - p_67p_{76} - p_{78}p_{87})](m_{20} + m_{25})
+ [p_3(1 - p_67p_{76} - p_{78}p_{87})](m_{30} + m_{34} + m_{35} + m_{36}) + (m_{40} + m_{45})
\[ [p_{03}p_{34}(1 - p_67p_{76} - p_{78}p_{87})] + [p_{03}p_{36}(1 - p_{78}p_{87}) + p_{07}p_{76} + p_{01}p_{18}p_{87}p_{76}](m_{67} + m_{69}) + [p_{03}p_{36}p_{67} + p_{01}p_{18}p_{87} + p_{07}](m_{70} + m_{76} + m_{78})
+ (m_{87} + m_{89})[p_{07}p_{78} + p_{01}p_{18} + p_{03}p_{36}p_{67}p_{78} - p_{01}p_{18}p_{67}p_{76}]
\]

\[ = (1 - p_67p_{76} - p_{78}p_{87})[\mu_0 + \mu_1p_{01} + \mu_2p_{01}p_{12} + \mu_3p_{03} + \mu_4p_{03}p_{34}] + \mu_6
\[ [p_{03}p_{36}(1 - p_{78}p_{87}) + p_{07}p_{76} + p_{01}p_{18}p_{87}p_{76}]\mu_7[p_{03}p_{36}p_{67} + p_{01}p_{18}p_{87} + p_{07}] + \mu_8[p_{07}p_{78} + p_{01}p_{18} + p_{03}p_{36}p_{67}p_{78} - p_{01}p_{18}p_{67}p_{76}]
\]

MTSF = \frac{D_1'(0)-N_1'(0)}{D_1(0)} \tag{12}

Where \( D_1(0) \) is given by (10)

Now by substituting the values of \( \mu_i \)'s and \( p_{ij} \)'s in (12) we get the expressions for numerator and denominator of MTSF as

\[
\frac{D_1'(0)-N_1'(0)}{D_1(0)} = \frac{A_1}{B_1}, \text{ where}
\]

\[
A_1 = [(\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1)(\alpha_1 + \alpha_2 + \gamma_3) - (\alpha_2 + \gamma_1)\alpha_2\gamma_2 - \alpha_1\gamma_1(\alpha_1 + \gamma_2)][(\alpha_2 + \gamma_1)
\]

\[
(\alpha_1 + \alpha_3 + \beta + \gamma_2)((\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_2 + \gamma_1) + \alpha_1\beta) + (\alpha_2 + \alpha_3 + \beta + \gamma_1)
\]

\[
(\alpha_2 + \gamma_1)[\alpha_2(\alpha_1 + \gamma_2) + \alpha_2\beta] + (\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1)\alpha_2\alpha_3[\alpha_1\gamma_1(\alpha_1 + \alpha_3 + \beta + \gamma_2)
\]

\[
(\alpha_2 + \alpha_3 + \beta + \gamma_1)((\alpha_1 + \gamma_2)(\alpha_1 + \alpha_2 + \gamma_3) + (\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_2 + \gamma_1) - \alpha_1\gamma_1)
\]

\[
(\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1)\alpha_3[\alpha_2\gamma_2(\alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_2 + \gamma_1) + (\alpha_1 + \alpha_3 + \beta + \gamma_2)[\alpha_1\gamma_1 +
\]

\[
(\alpha_2 + \gamma_1)(\alpha_2 + \gamma_2)) + (\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1)\alpha_1\alpha_3[(\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_2\gamma_2 + (\alpha_1 + \gamma_2)
\]

\[
(\alpha_1 + \alpha_2 + \gamma_3) + (\alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_1 + \gamma_2) + \alpha_2\gamma_2(\alpha_2 + \alpha_3 + \beta + \gamma_1)
\]

128
\[
B_1 = [(\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1)(\alpha_1 + \alpha_2 + \gamma_3) - (\alpha_2 + \gamma_1)\alpha_2\gamma_2 - \alpha_1\gamma_1 (\alpha_1 + \gamma_2)]\{(\alpha_1 + \gamma_2) \\
(\alpha_2 + \gamma_1)(\alpha_1 + \alpha_2 + \alpha_3)(\alpha_2 + \alpha_3 + \beta + \gamma_1) - \alpha_1\gamma_1(\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1) - (\alpha_1 + \gamma_2) \\
(\alpha_2 + \gamma_1 - \beta)\alpha_1\gamma_1(\alpha_1 + \alpha_3 + \beta + \gamma_2) - (\alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_1 + \gamma_2 - \beta)\alpha_2\gamma_2 \\
(\alpha_2 + \gamma_1)] - (\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1)\alpha_3[(\alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_2 + \gamma_1)(\alpha_1(\alpha_1 + \gamma_2) \\
(\alpha_1 + \alpha_3 + \beta + \gamma_2) + \alpha_2\gamma_2\gamma_3) + \alpha_1\gamma_1\gamma_3(\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_1 + \gamma_2)]
\]

### 4.6 Availability Analysis

Define \( A_1(t) \) as the probability that the system is in up-state at epoch ‘t’ when it initially started from regenerative state \( S_1 \). To obtain recurrence relations among different point wise availabilities we use the simple probabilistic arguments.

As an illustration \( A_0(t) \) is the sum of the following probabilities:

1. The system remains up in state \( S_0 \) without making any transition to any other regenerative state up to time ‘t’, the probability of this event equals

\[
Z_0(t) = \exp[-(\alpha_1 + \alpha_2 + \alpha_3)t]
\]

2. The system transits from state \( S_0 \) to state \( S_1 \) during \((u, u + du), u \leq t \) and then starting at epoch \( u \) from \( S_1 \) it is available for remaining time \((t - u)\) the probability of this event is:

\[
\int_0^t q_{01}(u)A_1(t - u)\,du = q_{01}(t)\otimes A_1(t).
\]

3. The system transits from state \( S_0 \) to state \( S_3 \) during \((u, u + du), u \leq t \) and then starting at epoch \( u \) from \( S_3 \) it is available for remaining time \((t - u)\) the probability of this event is:

\[
\int_0^t q_{03}(u)A_3(t - u)\,du = q_{03}(t)\otimes A_3(t).
\]

4. The system transits from state \( S_0 \) to state \( S_7 \) during \((u, u + du), u \leq t \) and then starting at epoch \( u \) from \( S_7 \) it is available for remaining time \((t - u)\) the probability of this event is:
\[ \int_0^t q_{07}(u)A_7(t - u)\,du = q_{07}(t)\odot A_7(t). \]

Therefore \( A_0(t) \) becomes

\[ A_0(t) = Z_0(t) + q_{01}(t)\odot A_1(t) + q_{03}(t)\odot A_3(t) + q_{07}(t)\odot A_7(t) \]

By similar arguments, we have

\[
\begin{align*}
A_1(t) & = Z_1(t) + q_{10}(t)\odot A_0(t) + q_{12}(t)\odot A_2(t) + q_{15}(t)\odot A_5(t) + q_{18}(t)\odot A_8(t) \\
A_2(t) & = Z_2(t) + q_{20}(t)\odot A_0(t) + q_{25}(t)\odot A_5(t) \\
A_3(t) & = Z_3(t) + q_{30}(t)\odot A_0(t) + q_{34}(t)\odot A_4(t) + q_{35}(t)\odot A_5(t) + q_{36}(t)\odot A_6(t) \\
A_4(t) & = Z_4(t) + q_{40}(t)\odot A_0(t) + q_{45}(t)\odot A_5(t) \\
A_5(t) & = q_{53}(t)\odot A_3(t) \\
A_6(t) & = Z_6(t) + q_{67}(t)\odot A_7(t) + q_{69}(t)\odot A_9(t) \\
A_7(t) & = Z_7(t) + q_{70}(t)\odot A_0(t) + q_{76}(t)\odot A_6(t) + q_{78}(t)\odot A_8(t) \\
A_8(t) & = Z_8(t) + q_{87}(t)\odot A_7(t) + q_{89}(t)\odot A_9(t) \\
A_9(t) & = q_{96}(t)\odot A_6(t)
\end{align*}
\]

Taking the Laplace transform of above equations, we get

\[
\begin{align*}
A_0^*(s) & = Z_0^*(s) + q_{01}^*(s)A_1^*(s) + q_{03}^*(s)A_3^*(s) + q_{07}^*(s)A_7^*(s) \\
A_1^*(s) & = Z_1^*(s) + q_{10}^*(s)A_0^*(s) + q_{12}^*(s)A_2^*(s) + q_{15}^*(s)A_5^*(s) + q_{18}^*(s)A_8^*(s) \\
A_2^*(s) & = Z_2^*(s) + q_{20}^*(s)A_0^*(s) + q_{25}^*(s)A_5^*(s) \\
A_3^*(s) & = Z_3^*(s) + q_{30}^*(s)A_0^*(s) + q_{34}^*(s)A_4^*(s) + q_{35}^*(s)A_5^*(s) + q_{46}^*(s)A_6^*(s) \\
A_4^*(s) & = Z_4^*(s) + q_{40}^*(s)A_0^*(s) + q_{45}^*(s)A_5^*(s) \\
A_5^*(s) & = q_{43}^*(s)A_3^*(s) \\
A_6^*(s) & = Z_6^*(s) + q_{67}^*(s)A_7^*(s) + q_{69}^*(s)A_9^*(s) \\
A_7^*(s) & = Z_7^*(s) + q_{70}^*(s)A_0^*(s) + q_{76}^*(s)A_6^*(s) + q_{78}^*(s)A_8^*(s) \\
A_8^*(s) & = Z_8^*(s) + q_{87}^*(s)A_7^*(s) + q_{89}^*(s)A_9^*(s) \\
A_9^*(s) & = q_{96}^*(s)A_6^*(s)
\end{align*}
\]

On solving above equations, the Laplace-transformation of the point wise availability is

\[ A_0^*(s) = \frac{N_2(s)}{D_2(s)} \]
where

\[
N_2(s) = [(1 - q_{69}q_{96})*q_{78}q_{87} - q_{67}q_{78} - q_{89}q_{96}][(1 - q_{34}q_{45}q_{53} - q_{35}q_{53})
\]
\[+(Z_{6} + Z_{1q_{10}} + Z_{2q_{12}q_{12}}) + (Z_{3} + Z_{4q_{34}})(q_{01}q_{12}q_{25}q_{53} + q_{01}q_{15}q_{53} + q_{03})
\]+[Z_{6}(1 - q_{78}q_{87}) + Z_{7}q_{36}q_{67} + Z_{6}q_{36}q_{67}q_{78}]
\[+ (1 - q_{34}q_{45}q_{53} - q_{35}q_{53})Z_{6}(q_{01}q_{18}(q_{87}q_{76} + q_{89}q_{96}) + q_{07}(q_{76} + q_{78}q_{89}q_{96}))+Z_{7}(q_{01}q_{18}q_{87} + q_{01}q_{18}q_{96}(q_{87}q_{69} - q_{67}q_{69}) + q_{07}(1 - q_{69}q_{96})) + Z_{8}((1 - q_{67}q_{76} - q_{69}q_{96})q_{01}q_{18} + q_{07}q_{78}(1 - q_{69}q_{96}))
\]

\[N_{2}(s) = [(1 - q_{69}q_{96})*(1 - q_{78}q_{87}) - q_{67}q_{78} - q_{67}q_{78}q_{89}q_{96}][(1 - q_{34}q_{45}q_{53} - q_{35}q_{53})
\]
\[+(1 - q_{01}q_{10} - q_{01}q_{12}q_{20}) - (q_{01}q_{30} + q_{01}q_{34}q_{40})(q_{12}q_{25}q_{53} + q_{15}q_{53}) - q_{03}q_{30}
\] - q_{03}q_{34}q_{40} + q_{70}q_{01}q_{18}(1 - q_{34}q_{45}q_{53} - q_{35}q_{53}) \[q_{87} + q_{96}(q_{67}q_{89} - q_{87}q_{69}) - \]
\[+(1 - q_{34}q_{45}q_{53} - q_{35}q_{53})q_{07}q_{70}(1 - q_{69}q_{96}) - q_{36}q_{67}q_{70}q_{03} + q_{01}(q_{12}q_{25}q_{53}
\] + q_{15}q_{53}])

The steady state availability of the system will be given by

\[A_{0} = \lim_{t \to \infty} A_{0}(t) = \lim_{s \to 0} s A_{0}(s) = N_{2}(0)/D_{2}(0)
\]

As we know that, \(q_{ij}(t)\) is the pdf of the time of transition from state \(S_{i}\) to \(S_{j}\) and \(q_{ij}(t)dt\) is the probability of transition from state \(S_{i}\) to \(S_{j}\) during the interval \((t, t + dt)\), thus

\[q_{ij}^{*}(s)|_{s=0} = q_{ij}^{*}(0) = p_{ij}
\]

and

\[-q_{ij}^{*}(s)|_{s=0} = m_{ij}
\]

Also we know that

\[\lim_{s \to 0} Z_{i}^{*}(s) = f_{0}^\infty Z_{i}(t)dt = \mu_{i}
\]

Therefore, we have

\[Z_{0}^{*}(s) = \frac{1}{\beta_{1} + \beta_{2} + \beta_{3}} = \mu_{0}
\]

\[Z_{1}^{*}(s) = \frac{1}{\beta_{1} + \beta_{2} + \beta_{3}} = \mu_{1}
\]

\[Z_{2}^{*}(s) = \frac{1}{\gamma_{1} + \gamma_{2}} = \mu_{2}
\]

\[Z_{3}^{*}(s) = \frac{1}{\gamma_{2} + \beta_{1} + \beta_{3}} = \mu_{3}
\]
Using above relations and relations in (3), the expressions of $N_2(0)$ and $D_2(0)$ becomes

\[
N_2(0) = \left[ p_{67}(1 - p_{78}p_{87}) - p_{67}p_{76} - p_{67}p_{78}p_{89} \right] \left[ (1 - p_{34}p_{45} - p_{35}) (\mu_0 + \mu_1 p_{10} + \mu_2 p_{12} + \mu_3 + \mu_4 p_{34}) (p_{01} p_{12} p_{25} + p_{01} p_{15} + p_{03}) \right] + [\mu_6 p_{36} (1 - p_{78} p_{87}) \\
\mu_7 p_{36} p_{67} + \mu_8 p_{36} p_{67} p_{78} (p_{01} p_{12} p_{25} + p_{01} p_{15} + p_{03}) + (1 - p_{34} p_{45} - p_{35}) \\
[\mu_6 (p_{01} p_{18} (p_{87} p_{76} + p_{89}) + p_{07} (p_{76} + p_{78} p_{89})) + \mu_7 (p_{01} p_{18} p_{87} + p_{07} p_{67} + p_{01} p_{18} (p_{67} p_{89} - p_{87} p_{69})) + \mu_8 \{(1 - p_{67} p_{76} - p_{69}) p_{01} p_{18} + p_{07} p_{78} p_{67}] \\
\right] (19)
\]

and

\[
D_2(0) = [p_{67}(1 - p_{78} p_{87}) - p_{67} p_{76} - p_{67} p_{78} p_{89} [1 - p_{34} p_{45} - p_{35} + p_{01} p_{10} p_{34} p_{45} - \\
\mu_1 p_{10} + p_{01} p_{10} p_{35} - p_{01} p_{12} p_{20} + p_{01} p_{12} p_{20} p_{34} p_{45} + p_{01} p_{12} p_{20} p_{35} - p_{03} p_{30} \\
- p_{30} p_{01} p_{12} p_{25} - p_{30} p_{01} p_{15} + p_{34} p_{40} p_{01} p_{12} p_{25} - p_{34} p_{40} p_{01} p_{15} + p_{34} p_{40} p_{01}] \\
- p_{70} p_{01} p_{12} p_{25} p_{36} p_{67} - p_{70} p_{01} p_{15} p_{36} p_{67} - p_{70} p_{01} p_{18} p_{67} + p_{70} p_{01} p_{34} p_{45} p_{18} \\
p_{87} - (p_{70} p_{01} p_{18} - p_{70} p_{01} p_{34} p_{45} p_{18} - p_{70} p_{01} p_{35} p_{18}) (p_{67} p_{9} - p_{87} p_{69}) + p_{70} \\
p_{01} p_{35} p_{18} p_{67} + p_{07} p_{70} p_{34} p_{45} p_{67} + p_{07} p_{70} p_{35} p_{53} p_{67} - p_{07} p_{70} p_{67} - p_{70} p_{03} p_{36} p_{67} \\
\right] = p_{67} p_{70} \left[ 1 - p_{34} p_{45} - p_{35} + p_{01} p_{10} p_{34} p_{45} - p_{01} p_{10} p_{35} - p_{01} p_{12} p_{20} \\
+ p_{01} p_{12} p_{20} p_{34} p_{45} + p_{01} p_{12} p_{20} p_{35} - p_{03} p_{30} - p_{30} p_{01} p_{12} p_{25} - p_{30} p_{01} p_{15} - \\
p_{34} p_{40} p_{01} p_{12} p_{25} - p_{34} p_{40} p_{01} p_{15} + p_{34} p_{40} p_{03} - p_{70} p_{01} p_{12} p_{25} p_{36} p_{67} - \\
p_{70} p_{01} p_{15} p_{36} p_{67} - p_{67} p_{70} p_{01} p_{18} p_{67} - p_{67} p_{70} p_{01} p_{18} p_{89} + p_{67} p_{70} p_{01} p_{34} p_{45} \\
p_{18} p_{87} - p_{67} p_{70} p_{01} p_{34} p_{45} p_{18} p_{89} - p_{67} p_{70} p_{01} p_{35} p_{18} p_{87} + p_{67} p_{70} p_{01} p_{35} \\
p_{18} p_{89} + p_{07} p_{70} p_{34} p_{45} p_{67} + p_{07} p_{70} p_{35} p_{53} p_{67} - p_{07} p_{70} p_{67} - p_{70} p_{03} p_{36} p_{67} \\
\right] = p_{67} p_{70} \left[ 1 - p_{34} p_{45} - p_{35} - p_{01} p_{10} + p_{01} p_{10} p_{34} p_{45}\{(p_{10} + p_{18}) + p_{01} p_{10} p_{35} - \\
p_{01} p_{12} p_{20} + p_{01} p_{12} p_{34} p_{45} - p_{01} p_{12} p_{25} p_{34} p_{45} + p_{01} p_{12} p_{20} p_{35} - p_{03} p_{30} - p_{30} p_{01} p_{12} p_{25} - p_{30} p_{01} p_{15} - \\
p_{34} p_{40} p_{01} p_{12} p_{25} - p_{34} p_{40} p_{01} p_{15} - p_{34} p_{40} p_{03} - p_{70} p_{01} p_{12} p_{25} p_{36} p_{67} - \\
p_{70} p_{01} p_{15} p_{36} p_{67} - p_{67} p_{70} p_{01} p_{18} p_{67} - p_{67} p_{70} p_{01} p_{18} p_{89} + p_{67} p_{70} p_{01} p_{34} p_{45} \\
p_{18} p_{87} - p_{67} p_{70} p_{01} p_{34} p_{45} p_{18} p_{89} - p_{67} p_{70} p_{01} p_{35} p_{18} p_{87} + p_{67} p_{70} p_{01} p_{35} \\
p_{18} p_{89} + p_{07} p_{70} p_{34} p_{45} p_{67} + p_{07} p_{70} p_{35} p_{53} p_{67} - p_{07} p_{70} p_{67} - p_{70} p_{03} p_{36} p_{67} \\
\right]
\]
\[ = p_{67} p_{70} [1 - p_{34} p_{45} - p_{35} + p_{01} p_{34} p_{45} - p_{01} p_{15} (p_{34} + p_{36} + p_{35}) - p_{01} p_{12} p_{25} + p_{01} p_{12} p_{25} p_{34} p_{40} - p_{01} p_{10} + p_{01} p_{10} p_{35} - p_{01} p_{12} + p_{01} p_{10} p_{25} + p_{01} p_{10} p_{35} - p_{03} p_{30} - p_{34} p_{40} p_{01} p_{12} p_{25} - p_{34} p_{40} p_{03} - p_{01} p_{18} + p_{01} p_{35} p_{18} + p_{07} p_{34} p_{45} + p_{07} p_{35} - p_{03} p_{36} - p_{07}] \]

\[ = p_{67} p_{70} [1 - p_{34} p_{45} (1 - p_{01} + p_{07}) - p_{01} + p_{01} p_{35} (p_{15} + p_{10} + p_{12} + p_{18}) - p_{35} - p_{03} p_{30} - p_{34} p_{40} p_{03} + p_{07} p_{35} - p_{03} p_{36} - p_{07}] \]

\[ = p_{67} p_{70} [1 - p_{03} p_{34} p_{45} - p_{01} + p_{01} p_{35} - p_{35} - p_{03} p_{30} - p_{34} p_{40} p_{03} + p_{07} p_{35} - p_{03} p_{36} - p_{07}] \]

\[ = p_{67} p_{70} [1 - p_{01} - p_{35} (1 - p_{01} + p_{07}) - p_{03} p_{34} - p_{03} p_{30} + p_{07} p_{35} - p_{03} p_{36} - p_{07}] \]

\[ = p_{67} p_{70} [1 - p_{01} - p_{03} (p_{35} + p_{34} + p_{30} + p_{36}) - p_{07}] \]

\[ = p_{67} p_{70} [1 - p_{01} - p_{03} - p_{07}] \]

\[ = 0, \text{ as it should be} \]

The steady state probability that the system will be up in the long run is given by

\[ A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} s A_0^s(s) \]

\[ A_0 = \lim_{s \to 0} \frac{s N_2(s)}{D_2(s)} \]

As \( s \to 0 \), \( D_2(s) \) becomes zero. Thus above equation becomes indeterminate form.

Hence on using L’Hospital’s rule, \( A_0 \) becomes

\[ A_0 = \frac{N_2(0)}{D_2'(0)} \quad (20) \]

Where \( N_2(0) \) is given by equation (19)

To obtain \( D_2'(0) \), we collect the coefficient of relevant \( m_{ij}'s \) where \( m_{ij}'s \) is the mean elapsed time of the system in state \( S_i \) before transiting to state \( S_j \).

In notation

\[ m_{ij} = -q_{ij}'(s)|_{s=0} \]
and

\[ \sum m_{ij} = \mu_i \]

Coefficient of \( m_{01} = [p_{67}p_{70}][p_{10} - p_{10}p_{34}p_{45} - p_{10}p_{35} + p_{12}p_{20} - p_{12}p_{20}p_{34}p_{45} - p_{12}p_{20}p_{35} + p_{12}p_{25}p_{30} + p_{15}p_{30} + p_{34}p_{40}p_{12}p_{25} + p_{34}p_{40}p_{15}] + p_{12}p_{25}p_{35} + p_{36}p_{67}p_{70} + p_{15}p_{36}p_{67}p_{70} - p_{18}p_{34}p_{45}p_{87}p_{70} + p_{70}p_{18}(p_{67}p_{89} - p_{87}p_{69}) - p_{18}p_{34}p_{45}p_{70}(p_{67}p_{89} - p_{87}p_{69}) + p_{18}p_{87}p_{70} - p_{18}p_{35}p_{70}(p_{67}p_{89} - p_{87}p_{69}) - p_{18}p_{87}p_{70}p_{35} \]

\[ = [p_{67}p_{70}][p_{10} - p_{10}p_{34}p_{45} - p_{10}p_{35} + p_{12}p_{20} - p_{12}p_{20}p_{34}p_{45} - p_{12}p_{20}p_{35} + p_{12}p_{25}p_{30} + p_{15}p_{30} + p_{34}p_{40}p_{12}p_{25} + p_{34}p_{40}p_{15}] + p_{12}p_{25}p_{36} + p_{67}p_{70} + p_{15}p_{36}p_{67}p_{70} + p_{18}p_{67}p_{70} - p_{18}p_{34}p_{45}p_{67}p_{70} - p_{18}p_{35}p_{67}p_{70} \]

\[ = [p_{67}p_{70}][p_{10} - p_{10}p_{34}(p_{10} + p_{12} + p_{18} + p_{15}) - p_{10}p_{35} + p_{12}p_{20} + p_{12}p_{25}p_{34} - p_{12}p_{20}p_{35} + p_{12}p_{25}p_{30} + p_{15}(p_{30} + p_{34}p_{36}) + p_{18} + p_{12}p_{25}p_{36} - p_{18}p_{35}] \]

\[ = [p_{67}p_{70}][p_{10} + p_{15} + p_{18} + p_{34}p_{45} - p_{10}p_{35} + p_{12}p_{20} - p_{12}p_{35} + p_{12}p_{25}(p_{30} + p_{34}p_{35} + p_{36}) - p_{12}p_{35} - p_{15}p_{35} - p_{18}p_{35}] \]

\[ = [p_{67}p_{70}][1 - p_{12} - p_{34}p_{45} + p_{35}(p_{10} + p_{12} + p_{18} + p_{15}) + p_{12}] \]

\[ = [p_{67}p_{70}][1 - p_{35} - p_{34}p_{45}] \]

Coefficient of \( m_{03} = [p_{67}p_{70}][1 - p_{35} - p_{34}p_{45}] \]

Coefficient of \( m_{07} = [p_{67}p_{70}][1 - p_{35} - p_{34}p_{45}] \]

Coefficient of \( m_{10} = [p_{67}p_{70}][p_{01} - p_{01}p_{34}p_{45} - p_{01}p_{35}] \]

Coefficient of \( m_{12} = [p_{67}p_{70}][p_{01}p_{20} - p_{01}p_{20}p_{34}p_{45} - p_{01}p_{20}p_{35} + p_{01}p_{30}p_{25} + p_{01}p_{25}p_{34}p_{40} + p_{01}p_{25}p_{36}] \]

\[ = [p_{67}p_{70}][p_{01}p_{20} - p_{01}p_{34}p_{45} + p_{01}p_{25}p_{34}p_{45} + p_{01}p_{25}p_{34}p_{40} - p_{01}p_{35} - p_{01}p_{25}(p_{30} + p_{35} + p_{36})] \]

\[ = [p_{67}p_{70}][p_{01} - p_{01}p_{25}p_{34} - p_{01}p_{34}p_{45} - p_{01}p_{35} + p_{01}p_{25}p_{34} + \]
\[ \begin{align*}
\text{Coefficient of } m_{15} &= [p_{67}p_{70}][p_{01}p_{34}p_{40} - p_{01}p_{34}p_{45} - p_{01}p_{35}] \\
&= [p_{67}p_{70}][p_{01}p_{30}p_{53} + p_{01}p_{34}p_{40}p_{53} - p_{01}p_{53}p_{36}] \\
&= [p_{67}p_{70}][p_{01} - p_{01}p_{34} - p_{01}p_{35} + p_{01}p_{34} - p_{01}p_{34}p_{45}] \\
&= [p_{67}p_{70}][p_{01} - p_{01}p_{34}p_{45} - p_{01}p_{35}] \\
\text{Coefficient of } m_{18} &= [p_{70}p_{01}(1 - p_{34}p_{45} - p_{35})][p_{87} + p_{96}(p_{67}p_{89} - p_{67}p_{69})] \\
&= [p_{70}p_{01}p_{67} - p_{70}p_{01}p_{34}p_{45}p_{67} - p_{70}p_{01}p_{35}p_{67}] \\
&= [p_{67}p_{70}][p_{01} - p_{01}p_{34}p_{45} - p_{01}p_{35}] \\
\text{Coefficient of } m_{20} &= [p_{67}p_{70}][p_{01}p_{12} - p_{01}p_{12}p_{34}p_{45} - p_{01}p_{12}p_{35}] \\
&= [p_{67}p_{70}p_{01}p_{12}][1 - p_{34}p_{45} - p_{01}p_{35}] \\
\text{Coefficient of } m_{25} &= [p_{67}p_{70}][p_{01}p_{12}p_{34}p_{40} + p_{01}p_{12}p_{30}] + p_{01}p_{12}p_{36}p_{67}p_{70} \\
&= [p_{01}p_{12}p_{67}p_{70}][p_{34}p_{40} + p_{30} + p_{36}] \\
&= [p_{67}p_{70}p_{01}p_{12}][1 - p_{34}p_{45} - p_{01}p_{35}] \\
\text{Coefficient of } m_{30} &= [p_{67}p_{70}][p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{3}] \\
\text{Coefficient of } m_{34} &= [p_{67}p_{70}][p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{3}] \\
\text{Coefficient of } m_{35} &= [p_{67}p_{70}][1 - p_{01}p_{10} + p_{01}p_{12}p_{20}] - p_{01}p_{18}p_{70}[p_{87} + p_{67}p_{89} - p_{69}p_{87}] - p_{67}p_{70}p_{07} \\
&= [p_{67}p_{70}][p_{03} + p_{10} - p_{01}(p_{10} + p_{12} + p_{18}) + p_{01}p_{12}p_{25}] \\
&= [p_{67}p_{70}][p_{03} + p_{10} - p_{01}(1 - p_{15}) + p_{01}p_{12}p_{25}] \\
&= [p_{67}p_{70}][p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{03}] \\
\text{Coefficient of } m_{36} &= [p_{67}p_{70}][p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{03}] \\
\text{Coefficient of } m_{40} &= [p_{67}p_{70}][p_{01}p_{12}p_{25}p_{34} + p_{01}p_{34}p_{15} + p_{03}p_{34}] \\
&= [p_{34}p_{67}p_{70}][p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{03}] \\
\end{align*} \]
Coefficient of $m_{45} = [p_{67}p_{70}][p_{34} - p_{01}p_{10}p_{34} - p_{01}p_{12}p_{25}p_{34}] - p_{07}p_{70}p_{34}p_{67} - [p_{87} + (p_{67}p_{89} - p_{87}p_{69})]p_{01}p_{18}p_{34}p_{70} = [p_{67}p_{70}][p_{34}(1 - p_{07}) - p_{01}p_{10}p_{34} - p_{01}p_{12}p_{20}p_{34} - p_{01}p_{18}p_{34}] = [p_{67}p_{70}][p_{01}p_{34} - p_{01}p_{34} - p_{01}p_{15}p_{34} - p_{01}p_{12}p_{25}p_{34} - p_{03}p_{34}] = [p_{34}p_{67}p_{70}][p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{03}]$

Coefficient of $m_{53} = [p_{67}p_{70}][p_{34}p_{45} + p_{35} - p_{01}p_{10}p_{34}p_{45} - p_{01}p_{12}p_{20}p_{34}p_{45} - p_{01}p_{10}p_{35} - p_{01}p_{12}p_{20}p_{35} + p_{01}p_{12}p_{25}p_{30} + p_{01}p_{15}p_{30} + p_{01}p_{12}p_{25}p_{34}p_{40} + p_{01}p_{15}p_{34}p_{40}] + p_{01}p_{12}p_{25}p_{36}p_{67}p_{70} + p_{01}p_{15}p_{36}p_{67}p_{70} + p_{01}p_{18}p_{70}[p_{87} + (p_{67}p_{89} - p_{87}p_{69})][p_{35} + p_{34}p_{45}] - p_{07}p_{67}p_{70}[p_{35} + p_{34}p_{45}] = [p_{35} + p_{34}p_{45}]p_{67}p_{70}[1 - p_{01}p_{10} - p_{07} - p_{01}p_{12}p_{20}] + [p_{15} + p_{12}p_{25}]p_{67}p_{70}[p_{01}p_{30} + p_{01}p_{34}p_{40} + p_{01}p_{36}] + p_{01}p_{18}p_{70}[p_{87} + (p_{67}p_{89} - p_{87}p_{69})][p_{35} + p_{34}p_{45}]

Coefficient of $m_{67} = [p_{78}p_{89} + p_{76}][1 - p_{34}p_{45} - p_{35} + p_{01}p_{10} - p_{01}p_{10}p_{34}p_{45} - p_{01}p_{10}p_{35} + p_{01}p_{12}p_{20} + p_{01}p_{12}p_{20}p_{34}p_{45} + p_{01}p_{12}p_{20}p_{35}] + p_{01}p_{12}p_{25}p_{30} + p_{01}p_{15}p_{30} - p_{03}p_{30} - p_{01}p_{12}p_{25}p_{34}p_{40} - p_{01}p_{15}p_{34}p_{40} - p_{01}p_{34}p_{40} + p_{01}p_{12}p_{25}p_{36}p_{67}p_{70} + p_{03}p_{36}p_{70} + p_{01}p_{15}p_{36}p_{67}p_{70} + p_{01}p_{18}p_{70}p_{89}(1 - p_{34}p_{45} - p_{35}) = [p_{78}p_{89} + p_{76}][p_{07} + p_{01}p_{18}][1 - p_{34}p_{45} - p_{35}] + p_{03}p_{36} + p_{01}p_{36}(p_{15} + p_{12}p_{25}) + p_{01}p_{36}p_{67}p_{70}(p_{15} + p_{12}p_{25}) + p_{03}p_{36}p_{70} + p_{01}p_{18}p_{70}p_{89}(1 - p_{34}p_{45} - p_{35})

Coefficient of $m_{69} = [p_{78}p_{89} + p_{76}][(p_{07} + p_{01}p_{18})(1 - p_{34}p_{45} - p_{35}) + p_{03}p_{36} + p_{01}p_{36}(p_{15} + p_{12}p_{25}) + p_{01}p_{36}p_{67}p_{70}(p_{15} + p_{12}p_{25}) + p_{03}p_{36}p_{70} + p_{01}p_{18}p_{70}p_{89}(1 - p_{34}p_{45} - p_{35})

Coefficient of $m_{70} = [p_{67}][1 - p_{34}p_{45} - p_{35} + p_{01}p_{10} - p_{01}p_{10}p_{34}p_{45} - p_{01}p_{10}p_{35} - p_{01}p_{12}p_{20} + p_{01}p_{12}p_{20}p_{34}p_{45} + p_{01}p_{12}p_{20}p_{35} - p_{01}p_{12}p_{25}p_{30} -
\[ p_{01}p_{15}p_{30} - p_{03}p_{30} - p_{01}p_{12}p_{25}p_{34}p_{40} - p_{01}p_{15}p_{34}p_{40} - p_{03}p_{34}p_{40} \]
\[ = [p_{67}][1 - p_{30} + p_{34} + p_{35} - p_{07}p_{34}p_{45} + p_{07} - p_{36} + p_{03}p_{36} - p_{07}p_{35} + p_{01}p_{18} - p_{01}p_{18}p_{35} + p_{01}p_{15}p_{36} + p_{01}p_{12}p_{25}p_{36} - p_{01}p_{18}p_{34}p_{45} \]
\[ = [p_{67}][p_{01}p_{36}(p_{15} + p_{12}p_{25}) + (p_{07} + p_{01}p_{18})(1 - p_{34}p_{45} - p_{35}) + p_{03}p_{36}] \]

Coefficient of \( m_{76} = [p_{67}][p_{01}p_{36}(p_{15} + p_{12}p_{25}) + (p_{07} + p_{01}p_{18})(1 - p_{34}p_{45} - p_{35}) + p_{03}p_{36}] \)

Coefficient of \( m_{78} = [p_{67}][p_{01}p_{36}(p_{15} + p_{12}p_{25}) + (p_{07} + p_{01}p_{18})(1 - p_{34}p_{45} - p_{35}) + p_{03}p_{36}] \)

Coefficient of \( m_{87} = [p_{67}p_{78}][p_{01}p_{36}(p_{15} + p_{12}p_{25}) + (p_{07} + p_{01}p_{18})(1 - p_{34}p_{45} - p_{35}) + p_{03}p_{36}] \)

Coefficient of \( m_{89} = [p_{67}p_{78}][p_{01}p_{36}(p_{15} + p_{12}p_{25}) + (p_{07} + p_{01}p_{18})(1 - p_{34}p_{45} - p_{35}) + p_{03}p_{36}] \)

Coefficient of \( m_{96} = [p_{67}p_{78}p_{89} + p_{69}(1 - p_{78}p_{87})][p_{01}p_{12}p_{25}p_{36} + p_{01}p_{15}p_{25} + p_{01}p_{18} - p_{01}p_{18}p_{34}p_{45} - p_{01}p_{18}p_{35} - p_{07}p_{34}p_{45} - p_{07}p_{35} + p_{03}p_{36} + p_{07}] + p_{01}p_{18}p_{70}(1 - p_{34}p_{45} - p_{35})(p_{67}p_{89} - p_{87}p_{69}) + p_{07}p_{70}p_{69}(1 - p_{35} - p_{34}p_{45}) \]
\[ = [p_{67}p_{78}p_{89} + p_{69}(1 - p_{78}p_{87})][p_{01}p_{36}(p_{15} + p_{12}p_{25}) + p_{03}p_{36} + (p_{07} + p_{01}p_{18})(1 - p_{34}p_{45} - p_{35})][p_{01}p_{18}p_{70}(p_{67}p_{89} - p_{87}p_{69}) + p_{07}p_{70}p_{69}(1 - p_{34}p_{45} - p_{35})] \]

Therefore, on collecting the above coefficients and using the relations (3), Denominator of (20) becomes

Therefore \( D_2'(0) = [1 - p_{34}p_{45} - p_{35}]p_{67}p_{70}[\mu_0 + \mu_1p_{01} + \mu_1p_{01}p_{12}] + (\mu_0 + \mu_4p_{34}) \)
Now by substituting the values of $\mu_i$'s and $p_{ij}$'s in (20) we get the expressions for numerator and denominator of steady state availability as

\[N_2(0) = (\gamma_2 \gamma_3)(\alpha_2 + \gamma_1) \gamma_1[\{(\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_1 + \gamma_2) - \alpha_1 \beta - \alpha_1(\alpha_1 + \gamma_2)\}(\alpha_2 + \gamma_1)\]
\[(\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1(\alpha_2 + \gamma_1) + \alpha_1 \beta + \{\alpha_1(\alpha_1 + \gamma_2) + \beta\}{\alpha_2 \beta + (\alpha_2 + \gamma_1)}\]
\[(\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1)\alpha_2 + \gamma_1\gamma_1[\{(\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_1 + \alpha_2 + \gamma_1) - \alpha_1 \gamma_1\} + \alpha_2 \alpha_3 \gamma_2 + \]
\[(\alpha_2 + \gamma_1)\gamma_3 \gamma_2[\alpha_2 \beta + \alpha_2(\alpha_2 + \gamma_1)(\alpha_1 + \gamma_2) + \alpha_2 \alpha_3 + \beta + \gamma_1)]\]
\[\{(\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_1 + \gamma_2) - \alpha_1(\alpha_1 + \gamma_2 + \beta)\}[\alpha_1 \alpha_3 \gamma_2 + \alpha_1 \alpha_3(\alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_1 + \gamma_2) + \alpha_1 \alpha_3 \gamma_2(\alpha_2 + \gamma_1) - \gamma_1(\alpha_1 + \gamma_2)] + \alpha_1 \alpha_3 \]
\[(\alpha_1 + \alpha_2 + \gamma_1) - \alpha_2 \gamma_2 - \alpha_1(\alpha_1 + \alpha_2 + \gamma_3)] + (\alpha_2 + \alpha_3 + \beta + \gamma_1)\]
\[\alpha_1 \alpha_3 \gamma_2\]

\[D_2(0) = (\alpha_1 + \alpha_2 + \alpha_3)\gamma_1(\alpha_2 + \gamma_1)[(\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_1 + \gamma_2) - \alpha_1 \beta - \alpha_1(\alpha_1 + \gamma_2)]\]
\[(\alpha_2 + \alpha_3 + \beta + \gamma_1)\gamma_2 \gamma_3(\alpha_2 + \gamma_1) + \alpha_1(\alpha_2 + \gamma_1) + \alpha_1 \beta + \gamma_1 \gamma_2 \gamma_3 \gamma_1(\alpha_1 + \gamma_2 + \beta)\]
\[(\alpha_2 + \gamma_1)[\alpha_2 \beta + \alpha_1 \alpha_2(\alpha_2 + \gamma_1)(\alpha_2 + \gamma_1) + \alpha_2 \alpha_3 + \beta + \gamma_1] + (\alpha_2 + \gamma_1)\]
\[[(\alpha_1 + \alpha_2 + \gamma_1) + \alpha_1 \beta) \gamma_2 \gamma_3(\{(\alpha_1 + \alpha_2 + \alpha_3)(\alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_2 + \gamma_1) - \alpha_1 \gamma_1\)
\[\alpha_2 + \gamma_1) - \alpha_3(\alpha_2 + \gamma_1)(\alpha_2 + \alpha_3 + \beta + \gamma_1) - \alpha_1 \gamma_1 \beta) + \alpha_1 \alpha_2 \gamma_2 \gamma_3(\alpha_2 + \gamma_1) + (\alpha_2 + \gamma_1)\]
\[\{(\alpha_1 + \alpha_2 + \gamma_2) + \alpha_1 \alpha_3 \gamma_3 \gamma_3(\alpha_1(\alpha_1 + \gamma_2 + \beta)\}[\gamma_2(\alpha_2 + \gamma_1)] + \gamma_1 \alpha_1 \alpha_3 \]
\[(\alpha_1 + \gamma_2) + \alpha_2(\alpha_2 + \gamma_1 + \beta) + \gamma_1 \gamma_2(\alpha_2 + \gamma_1) + \alpha_2 \alpha_3 \gamma_1(\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1 \alpha_3 \gamma_1(\alpha_1 + \gamma_2)\]
\[(\alpha_1 + \alpha_2 + \gamma_2) - \alpha_1(\alpha_1 + \gamma_2) - \alpha_1 \beta) + \alpha_2 \alpha_3(\alpha_2 + \alpha_3 + \beta + \gamma_1)\]
\[\alpha_2 + \gamma_1)\]
\[\{(\alpha_1 + \alpha_2 + \gamma_1) + \alpha_1 \beta) \gamma_2 \gamma_3(\{(\alpha_1 + \alpha_2 + \alpha_3)(\alpha_2 + \alpha_3 + \beta + \gamma_1)(\alpha_2 + \gamma_1) - \alpha_1 \gamma_1\)
\[\alpha_2 + \gamma_1) - \alpha_3(\alpha_2 + \gamma_1)(\alpha_2 + \alpha_3 + \beta + \gamma_1) - \alpha_1 \gamma_1 \beta) + \alpha_1 \alpha_2 \gamma_2 \gamma_3(\alpha_2 + \gamma_1) + (\alpha_2 + \gamma_1)\]
\[\{(\alpha_1 + \alpha_2 + \gamma_2) + \alpha_1 \alpha_3 \gamma_3(\alpha_1(\alpha_1 + \gamma_2 + \beta)\}[\gamma_2(\alpha_2 + \gamma_1)] + \gamma_1 \alpha_1 \alpha_3 \]
\[(\alpha_1 + \gamma_2) + \alpha_2(\alpha_2 + \gamma_1 + \beta) + \gamma_1 \gamma_2(\alpha_2 + \gamma_1) + \alpha_2 \alpha_3 \gamma_1(\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1 \alpha_3 \gamma_1(\alpha_1 + \gamma_2)\]
\[(\alpha_1 + \alpha_2 + \gamma_2) - \alpha_1(\alpha_1 + \gamma_2) - \alpha_1 \beta) + \alpha_2 \alpha_3(\alpha_2 + \alpha_3 + \beta + \gamma_1)\]
\[\alpha_2 + \gamma_1)\]} + \gamma_1 \gamma_2(\alpha_2 + \gamma_1) + \alpha_1 \alpha_2 \gamma_2 \gamma_3(\alpha_2 + \gamma_1) + \alpha_1 \alpha_2 \gamma_2 \gamma_3(\alpha_2 + \gamma_1)\]
\[(\alpha_1 + \alpha_2 + \gamma_3) - \alpha_1 \gamma_1)] + \gamma_1(\alpha_2 + \gamma_1)[\alpha_1 \alpha_3 \gamma_2 \gamma_3(\alpha_2(\alpha_2 + \gamma_1 + \beta) + \alpha_1 \alpha_3 \gamma_3 \gamma_3(\alpha_1 + \gamma_2 + \beta) + \alpha_2 \alpha_3(\alpha_1 + \gamma_2) + \alpha_2 \alpha_3(\alpha_1 + \gamma_2)\]
\[\alpha_1(\alpha_1 + \gamma_2 + \beta)\] + \alpha_2 \alpha_3 \gamma_3(\alpha_1 + \gamma_2)(\alpha_2 + \gamma_1)\]
(\alpha_2 + \alpha_3 + \beta + \gamma_1) + (\alpha_2 + \gamma_1)[\alpha_1\alpha_3\gamma_3(\alpha_1 + \gamma_2)(\gamma_2 - \gamma_1)(\alpha_1 + \alpha_3 + \beta + \gamma_2) \\
(\alpha_1 + \gamma_2) + \alpha_1\alpha_3\gamma_3((\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_1 + \gamma_2) - \alpha_1(\alpha_1 + \gamma_2 + \beta))(\alpha_2 + \gamma_1) \\
(\alpha_2 + \alpha_3 + \beta + \gamma_1)

4.7 BUSY PERIOD ANALYSIS

Let \( Z_4(t) \) be the probability that the system up initially in regenerative state \( S_i \) remains up continuously till time \( t \) without passing through any other regenerative state or returning to itself.

Therefore we have

\[
\begin{align*}
Z_0(t) &= \exp[-(\alpha_1 + \alpha_2 + \alpha_3)t] \\
Z_1(t) &= \exp[-(\gamma_1 + \beta + \alpha_2 + \alpha_3)t] \\
Z_2(t) &= \exp[-(\gamma_1 + \alpha_2)t] \\
Z_3(t) &= \exp[-(\gamma_2 + \beta + \alpha_1 + \alpha_3)t] \\
Z_4(t) &= \exp[-(\gamma_2 + \alpha_1)t] \\
Z_5(t) &= \exp[-(\gamma_1)t] \\
Z_6(t) &= \exp[-(\gamma_2 + \alpha_1)t] \\
Z_7(t) &= \exp[-(\gamma_3 + \alpha_1 + \alpha_2)t] \\
Z_8(t) &= \exp[-(\gamma_1 + \alpha_2)t] \\
Z_9(t) &= \exp[-(\gamma_1)t]
\end{align*}
\]  

\hspace{2cm} (22)

Let \( B_i(t) \) be the probability that the system is under repair at epoch \( t \) given that the system entered regenerative state \( S_i \) at \( t = 0 \). Now we will determine these probabilities. To illustrate the calculations we consider \( B_0(t) \) and similar arguments may be employed for other probabilities.

\( B_0(t) \) consists of the sum of the following independent contingencies and their respective probabilities:

1. The system transits to state \( S_1 \) from \( S_0 \) during \( (u, u + du) \), \( u \leq t \) and then repairman may be found busy at epoch \( (t - u) \), starting from \( S_1 \), the probability of this event is

\[
\int_0^t q_{01}(t)B_1(t - u)du = q_{01}(t)\circ B_1(t)
\]

2. The system transits to state \( S_3 \) from \( S_0 \) during \( (u, u + du) \), \( u \leq t \) and then repairman may be found busy at epoch \( (t - u) \), starting from \( S_3 \), the probability of this event is

\[
\int_0^t q_{03}(t)B_3(t - u)du = q_{03}(t)\circ B_3(t)
\]

3. The system transits to state \( S_7 \) from \( S_0 \) during \( (u, u + du) \), \( u \leq t \) and then repairman may be found busy at epoch \( (t - u) \), starting from \( S_2 \), the probability of this event is

\[
\int_0^t q_{07}(t)B_7(t - u)du = q_{07}(t)\circ B_7(t)
\]

139
The expression for $B_0(t)$ becomes

$$B_0(t) = q_{01}(t) B_1(t) + q_{03}(t) B_3(t) + q_{07}(t) B_7(t)$$

By similar arguments we have

$$
\begin{align*}
B_1(t) &= Z_1(t) + q_{10}(t) B_0(t) + q_{12}(t) B_2(t) + q_{15}(t) B_5(t) + q_{18}(t) B_8(t) \\
B_2(t) &= Z_2(t) + q_{20}(t) B_0(t) + q_{25}(t) B_5(t) \\
B_3(t) &= Z_3(t) + q_{30}(t) B_0(t) + q_{34}(t) B_4(t) + q_{35}(t) B_5(t) + q_{36}(t) B_6(t) \\
B_4(t) &= Z_4(t) + q_{40}(t) B_0(t) + q_{45}(t) B_5(t) \\
B_5(t) &= Z_5(t) + q_{53}(t) B_3(t) \\
B_6(t) &= Z_6(t) + q_{67}(t) B_7(t) + q_{69}(t) B_9(t) \\
B_7(t) &= Z_7(t) + q_{70}(t) B_0(t) + q_{76}(t) B_6(t) + q_{78}(t) B_9(t) \\
B_8(t) &= Z_8(t) + q_{87}(t) B_7(t) + q_{89}(t) B_9(t) \\
B_9(t) &= Z_9(t) + q_{96}(t) B_6(t)
\end{align*}
$$

(23)

Taking Laplace transform of these relations, we get

$$
\begin{align*}
B'_0(s) &= q_{01}(s) B'_1(s) + q_{03}(s) B'_3(s) + q_{07}(s) B'_7(s) \\
B'_1(s) &= Z'_1(s) + q_{10}(s) B'_0(s) + q_{12}(s) B'_2(s) + q_{15}(s) B'_5(s) + q_{18}(s) B'_8(s) \\
B'_2(s) &= Z'_2(s) + q_{20}(s) B'_0(s) + q_{25}(s) B'_5(s) \\
B'_3(s) &= Z'_3(s) + q_{30}(s) B'_0(s) + q_{34}(s) B'_4(s) + q_{35}(s) B'_5(s) + q_{36}(s) B'_6(s) \\
B'_4(s) &= Z'_4(s) + q_{40}(s) B'_0(s) + q_{45}(s) B'_5(s) \\
B'_5(s) &= Z'_5(s) + q_{53}(s) B'_3(s) \\
B'_6(s) &= Z'_6(s) + q_{67}(s) B'_7(s) + q_{69}(s) B'_9(s) \\
B'_7(s) &= Z'_7(s) + q_{70}(s) B'_0(s) + q_{76}(s) B'_6(s) + q_{78}(s) B'_9(s) \\
B'_8(s) &= Z'_8(s) + q_{87}(s) B'_7(s) + q_{89}(s) B'_9(s) \\
B'_9(s) &= Z'_9(s) + q_{96}(s) B'_6(s)
\end{align*}
$$

(24)

On solving above equations for $B'_0(s)$ we get

$$B'_0(s) = \frac{N_3(s)}{D_3(s)}
$$

(25)

Where

$$
\begin{align*}
N_3(s) &= [(1 - q_{69} q_{56})(1 - q_{78} q_{87}) - q_{67} q_{76} - q_{67} q_{78} q_{89} q_{56}][(1 - q_{34} q_{45} q_{53} \\
- q_{35} q_{53}) (Z'_1 q'_{01} + Z'_2 q'_{01} q'_{12}) + (Z'_3 + Z'_4 q_{34}) (q'_{01} q_{12} q_{25} q_{53} + q'_{01} q_{15} q_{53}]
\end{align*}
$$
and

\[ D_3(s) \text{ is same as } D_2(s) \text{ as obtained in availability is given by (17) } \]

Thus in the long run, the fraction of time for which system is under repair is given by

\[ B_0 = \lim_{t \to \infty} B_0(t) = \lim_{s \to 0} sB_0(s) = \frac{N_3(0)}{D_2'(0)}, \text{ where} \]

\[ N_3(0) = [p_{67}(1 - p_{78}p_{87}) - p_{67}p_{76} - p_{67}p_{78}p_{89}][((1 - p_{34}p_{45} - p_{35})(\mu_1p_{10} + 
\mu_2p_{12}p_{12}) + (\mu_3 + \mu_4p_{34})(p_{01}p_{12}p_{12} + p_{01}p_{12} + p_{03}) + \mu_5(p_{01}p_{12}p_{25} - p_{01}p_{12} - p_{03}p_{35} - p_{03}p_{34}p_{45}) + (p_{01}p_{12}p_{25} + p_{01}p_{12} + p_{03})][\mu_6p_{36}(1 - p_{78}p_{87}) + \mu_7p_{36}p_{67} + \mu_8p_{36}p_{67}p_{78} + \mu_9p_{36}p_{67}p_{78}p_{89} - p_{69}(1 - p_{78}p_{87})][p_{01}p_{12}p_{25} + p_{01}p_{12} + p_{03}] + (1 - p_{34}p_{45} - p_{35})[\mu_6p_{01}p_{18}p_{87} + p_{07}(p_{76} + p_{78}p_{89})] + \mu_7[p_{01}p_{12}p_{87} + p_{01}p_{18}p_{96}(p_{67}p_{89} + p_{67}p_{69}) + p_{07}p_{67}p_{78}p_{67}] + \mu_9[p_{01}p_{12}p_{87}p_{67}p_{69} + p_{01}p_{18}p_{89}(1 - p_{67}p_{76}) + p_{07}(p_{78}p_{89} + p_{67}p_{76})]] \]

and

\[ D_3'(0) = D_2'(0) \text{ is already obtained and is given by relation (21).} \]

Now by substituting the values of \( \mu_i \)'s and \( p_{ij} \)'s in (26) we get the expressions for numerator of and denominator of \( B_0 \) as:

\[ N_3(0) = \gamma_1[y_2[((\alpha_1 + \alpha_2 + \alpha_3)(\alpha_2 + \gamma_1) - \alpha_1\gamma_1)] - \alpha_2y_2(\alpha_2 + \gamma_1)\alpha_1\alpha_2y_2][((\alpha_1 + \gamma_2) + (\alpha_1 + \alpha_3 + \beta + \gamma_2) - \alpha_1\beta - (\alpha_1 + \gamma_2)](\alpha_2 + \gamma_1 + \beta) + \{\alpha_1(\alpha_1 + \gamma_2) + \beta\} + \{\alpha_2(\alpha_2 + \gamma_1)(\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1)\} + (\alpha_1 + \gamma_2)[\alpha_2\beta + \alpha_2(\alpha_2 + \gamma_1)] (\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1)]\{\alpha_3\gamma_1((\alpha_1 + \alpha_2 + \gamma_2)(\alpha_2 + \gamma_1) - \alpha_1\gamma_1) + \alpha_3\gamma_1\gamma_2 \]
\[
\begin{align*}
\alpha_2 + \gamma_1 &+ \alpha_2 \alpha_3 \gamma_1 y_2 + \alpha_1 \alpha_2 \alpha_3 \gamma_2 - \alpha_1 \{(\alpha_1 + \alpha_2 + \gamma_3)(\alpha_2 + \gamma_1) - \alpha_1 \gamma_1\} + \gamma_1 \\
\alpha_2 + \gamma_1 \alpha_1 + \alpha_3 + \beta + \gamma_2 \alpha_1 + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2 + \beta) &][\{(\alpha_1 + \alpha_2 + \gamma_1 + \beta) + \alpha_1 \alpha_3 \gamma_1 (\alpha_2 + \gamma_1) + (\alpha_2 + \alpha_3 + \beta + \gamma_1)\} + \alpha_1 \alpha_3 \gamma_2 (\alpha_2 + \gamma_1) - \alpha_1 (\alpha_1 + \gamma_2 + \beta) + \gamma_1 (\alpha_1 + \gamma_2 + \beta)
\end{align*}
\]

\[
D_3'(0) = (\alpha_1 + \alpha_2 + \alpha_3) \gamma_1 (\alpha_2 + \gamma_1) [(\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_1 + \gamma_2) - \alpha_1 \beta - \alpha_1 (\alpha_1 + \gamma_2)]]
\]

\[
(\alpha_2 + \alpha_3 + \beta + \gamma_1) \gamma_2 \gamma_3 (\alpha_2 + \gamma_1) + \alpha_1 (\alpha_2 + \gamma_1) + \alpha_1 \beta) + \gamma_1 \gamma_2 \gamma_3 (\alpha_1 + \gamma_2 + \beta)
\]

\[
\gamma_2 (\alpha_2 + \gamma_1) [\alpha_2 \beta + \alpha_1 \alpha_2 (\alpha_2 + \gamma_1) \alpha_1 (\alpha_2 + \gamma_1) (\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1 \alpha_3 \gamma_2 (\alpha_2 + \gamma_1) - \alpha_1 \gamma_1
\]

\[
\alpha_2 + \gamma_1 - \alpha_3 (\alpha_2 + \gamma_1) (\alpha_2 + \gamma_1) (\alpha_2 + \alpha_3 + \beta + \gamma_1) - \alpha_1 \gamma_1 \beta) + \alpha_1 \alpha_2 \gamma_2 \gamma_3 (\alpha_2 + \gamma_1 + \beta)
\]

\[
(\alpha_1 + \alpha_2 + \gamma_1 + \beta) + (\alpha_2 + \gamma_1) [\alpha_3 (\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1 \alpha_3] \{(\alpha_1 + \gamma_2)
\]

\[
(\alpha_1 + \alpha_3 + \beta + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2) - \alpha_1 \beta) + \alpha_2 \alpha_3 (\alpha_1 + \gamma_2) (\alpha_2 + \alpha_3 + \beta + \gamma_1)
\]

\[
(\alpha_2 + \gamma_1) + [\alpha_2 \gamma_1 (\alpha_1 + \alpha_2 + \gamma_1) + \gamma_1 \gamma_2 (\alpha_2 + \gamma_1) + \alpha_1 \alpha_2 \gamma_2 + \alpha_1 \{(\alpha_2 + \gamma_1)
\]

\[
(\alpha_1 + \alpha_2 + \gamma_3 - \alpha_1 \gamma_1) + \gamma_1 (\alpha_2 + \gamma_1) [\alpha_1 \alpha_3 \gamma_2 \gamma_3 (\alpha_2 + \alpha_1 + \gamma_1 + \beta)] + \alpha_1 \alpha_2 \alpha_3 \gamma_3
\]

\[
(\alpha_1 + \alpha_3 + \beta + \gamma_2) (\alpha_1 + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2 + \beta) + \alpha_2 \alpha_3 \gamma_3 (\alpha_1 + \gamma_2) (\alpha_2 + \gamma_1)
\]

\[
(\alpha_2 + \alpha_3 + \beta + \gamma_1) + (\alpha_2 + \gamma_1) [\alpha_1 \alpha_3 \gamma_3 (\alpha_1 + \gamma_2) (\gamma_2 - \gamma_1) (\alpha_1 + \alpha_3 + \beta + \gamma_2)
\]

\[
(\alpha_1 + \gamma_2) + \alpha_1 \alpha_3 \gamma_3 (\alpha_1 + \alpha_3 + \beta + \gamma_2) (\alpha_1 + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2 + \beta) (\alpha_2 + \gamma_1)
\]

\[
\alpha_2 + \alpha_3 + \beta + \gamma_1
\]

142
4.8 EXPECTED NUMBER OF VISITS BY THE REPAIR FACILITIES

Let us define $V_i(t)$ as the expected no. of repairs of the unit during the time interval(0, t] when the system initially starts from regenerative state $S_i$. Using the definition of $V_i(t)$, the recursive relation among $V_i(t)$'s can be easily developed

$$V_0(t) = Q_{01}(t)@V_1(t) + Q_{03}(t)@V_3(t) + Q_{07}(t)@V_7(t)$$
$$V_1(t) = Q_{10}(t)@[1 + V_0(t)] + Q_{12}(t)@V_2(t) + Q_{13}(t)@V_3(t) + Q_{15}(t)@V_5(t)$$
$$V_2(t) = Q_{20}(t)@[1 + V_0(t)] + Q_{25}(t)@V_5(t)$$
$$V_3(t) = Q_{30}(t)@[1 + V_0(t)] + Q_{34}(t)@V_4(t) + Q_{35}(t)@V_5(t) + Q_{36}(t)@V_6(t)$$
$$V_4(t) = Q_{40}(t)@[1 + V_0(t)] + Q_{45}(t)@V_5(t)$$
$$V_5(t) = Q_{53}(t)@[1 + V_3(t)]$$
$$V_6(t) = Q_{67}(t)@[1 + V_7(t)] + Q_{69}(t)@V_9(t)$$
$$V_7(t) = Q_{70}(t)@[1 + V_0(t)] + Q_{76}(t)@V_6(t) + Q_{78}(t)@V_8(t)$$
$$V_8(t) = Q_{87}(t)@[1 + V_7(t)] + Q_{89}(t)@V_9(t)$$
$$V_9(t) = Q_{96}(t)@[1 + V_6(t)]$$

Taking Laplace Stieltjes transformation of these relations, we get

$$\tilde{V}_0(s) = \tilde{Q}_{01}(s)\tilde{V}_1(s) + \tilde{Q}_{03}(s)\tilde{V}_3(s) + \tilde{Q}_{07}(s)\tilde{V}_7(s)$$
$$\tilde{V}_1(s) = \tilde{Q}_{10}(s)[1 + \tilde{V}_0(s)] + \tilde{Q}_{12}(s)\tilde{V}_2(s) + \tilde{Q}_{13}(s)\tilde{V}_3(s) + \tilde{Q}_{15}(s)\tilde{V}_5(s)$$
$$\tilde{V}_2(s) = \tilde{Q}_{20}(s)[1 + \tilde{V}_0(s)] + \tilde{Q}_{25}(s)\tilde{V}_5(s)$$
$$\tilde{V}_3(s) = \tilde{Q}_{30}(s)[1 + \tilde{V}_0(s)] + \tilde{Q}_{34}(s)\tilde{V}_4(s) + \tilde{Q}_{35}(s)\tilde{V}_5(s) + \tilde{Q}_{36}(s)\tilde{V}_6(s)$$
$$\tilde{V}_4(s) = \tilde{Q}_{40}(s)[1 + \tilde{V}_0(s)] + \tilde{Q}_{45}(s)\tilde{V}_5(s)$$
$$\tilde{V}_5(s) = \tilde{Q}_{53}(s)[1 + \tilde{V}_3(s)]$$
$$\tilde{V}_6(s) = \tilde{Q}_{67}(s)[1 + \tilde{V}_7(s)] + \tilde{Q}_{69}(s)\tilde{V}_9(s)$$
$$\tilde{V}_7(s) = \tilde{Q}_{70}(s)[1 + \tilde{V}_0(s)] + \tilde{Q}_{76}(s)\tilde{V}_6(s) + \tilde{Q}_{78}(s)\tilde{V}_8(s)$$
$$\tilde{V}_8(s) = \tilde{Q}_{87}(s)[1 + \tilde{V}_7(s)] + \tilde{Q}_{89}(s)\tilde{V}_9(s)$$
$$\tilde{V}_9(s) = \tilde{Q}_{96}(s)[1 + \tilde{V}_6(s)]$$

By solving the above equations for $\tilde{V}_0(s)$ the Laplace Stieltjes transformation of the expected no of visits by the service facility ig given by

$$\tilde{V}_0(s) = \frac{N_4(s)}{D_4(s)}$$

$$N_4(s) = [(1 - \tilde{Q}_{69}\tilde{Q}_{96})(1 - \tilde{Q}_{78}\tilde{Q}_{87}) - \tilde{Q}_{67}\tilde{Q}_{76} - \tilde{Q}_{67}\tilde{Q}_{78}\tilde{Q}_{89}\tilde{Q}_{96}][(1 - \tilde{Q}_{34}\tilde{Q}_{45}\tilde{Q}_{53} - \tilde{Q}_{35}}$$
\[\tilde{Q}_{53}(\tilde{Q}_{01}\tilde{Q}_{10} + \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{20}) + (\tilde{Q}_{30} + \tilde{Q}_{34}\tilde{Q}_{40})(\tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{25}\tilde{Q}_{53} + \tilde{Q}_{01}\tilde{Q}_{15}\tilde{Q}_{53} + \\
\tilde{Q}_{03} + \tilde{Q}_{53}(\tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{25} + \tilde{Q}_{01}\tilde{Q}_{15} - \tilde{Q}_{30}\tilde{Q}_{34}\tilde{Q}_{45} - \tilde{Q}_{30}\tilde{Q}_{35})] + (\tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{15}\tilde{Q}_{53} + \\
\tilde{Q}_{03} + \tilde{Q}_{01}\tilde{Q}_{15}\tilde{Q}_{53})[\tilde{Q}_{06}\tilde{Q}_{36}(1 - \tilde{Q}_{78}\tilde{Q}_{87}) + \tilde{Q}_{36}\tilde{Q}_{67}\tilde{Q}_{70} + \tilde{Q}_{36}\tilde{Q}_{67}\tilde{Q}_{78}\tilde{Q}_{87} + \tilde{Q}_{36} \\
\tilde{Q}_{96}(\tilde{Q}_{67}\tilde{Q}_{78}\tilde{Q}_{89} - \tilde{Q}_{69}(1 - \tilde{Q}_{78}\tilde{Q}_{87}))] + (1 - \tilde{Q}_{34}\tilde{Q}_{45}\tilde{Q}_{53} - \tilde{Q}_{35}\tilde{Q}_{53})[\tilde{Q}_{67}\{\tilde{Q}_{01}\tilde{Q}_{18} \\
(\tilde{Q}_{87}\tilde{Q}_{76} + \tilde{Q}_{89}\tilde{Q}_{96}) + \tilde{Q}_{07}(\tilde{Q}_{76} + \tilde{Q}_{96}\tilde{Q}_{87}\tilde{Q}_{99})\} + \tilde{Q}_{07}(\tilde{Q}_{76} + \tilde{Q}_{96}\tilde{Q}_{87}) + \tilde{Q}_{01}\tilde{Q}_{18}\tilde{Q}_{87}[(1 - \tilde{Q}_{67}\tilde{Q}_{76} - \tilde{Q}_{69}\tilde{Q}_{96}) \\
+ \tilde{Q}_{07}\tilde{Q}_{78}(1 - \tilde{Q}_{69}\tilde{Q}_{96})] + \tilde{Q}_{96}\{\tilde{Q}_{01}\tilde{Q}_{18}\tilde{Q}_{87}\tilde{Q}_{76}\tilde{Q}_{69} + \tilde{Q}_{01}\tilde{Q}_{18}\tilde{Q}_{89}(1 - \tilde{Q}_{67}\tilde{Q}_{76}) + \\
\tilde{Q}_{07}(\tilde{Q}_{78}\tilde{Q}_{89} + \tilde{Q}_{67}\tilde{Q}_{76})\}]
\]

and

\[D_4(s) \text{ can be obtained by replacing } q_{ij}^* \text{'s by } \tilde{Q}_{ij} \text{'s equation (17)}\]

In steady state the number of visits per unit time is given by

\[N_0 = \lim_{t \to \infty} N(t) = \lim_{s \to 0} s\tilde{N}_0(s) = \frac{N_4(0)}{D_4(0)} \quad (30)\]

\[N_4(0) = [p_{67}(1 - p_{78}p_{87}) - p_{67}p_{76} - p_{67}p_{78}p_{89}p_{96}][(1 - p_{34}p_{45} - p_{35}) (p_{01}p_{10} + \\
p_{01}p_{12}p_{20}) + (p_{30} + p_{34}p_{40})(p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{03}) + (p_{01}p_{12}p_{25} - \\
p_{01}p_{15} - p_{03}p_{35} - p_{03}p_{34}p_{45})] + (p_{01}p_{12}p_{25} + p_{01}p_{15} + p_{03})[p_{36}p_{67}(1 - \\
p_{78}p_{87}) + p_{36}p_{67}p_{70} + p_{36}p_{67}p_{78}p_{87} + p_{36}(p_{67}p_{78}p_{89} - p_{69}(1 - p_{78}p_{87}))] + \\
(1 - p_{34}p_{45} - p_{35})p_{67}\{p_{01}p_{18}(p_{87}p_{76} + p_{89}) + p_{07}(p_{76} + p_{78}p_{89})\} + p_{70} \\
\{p_{01}p_{18}p_{87} + p_{01}p_{18}(p_{67}p_{89} + p_{87}p_{69}) + p_{07}p_{67}\} + p_{87}[(1 - p_{67}p_{76} - p_{69}) \\
p_{01}p_{18} + p_{07}p_{78}p_{67}] + p_{01}p_{18}p_{87}p_{76}p_{69} + p_{01}p_{18}p_{89}(1 - p_{67}p_{76}) + (p_{78}p_{89} \\
+ p_{67}p_{76})]\]

and \(D_4'(0) = D_2'(0)\) is already specified in eq (21)

Now by substituting the values of \(\mu_i\)'s and \(p_{ij}\)'s in (30) we get the expressions for numerator of and denominator of \(V_0\) as:

\[N_4(0) = \gamma_1[\gamma_2((\alpha_1 + \alpha_2 + \alpha_3)(\alpha_2 + \gamma_1) - \alpha_1\gamma_1) - \alpha_2\gamma_2(\alpha_2 + \gamma_1)\alpha_1\alpha_2\gamma_2][((\alpha_1 + \gamma_2) \\
(\alpha_1 + \alpha_3 + \beta + \gamma_2) - \alpha_1\beta - \alpha_1(\alpha_1 + \gamma_2)]\alpha_1\gamma_1(\alpha_2 + \gamma_1 + \beta) + \gamma_2(\alpha_1(\alpha_1 + \gamma_2) + \\
\]

144
\[ \beta \{ \alpha_2 \beta + \alpha_2 (\alpha_2 + \gamma_1)(\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1) \} + (\alpha_1 + \gamma_2)[\alpha_2 \beta + \alpha_2 (\alpha_2 + \gamma_1)] \\
(\alpha_1 + \alpha_2 + \alpha_3 + \beta + \gamma_1) [\alpha_3 \gamma_2 [(\alpha_1 + \alpha_2 + \gamma_3)(\alpha_2 + \gamma_1) - \alpha_1 \gamma_1] + \alpha_3 \gamma_1 \gamma_2 \\
(\alpha_2 + \gamma_1) + \alpha_2 \alpha_3 \gamma_1 \gamma_2 + \alpha_1 \alpha_2 \alpha_3 \gamma_2 - \alpha_1 [(\alpha_1 + \alpha_2 + \gamma_3)(\alpha_2 + \gamma_1) - \alpha_1 \gamma_1] \\
(\alpha_2 + \gamma_1)(\alpha_1 + \alpha_3 + \beta + \gamma_2)(\alpha_1 + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2 + \beta) [(\alpha_1 + \alpha_2 + \gamma_3)] \\
(\alpha_1 \alpha_2 \alpha_3 \gamma_2 + (\alpha_1 + \alpha_2 + \gamma_1) \alpha_2 \alpha_3 \gamma_2 (\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1 \alpha_3 \gamma_1 \gamma_3 \gamma_2 (\alpha_1 + \alpha_2 + \gamma_2) + (\alpha_2 + \alpha_3 + \beta + \gamma_1) \gamma_2 \gamma_3 + \alpha_1 \alpha_3 \gamma_2 (\alpha_2 + \gamma_1) \gamma_3 - (\alpha_1 + \gamma_3) \alpha_1 \alpha_3 \gamma_1 \gamma_3 + \alpha_3 \gamma_2 \gamma_3 (\alpha_2 + \gamma_1) \\
(\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1 \alpha_3 \gamma_1 [(\alpha_1 + \gamma_2) (\alpha_1 + \alpha_2 + \gamma_3) - \alpha_2 \gamma_2 - \alpha_1 (\alpha_1 + \alpha_2 + \gamma_3)] \\
+ (\alpha_2 + \alpha_3 + \beta + \gamma_1) \alpha_1 \alpha_3 \gamma_1 \gamma_2 + (\alpha_1 + \alpha_3 + \beta + \gamma_2) [\alpha_1 \alpha_2 \alpha_3 \gamma_1 \gamma_2 + \alpha_1 \alpha_2 \alpha_3 \gamma_3 (\alpha_1 + \gamma_2) \\
(\alpha_1 + \alpha_2 + \gamma_3) - \alpha_2 \gamma_2] + \{ \alpha_1 \alpha_2 \alpha_3 + \alpha_2 \alpha_3 \gamma_2 \} [(\alpha_1 + \gamma_2) (\alpha_2 + \alpha_3 + \beta + \gamma_1)] \\
\]

\[ D_4(0) = (\alpha_1 + \alpha_2 + \alpha_3) \gamma_1 (\alpha_2 + \gamma_1) [(\alpha_1 + \alpha_3 + \beta + \gamma_2) (\alpha_1 + \gamma_2) - \alpha_1 \beta - \alpha_1 (\alpha_1 + \gamma_2)] \\
(\alpha_2 + \alpha_3 + \beta + \gamma_1) \gamma_2 \gamma_3 (\alpha_2 + \gamma_1) + \alpha_1 (\alpha_2 + \gamma_1) + \alpha_1 \beta \gamma_1 \gamma_2 \gamma_3 (\alpha_1 + \gamma_2 + \beta) \\
(\alpha_2 + \gamma_1) [\alpha_2 \beta + \alpha_1 \alpha_2 (\alpha_2 + \gamma_1) \alpha_1 (\alpha_2 + \alpha_3 + \beta + \gamma_1)] + (\alpha_2 + \gamma_1) \\
[(\alpha_1 (\alpha_2 + \gamma_1) + \alpha_1 \beta) \gamma_2 \gamma_3 [(\alpha_1 + \alpha_2 + \alpha_3) (\alpha_2 + \alpha_3 + \beta + \gamma_1) (\alpha_2 + \gamma_1) - \alpha_1 \gamma_1 \\
(\alpha_2 + \gamma_1) - \alpha_3 (\alpha_2 + \gamma_1) (\alpha_2 + \alpha_3 + \beta + \gamma_1) - \alpha_1 \gamma_1 \beta] + \alpha_1 \alpha_2 \gamma_2 \gamma_3 (\alpha_2 + \gamma_1 + \beta) \\
{[(\alpha_1 + \gamma_2) \alpha_2 \gamma_2 + \gamma_2 \beta] + \alpha_1 \alpha_3 \gamma_3 [\alpha_1 (\alpha_1 + \gamma_2 + \beta)] (\gamma_2 (\alpha_2 + \gamma_1))] + \gamma_1 [\alpha_1 \alpha_3 \\
(\alpha_1 + \gamma_2) \alpha_2 (\alpha_2 + \gamma_1 + \beta) + (\alpha_2 + \gamma_1) [\alpha_3 (\alpha_2 + \alpha_3 + \beta + \gamma_1) + \alpha_1 \alpha_3] [(\alpha_1 + \gamma_2) \\
(\alpha_1 + \alpha_3 + \beta + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2) - \alpha_1 \beta] + \alpha_2 \alpha_3 (\alpha_1 + \gamma_2) (\alpha_2 + \alpha_3 + \beta + \gamma_1) \\
(\alpha_2 + \gamma_1)] + [\alpha_2 \gamma_1 (\alpha_1 + \alpha_2 + \gamma_1) + \gamma_1 \gamma_2 (\alpha_1 + \gamma_2) + \alpha_1 \alpha_2 \gamma_2 + \alpha_1 ((\alpha_2 + \gamma_1) \\
(\alpha_1 + \alpha_2 + \gamma_3) - \alpha_1 \gamma_1)] + \gamma_1 (\alpha_2 + \gamma_1) [\alpha_1 \alpha_3 \gamma_2 \gamma_3 (\alpha_2 + \gamma_1 + \beta)] + \alpha_1 \alpha_2 \alpha_3 \gamma_3 \\
(\alpha_1 + \alpha_3 + \beta + \gamma_2) (\alpha_1 + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2 + \beta)] + \alpha_2 \alpha_3 \gamma_3 (\alpha_1 + \gamma_2) (\alpha_2 + \gamma_1) \\
(\alpha_2 + \alpha_3 + \beta + \gamma_1)] + (\alpha_2 + \gamma_1) [\alpha_1 \alpha_3 \gamma_3 (\alpha_1 + \gamma_2) (\gamma_2 - \gamma_1) (\alpha_1 + \alpha_3 + \beta + \gamma_2) \\
(\alpha_1 + \gamma_2) + \alpha_1 \alpha_3 \gamma_3 [(\alpha_1 + \alpha_3 + \beta + \gamma_2) (\alpha_1 + \gamma_2) - \alpha_1 (\alpha_1 + \gamma_2 + \beta)] (\alpha_2 + \gamma_1) \\
(\alpha_2 + \alpha_3 + \beta + \gamma_1) \\
\]

145
4.9 COST ANALYSIS

The expected uptime and down time of the system and busy period of the repair man in \((0, t]\) are

\[
\mu_{\text{up}}(t) = \int_0^t A_0(u)\,du
\]

\[
\mu_{\text{dn}}(t) = t - \mu_{\text{up}}(t)
\]

and

\[
\mu_b = \int_0^t B_0(u)\,du
\]

So that

\[
\mu_{\text{up}}^*(s) = A_0^*(s)/s
\]

\[
\mu_{\text{dn}}^*(s) = 1/s^2 - \mu_{\text{up}}^*(s)
\]

and

\[
\mu_b^*(s) = B_0^*(s)/s
\]

The expected profits incurred in \((0, t]\) = expected total revenue in \((0, t]\) – expected total repair in \((0, t]\) – expected cost of visits by repairman in \((0, t]\).

\[
P = K_1\mu_{\text{up}}(t) - K_2\mu_b(t) - K_3V_0(t)
\]  \quad (31)

where

\[
K_1 = \text{Revenue per unit up time of the system.}
\]

\[
K_2 = \text{Cost per unit time for which the repair is busy.}
\]

\[
K_3 = \text{Cost per unit visits by the repairman.}
\]
4.10 GRAPHICAL STUDY OF SYSTEM BEHAVIOR

The behavior of MTSF and availability of the system is studied graphically in this section and to plot their graphs, the replacement and repair time distributions are also assumed to be distributed exponentially. The graphs of MTSF and that of availability are depicted with respect to the different parameters. It is observed that the MTSF decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF increases with increasing repair rates. Thus, we can conclude that the expected life of the system can be increased by increasing repair rate of the unit. Further, it is observed that the availability of the system gradually decreases with increasing failure rates irrespective of type of failure and increases with increasing repair rate of the unit.

For fixed values of the parameters $\alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2, \gamma_3$ and changing $\alpha_1$, TABLE-1 is obtained. TABLE-1: Effect of $\alpha_1$ and fixed parameters $\alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2$ and $\gamma_3$ on MTSF

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.02, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.04, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.06, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>24.1808</td>
<td>29.7904</td>
<td>35.2753</td>
</tr>
<tr>
<td>0.2</td>
<td>16.4361</td>
<td>20.1634</td>
<td>23.8154</td>
</tr>
<tr>
<td>0.3</td>
<td>13.6340</td>
<td>16.5858</td>
<td>19.4203</td>
</tr>
<tr>
<td>0.4</td>
<td>12.2143</td>
<td>14.7643</td>
<td>17.1671</td>
</tr>
<tr>
<td>0.5</td>
<td>11.3638</td>
<td>13.6724</td>
<td>15.8138</td>
</tr>
<tr>
<td>0.6</td>
<td>10.8002</td>
<td>12.9492</td>
<td>14.9170</td>
</tr>
<tr>
<td>0.7</td>
<td>10.4006</td>
<td>12.4369</td>
<td>14.2817</td>
</tr>
<tr>
<td>0.8</td>
<td>10.1032</td>
<td>12.0560</td>
<td>13.8094</td>
</tr>
<tr>
<td>0.9</td>
<td>9.87360</td>
<td>11.7622</td>
<td>13.4452</td>
</tr>
<tr>
<td>1.0</td>
<td>9.61260</td>
<td>11.5291</td>
<td>13.1562</td>
</tr>
</tbody>
</table>

In fig. 4.2, we plot MTSF w.r.t. $\alpha_1$ and fixed values of parameters $\alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2$ and $\gamma_3$. It is observed that MTSF of the system decreases w.r.t. $\alpha_1$ irrespective of the other parameters so that we conclude that expected life of the system increases with decreasing failure rate.
Behaviour of MTSF w.r.t $\alpha_1$ for different values of $\gamma_1$

Fig. 4.2
For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2, \gamma_3$ and changing $\gamma_1$, TABLE-2 is obtained

**TABLE- 2:** Effect of $\gamma_1$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2$ and $\gamma_3$ on MTSF.

<table>
<thead>
<tr>
<th>$\gamma_1$</th>
<th>$\alpha_1 = 0.25, \alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\alpha_1 = 0.50, \alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\alpha_1 = 0.75, \alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>58.3129</td>
<td>66.2270</td>
<td>76.3528</td>
</tr>
<tr>
<td>0.2</td>
<td>78.2350</td>
<td>86.6560</td>
<td>2.3580</td>
</tr>
<tr>
<td>0.3</td>
<td>90.9550</td>
<td>100.387</td>
<td>110.854</td>
</tr>
<tr>
<td>0.4</td>
<td>94.4440</td>
<td>110.275</td>
<td>124.974</td>
</tr>
<tr>
<td>0.5</td>
<td>96.7840</td>
<td>117.742</td>
<td>141.378</td>
</tr>
<tr>
<td>0.6</td>
<td>98.4610</td>
<td>123.583</td>
<td>155.86</td>
</tr>
<tr>
<td>0.7</td>
<td>99.7910</td>
<td>128.277</td>
<td>168.755</td>
</tr>
<tr>
<td>0.8</td>
<td>100.699</td>
<td>132.133</td>
<td>180.319</td>
</tr>
<tr>
<td>0.9</td>
<td>101.481</td>
<td>135.357</td>
<td>190.752</td>
</tr>
<tr>
<td>1.0</td>
<td>102.121</td>
<td>138.093</td>
<td>200.217</td>
</tr>
</tbody>
</table>

In fig. 4.3, we plot MTSF w.r.t. $\gamma_1$ and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2$ and $\gamma_3$. It is quiet clear that MTSF of the system increases w.r.t. $\gamma_1$ irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate.
Behaviour of MTSF w.r.t. $\gamma_1$ for different values of $\alpha_1$

Fig. 4.3
For fixed values of the parameters $\alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2, \gamma_3$ and changing $\alpha_1$, TABLE-3 is obtained.

TABLE-3: Effect of $\alpha_1$ and fixed parameters $\alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2$ and $\gamma_3$ on Availability.

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.02, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.04, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.06, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.0369860</td>
<td>0.0740521</td>
<td>0.1066440</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0186190</td>
<td>0.0371843</td>
<td>0.0542788</td>
</tr>
<tr>
<td>0.3</td>
<td>0.01181660</td>
<td>0.0235588</td>
<td>0.0345405</td>
</tr>
<tr>
<td>0.4</td>
<td>0.008333840</td>
<td>0.0166146</td>
<td>0.0244427</td>
</tr>
<tr>
<td>0.5</td>
<td>0.006256020</td>
<td>0.0124825</td>
<td>0.0184228</td>
</tr>
<tr>
<td>0.6</td>
<td>0.004897360</td>
<td>0.0097838</td>
<td>0.0144843</td>
</tr>
<tr>
<td>0.7</td>
<td>0.003952210</td>
<td>0.0079069</td>
<td>0.0117402</td>
</tr>
<tr>
<td>0.8</td>
<td>0.003264410</td>
<td>0.0065408</td>
<td>0.0097389</td>
</tr>
<tr>
<td>0.9</td>
<td>0.002761638</td>
<td>0.0055112</td>
<td>0.0082276</td>
</tr>
<tr>
<td>1.0</td>
<td>0.002345430</td>
<td>0.0047137</td>
<td>0.0070544</td>
</tr>
</tbody>
</table>

In fig. 4.4, we plot Availability w.r.t. $\alpha_1$ and fixed values of parameter $\alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2$ and $\gamma_3$. It is observed that Availability of the system decreases w.r.t. $\alpha_1$ irrespective of the other parameters. Therefore, we conclude that expected life of the system increases with decreasing failure rate.
Behaviour of Availability w.r.t. $\alpha_1$ for different values of $\gamma_1$

Fig. 4.4
For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2, \gamma_3$ and changing $\gamma_1$, TABLE-4 is obtained

TABLE- 4: Effect of $\gamma_1$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2$ and $\gamma_3$ on MTSF

<table>
<thead>
<tr>
<th>$\gamma_1$</th>
<th>$\gamma_1 = 0.025, \alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\gamma_1 = 0.50, \alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
<th>$\gamma_1 = 0.75, \alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_2 = 0.3, \gamma_3 = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.3330</td>
<td>0.2484</td>
<td>0.1933</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3814</td>
<td>0.3123</td>
<td>0.2586</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3944</td>
<td>0.3326</td>
<td>0.2821</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3995</td>
<td>0.3411</td>
<td>0.2925</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4020</td>
<td>0.3454</td>
<td>0.2978</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4032</td>
<td>0.3477</td>
<td>0.3008</td>
</tr>
<tr>
<td>0.7</td>
<td>0.4042</td>
<td>0.3492</td>
<td>0.3026</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4047</td>
<td>0.3500</td>
<td>0.3037</td>
</tr>
<tr>
<td>0.9</td>
<td>0.4050</td>
<td>0.3506</td>
<td>0.3045</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4053</td>
<td>0.3510</td>
<td>0.3050</td>
</tr>
</tbody>
</table>

In fig. 4.5, we plot Availability w.r.t. $\gamma_1$ and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2$ and $\gamma_3$. It is quite clear that Availability of the system increases w.r.t. $\gamma_1$ irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate.
Behaviour of Availability w. r. t. $\gamma_1$ for different values of $\alpha_1$

Fig. 4.5
For fixed values of the parameters $K_1, K_2, K_3, \alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2, \gamma_3$ and changing $\alpha_1$, TABLE-5 is obtained.

**TABLE-5: Effect of $\alpha_1$ and fixed parameters $K_1, K_2, K_3, \alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2$ and $\gamma_3$ on MTSF**

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>Profit Function</th>
<th>Profit Function</th>
<th>Profit Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.02, \gamma_2 = 0.3, \gamma_3 = 0.05, K_0 = 5000, K_1 = 100$</td>
<td>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.04, \gamma_2 = 0.3, \gamma_3 = 0.05, K_0 = 5000, K_1 = 100$</td>
<td>$\alpha_2 = 0.05, \alpha_3 = 0.02, \beta = 0.15, \gamma_1 = 0.06, \gamma_2 = 0.3, \gamma_3 = 0.05, K_0 = 5000, K_1 = 100$</td>
</tr>
<tr>
<td>0.1</td>
<td>176.68</td>
<td>360.353</td>
<td>521.849</td>
</tr>
<tr>
<td>0.2</td>
<td>86.826</td>
<td>178.847</td>
<td>263.487</td>
</tr>
<tr>
<td>0.3</td>
<td>53.848</td>
<td>112.019</td>
<td>166.346</td>
</tr>
<tr>
<td>0.4</td>
<td>37.099</td>
<td>78.0620</td>
<td>116.737</td>
</tr>
<tr>
<td>0.5</td>
<td>27.179</td>
<td>57.9060</td>
<td>87.196</td>
</tr>
<tr>
<td>0.6</td>
<td>20.737</td>
<td>44.7710</td>
<td>67.885</td>
</tr>
<tr>
<td>0.7</td>
<td>16.286</td>
<td>35.6530</td>
<td>54.439</td>
</tr>
<tr>
<td>0.8</td>
<td>13.067</td>
<td>29.0280</td>
<td>44.636</td>
</tr>
<tr>
<td>0.9</td>
<td>10.658</td>
<td>24.0440</td>
<td>37.236</td>
</tr>
<tr>
<td>1.0</td>
<td>8.8060</td>
<td>20.1890</td>
<td>31.492</td>
</tr>
</tbody>
</table>

In fig. 4.6, we plot Profit Function w.r.t. $\alpha_1$ and fixed values of parameter $K_1, K_2, K_3, \alpha_2, \alpha_3, \beta, \gamma_1, \gamma_2$ and $\gamma_3$. It is observed that Profit Function of the system decreases w.r.t. $\alpha_1$ irrespective of the other parameters. Therefore, we conclude Profit increases with decreasing failure rate.
Behaviour of Profit function w.r.t. $\alpha_1$ for different values of $\gamma_1$

Fig. 4.6
For fixed values of the parameters $K_1, K_2, K_3, \alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2, \gamma_3$ and changing $\gamma_1$, TABLE-6 is obtained.

**TABLE-6: Effect of $\gamma_1$ and fixed parameters $K_1, K_2, K_3, \alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2$ and $\gamma_3$ on MTSF**

<table>
<thead>
<tr>
<th>$\gamma_1$</th>
<th>Profit Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 0.025, \alpha_2 = 0.05,$</td>
<td>$\alpha_1 = 0.50, \alpha_2 = 0.05,$</td>
</tr>
<tr>
<td>$\alpha_3 = 0.02, \beta = 0.15,$</td>
<td>$\alpha_3 = 0.02, \beta = 0.15,$</td>
</tr>
<tr>
<td>$\gamma_2 = 0.3, \gamma_3 = 0.05,$</td>
<td>$\gamma_2 = 0.3, \gamma_3 = 0.05,$</td>
</tr>
<tr>
<td>$K_0 = 5000, K_1 = 100$</td>
<td>$K_0 = 5000, K_1 = 100$</td>
</tr>
<tr>
<td>$K_2 = 50$</td>
<td>$K_2 = 50$</td>
</tr>
<tr>
<td>0.1</td>
<td>317.914</td>
</tr>
<tr>
<td>0.2</td>
<td>493.318</td>
</tr>
<tr>
<td>0.3</td>
<td>580.767</td>
</tr>
<tr>
<td>0.4</td>
<td>627.18</td>
</tr>
<tr>
<td>0.5</td>
<td>653.43</td>
</tr>
<tr>
<td>0.6</td>
<td>669.046</td>
</tr>
<tr>
<td>0.7</td>
<td>678.695</td>
</tr>
<tr>
<td>0.8</td>
<td>684.816</td>
</tr>
<tr>
<td>0.9</td>
<td>688.765</td>
</tr>
<tr>
<td>1.0</td>
<td>691.329</td>
</tr>
</tbody>
</table>

In fig 4.7, we plot Profit Function w.r.t. $\gamma_1$ and fixed values of parameter $K_1, K_2, K_3, \alpha_1, \alpha_2, \alpha_3, \beta, \gamma_2$ and $\gamma_3$. It is observed that Profit Function of the system increases w.r.t. $\gamma_1$ irrespective of the other parameters. Therefore, we conclude Profit increases with increasing repair rate.

157
Fig. 4.7

Behaviour of Profit function w. r. t. $\gamma_1$ for different values of $\alpha_1$
Recommendations to the entrepreneur: On the basis of above analysis, it is recommended to the entrepreneur to increase the repair rates by enhancing the repair facilities which in turn ultimately lead to decrease in failure rate and increase in the profit of the firm.