PREFACE

The graph theory evolved in an attempt to solve famous bridge problem by the great Swiss mathematician, Leonhard Euler who used graphs to solve this problem in 1736. In 1848, A. Cayley used graph theory for the study of isomers of saturated hydrocarbons. During 1840 to 1850 a boost of research in graph theory followed the presentation of four color problem by A. De Morgan. A practical note to this was through the development of a game ‘Around The World’ by William Hamilton in 1859. A book by D.Konig on graph theory was the first of its kind to organize the work of several mathematicians and his own. Many eminent authors like Claud Berge, Paul Erdos, Frank Harary, Douglas West, Jonathan Gross and Jay Yellen have contributed to this science. The development of computer science and optimization techniques saw an unprecedented growth in this field. The graph theory and its applications have grown exponentially in the last century.

The graph serves as a mathematical model for any system involving a discrete arrangement of objects. Graph becomes aesthetic appealing due to its diagrammatic representation. Because of these the graph theory has been surprising large number of applications in almost all the fields and in variety of subjects like
computer science, physical science, biological science and social
science. A few samples of applications are stated.

(1) In chemistry, the understanding molecular structure of atoms
is set at easy by graph theory.

(2) Easy and optimized electrical network as well as commu-
nication network is possible using graphs. Thus it seems that the
graph theory enjoys the status of a beautiful QUEEN in the field of
science and technology.

Any field of investigation becomes more interesting when
there arises a number of problems that pose challenge to our mind
for their eventual solutions, more so when the field itself is just
emerging and a whole galore of seemingly related or even unre-
lated open problems provide motivation for research. Graph label-
ing is one such demanding field. Graph labeling was introduced in
the 1960s and has been used in diverse fields such as conflict reso-
lutions in social psychology, energy crises and the like. Quantita-
tive labeling of graph elements have been used in missile guidance
codes, radar location codes, coding theory, X-ray crystallography,
radio astronomy, circuit design, communication network etc. S.
M. Hedge has given a beautiful exposition of various applications
of graph labeling [8].
In 1967, Rosa [22] introduced a labeling called a β-valuation of a graph. Let \( G \) be a graph with \( q \) edges. An injective mapping \( f \) from the vertex set of \( G \) to the set of integers \( \{0, 1, 2, \ldots, q\} \) is said to be a β-valuation of \( G \) if \( f \) induces a label \( |f(x) - f(y)| \) for each edge \( xy \) of \( G \) and the resulting edge labels are distinct. In 1972, Golomb [5] has called such labeling graceful.

Ponraj, Vijaya Xavier Parthipan and Kala [17] introduced pair sum labeling and further studied in [18,19,20]. An injective map \( f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\} \) is said to be a pair sum labeling of a graph \( G(p, q) \) if the induced edge function \( f_e : E(G) \to \mathbb{Z} - \{0\} \) defined by \( f_e(uv) = f(u) + f(v) \) is one-one and \( f_e(E(G)) \) is either of the form \( \{\pm k_1, \pm k_2, \ldots, \pm k_q\} \) or \( \{\pm k_1, \pm k_2, \ldots, \pm k_{q-1}\} \cup \{\pm k_q\} \) according as \( q \) is even or odd. A graph with a pair sum labeling, it is called a pair sum graph. They proved that \( P_2 \times P_n, k_{1,n} \cup k_{1,m}, P_m \cup k_{1,n} \) are pair sum graphs.

Motivated by the results in [17], we define a new labeling called edge pair sum labeling. An injective map \( f : E(G) \to \{\pm 1, \pm 2, \ldots, \pm q\} \) is said to be an edge pair sum labeling of a graph \( G(V, E) \) if the induced vertex function \( f^* : V(G) \to \mathbb{Z} - \{0\} \) defined by \( f^*(v) = \sum_{e \in E_v} f(e) \) is one-one where \( E_v \) denotes the set of edges in \( G \) that are incident with a vertex \( v \) and \( f^*(V(G)) \) is either
of the form \( \{ \pm k_1, \pm k_2, \cdots, \pm k_{p/2} \} \) or \( \{ \pm k_1, \pm k_2, \cdots, \pm k_{p-1} \} \cup \{ \pm k_{p+1} \} \) according as \( p \) is even or odd. A graph with an edge pair sum labeling is called an *edge pair sum graph*.

In this thesis we deal with finite, undirected graphs without loops or multiple edges. For graph theoretic terminology we follow [6]. An excellent survey of graph labeling can be found in [4].

We present this thesis in six chapters.

Chapter 1, contains some basic definitions that are needed in the subsequent chapters.

In Chapter 2, we establish that the cycle related graphs such as wheel graph, subdivision of wheel graph, closed helm graph, one point union graph, cycle graph and complete bipartite graph admit edge pair sum labeling.

Chapter 3, deals with the study of edge pair sum labeling of trees. We prove that path graph, star graph, bistar graph, perfect binary tree, tree, spider graph are edge pair sum graphs.

In Chapter 4, we establish that edge pair sum labeling of graphs obtained by some operations namely union, composition and cartesian product.
In Chapter 5, we prove that the chain of graphs such as $N$ triangular snake, $N$ quadrilateral snake and total graph admit edge pair sum labeling.

In Chapter 6, we establish that some new families of graphs such as shadow graph, jelly fish, theta graph, flower graph, shell graph and butterfly graph admit edge pair sum labeling.