Appendix III
Appendix III

Graph Theoretic Modelling applied to String Theory and Process Physics

Appendix III presents the application of graph theoretic modelling to string theory, which is till date the leading contender for a unified theory of everything. Linear graph models have been developed for Lorentz Transformations, Light-cone Coordinates, Velocity and Momentum in Light-Cone Coordinates and Lorentz Invariance with extra dimensions. Process Physics, which has now been formulated as a far more general modelling paradigm that is capable of generating complex emergent behaviour is also discussed along with the simulation of iterator equation to various stages of quantum teleportation.

1. Lorentz Transformations

In special relativity, the speed of light is the same for all inertial observers. In comparing the coordinates of events, two inertial observers, called as Lorentz observers, find that the appropriate coordinate transformations mix space and time. The events are characterized by the values of four coordinates: a time coordinate \( t \) and three spatial coordinates \( x, y \) and \( z \). It is convenient to write these four coordinates as follows:

\[
x^\mu = (x^0, x^1, x^2, x^3) \equiv (ct, x, y, z)
\]  

(1)

The subscript \( \mu \) takes the four values 0, 1, 2 and 3. The \( x^\mu \) are space-time coordinates.

A graph representation for the above basis state vectors would be as shown in Figure 1.

![Field Graph Model for \( x^\mu = (x^0, x^1, x^2, x^3) \)](image)
In a Lorentz frame $S$, let us assume that two events are represented by the coordinates $x^\mu$ and $x^\mu + \Delta x^\mu$. We now consider a second Lorentz frame $S'$ in which the same two events are described by the coordinates $x'^\mu$ and $x'^\mu + \Delta x'^\mu$ respectively. In general, not only are the coordinates $x^\mu$ and $x'^\mu$ different, the coordinate differences $\Delta x^\mu$ and $\Delta x'^\mu$ are also different.

The graph representation for $x'^\mu$ is similar to $x^\mu$ and is shown in Figure 2.

However, the intervals $\Delta s^2$ and $\Delta s'^2$ defined as follows are the same:

$$-\Delta s^2 = -(\Delta x^0)^2 + (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2 \quad (2)$$

$$-\Delta s'^2 = -(\Delta x'^0)^2 + (\Delta x'^1)^2 + (\Delta x'^2)^2 + (\Delta x'^3)^2 \quad (3)$$

We have the invariant term as follows:

$$-(\Delta x^0)^2 + (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2 = -(\Delta x^0)^2 + (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2 \quad (4)$$

In other words,

$$\Delta s^2 = \Delta s'^2 \quad (5)$$

The minus sign is in front of the time-term $(\Delta x^0)^2$, as compared to the plus sign appearing before the spacelike differences $(\Delta x^1)^2$, $(\Delta x^2)^2$, and $(\Delta x^3)^2$. This sign encodes
the fundamental difference between time and space coordinates. The minus sign in the 
\((\Delta x^0)^2\) term implies that \(\Delta s^2 > 0\) for events that are separated in time, or timelike 
separated events for which

\[
(\Delta x^0)^2 > (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2 \tag{6}
\]

This discussion holds true for \(\Delta s^{i2}\) also. The history of a particle is represented in 
spacetime as a curve, the world-line of the particle. Any two events on the world-line of a 
particle are timelike separated, because no particle can move faster than light and 
therefore the distance light would have travelled in the time interval that separates the 
events must be larger than the space separation between the events. As an example, a 
person at the time he was born and at this moment are timelike separated events. A long 
time has passed but the person has not gone that far. Events connected by the world-line 
of a photon are said to be lightlike separated. For such a pair of events, we have \(\Delta s^2 = 0\), 
because in this case the two sides of Equation (6) are identical and the spatial separation 
between the events coincides with the distance that light would have travelled in the time 
that separates the events. Two events for which \(\Delta s^2 < 0\) are said to be spacelike 
separated.

2. Light-cone Coordinates

The light-cone coordinate system is very useful in studying string theory as the 
quantization of the relativistic string can be worked out most directly in these 
coordinates. The light-cone coordinates \(x^+\) and \(x^-\) are defined as two independent linear 
combinations of the time coordinate \(x^0\) and a chosen spatial coordinate, conventionally 
taken to be as \(x^1\). This is done as follows:

\[
\begin{bmatrix}
  x^+ \\
  x^-
  \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
  1 & 1 \\
  1 & -1
  \end{bmatrix} \begin{bmatrix}
  x^0 \\
  x^1
  \end{bmatrix} \tag{7}
\]

Coordinate 
System 
Transformation 
Operator Matrix
The graph representation for the coordinates $x^0$ and $x^1$ on one hand; and $x^+$ and $x^-$, on the other hand; are shown in Figure 3(a) and Figure 3(b) respectively and the transformation operator function between the two is shown in Figure 3(c).

![Diagram of Field Graph Models for Light-Cone Coordinates](image)

**Figure 3**  Field Graph Models for Light-Cone Coordinates

The coordinates $x^2$ and $x^3$ play no role in Equation (7). In the light-cone coordinate system, $(x^0, x^1)$ are traded for $(x^+, x^-)$, but the other two coordinates, $x^2$ and $x^3$ are kept. Thus, the complete set of light-cone coordinates is $(x^+, x^-, x^2, x^3)$.

The light cone at the given event is formed by all events that can be connected through light rays with the event as shown in Figure 4. When we observe the sky at night, we basically see only the past light cone within the entire spacetime. The past of the given event is formed by all events that can influence the event (that is, which can be connected by world lines within the past light cone to the given event).
Figure 4  Understanding the light cone coordinates

The present is the region between the two light cones. Points in an observer's present are inaccessible to her/him; only points in the past can send signals to the observer. In ordinary laboratory experience, using common units and methods of measurement, it may seem that we look at the present, but in fact there is always a delay time for light to propagate. For example, we see the Sun as it was about 8 minutes ago, not as it is "right now." Unlike Galilean/Newtonian theory, the present is thick; it is not a sheet but a volume. The future of the given event is formed by all events that can be reached through time-like curves lying within the future light cone.

3. Velocity

A central concept in special relativity is proper time. Proper time is a Lorentz invariant measure of time. If we consider a moving particle and two events along its trajectory, different Lorentz observers record different values for the time interval between the two events. If we now imagine that the moving particle is carrying a clock, the proper time elapsed is the time elapsed between the two events on that clock. By definition, it is an invariant as all observers of a particular clock must agree on the time elapsed on that clock.
Similarly, the differential $ds$ is a Lorentz invariant and can be used to construct new Lorentz vectors from old Lorentz vectors. For example, a velocity four-vector $u^\mu$ is obtained by taking the ratio of $dx^\mu$ and $ds$. Since $dx^\mu$ is a Lorentz vector and $ds$ is a Lorentz scalar, the ratio is also a Lorentz vector.

$$u^\mu = c \frac{dx^\mu}{ds} = c \left( \frac{d(ct)}{ds}, \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds} \right)$$

(8)

The factor of $c$ is included to give $u^\mu$ the units of velocity. In physical system theory, all the integrals and derivatives have got the same linear graph representation. Thus, with respect to the field graph in Figure 1, we can write

$$u^\mu = c \frac{dx^m}{ds}$$

or

$$\begin{bmatrix} u^0 \\ u^1 \\ u^2 \\ u^3 \end{bmatrix} = c \frac{d}{ds} \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} = c \frac{d}{ds} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

(9)

4. Momentum in Light-Cone Coordinates

In special relativity, there is a basic relationship between the rest mass $m$ of a point particle, its relativistic energy $E$, and its relativistic momentum $\vec{p}$. This relationship with respect to field graph in Figure 4 is given by:

$$\frac{E^2}{c^2} - \vec{p}.\vec{p} = m^2 c^2$$

(10)

Energy and momentum can be used to define a momentum four-vector as follows:

$$p^\mu = \left( p^0, p^1, p^2, p^3 \right) = \left( \frac{E}{c}, p_x, p_y, p_z \right)$$

(11)
The light-cone components $p^+$ and $p^−$ of the momentum Lorentz vector are obtained as follows:

\[ p^+ = \frac{1}{\sqrt{2}}(p^0 + p^1) = -p_− \]  \hspace{1cm} (12)

\[ p^- = \frac{1}{\sqrt{2}}(p^0 - p^1) = -p_+ \]  \hspace{1cm} (13)

With respect to the Input-Output Field Graphs in Figure 3(a); and output field graph in Figure 3(b); we can write

\[
\begin{bmatrix}
  p^+ &=& -p_-
  \\
  p^- &=& -p_+
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
  1 & 1 \\
  1 & -1
\end{bmatrix} \begin{bmatrix}
  p^0 \\
  p^1
\end{bmatrix}
\]

The block diagram representation for this is shown in Figure 5.

5. Lorentz Invariance with extra dimensions

If string theory is correct, we must entertain the possibility that spacetime has more than four dimensions. The number of time dimensions must be kept equal to one – it seems very difficult, if not altogether impossible, to construct a consistent theory with more than one time dimension. The extra dimensions must therefore be spatial. Lorentz invariance is a concept that admits a very natural generalization to spacetimes with additional dimensions.
As an example, if we have a world with one time-dimension and five spatial dimensions, then we would write

\[ x^\mu = \left( x^0, x^1, x^2, x^3, x^4, x^5 \right) \]  

(14)

The graph representation for such \( x^\mu \) (six-dimensional basis state vector) is shown in Figure 6.

![Six-dimensional Field Graph Model](image)

**Figure 6** Six-dimensional Field Graph Model for a world with one-time dimension and five spatial dimensions

In such a case, the Lorentz transformations are then defined as the linear changes of coordinates that leave \( ds^2 \) invariant, where \( ds^2 \) is defined as

\[ -ds^2 = -c^2 dt^2 + \left( dx^1 \right)^2 + \left( dx^2 \right)^2 + \left( dx^3 \right)^2 + \left( dx^4 \right)^2 + \left( dx^5 \right)^2 \]  

(15)
6. Process Physics

Originally based upon a pregeometric model of the Universe, Process Physics (Kitto 2007) has now been formulated as a far more general modelling paradigm that is capable of generating complex emergent behaviour. The dominant modelling methodology of physics is static. We model the universe as a $3 + 1$ dimensional pre-existing structure, where time shares a very similar status to that of space, despite our everyday experiences of the two phenomena as being remarkably different. Process Physics sprang out of an initial desire to explain from where such a structure could have emerged (i.e., to form a pregeometric model of the Universe). A particular aim was to explain the relationships between space, time and matter, especially with respect to how such phenomena could dynamically emerge from an underlying model.

An analysis of the structures emergent from within this model reveals that it is capable of dynamically generating apparently complex hierarchical behaviour. The roots of the model, which lie in quantum field theory (QFT), and indeed quantum theories are capable of describing a far wider class of systems than is generally considered to be the case. In particular, we try to show that QFT is capable of describing complex emergent behaviour.

7. The Low Level Process Physics Model

The initial low level model constructed within the Process Physics paradigm attempts to work around the static object driven methodology of reductionism by adopting a more relational approach. This is achieved by starting with a set of nodes for the sake of analysis, but instead of focussing upon the objects themselves, the connections between individual nodes are analysed. Thus, considering a set of $N$ nodes, we assume that they are connected in some way, with a connection strength between node $i$ and node $j$ given by the real value $B_{ij}$ (Figure 7). Nodes $i$ and $j$ are considered connected if they have a non-zero $B_{ij}$ value. Arrows indicate the sign of the $B_{ij}$ value. This structure of nodes and
their connections can be represented in an antisymmetric matrix, zero values represent no connection between nodes $i$ and $j$, while a nonzero value indicating a connection is represented here as $\pm 1$ depending upon the direction of the arrow drawn in the graph.

![Figure 7](image.png)

<table>
<thead>
<tr>
<th>node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>2</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

$\mathbf{B}_{ij} = \mathbf{B}$

**Figure 7** A simple relational structure of nodes and their connections.

We represent a set of these relational values as a square antisymmetric matrix $\mathbf{B}$. Antisymmetry ensures that $\mathbf{B}_{ii} = 0$, thus avoiding explicit node self-connection. Even with the pitfalls of object driven methodologies, we are still driven to talk in terms of nodes; it is very difficult to leave an object-based methodology behind. This is due to the very nature of our analytic techniques which have always been based upon the reductionist methodology. However, any system simply positing such a priori objects cannot be considered fundamental when modelling the Universe, as the explanation of these objects must lie outside the system being modelled. A partial solution to this dilemma arises if we recognise that the nodes can, in turn, be defined for the purposes of the model as a system of nodes. It becomes apparent that all nodes can be thought of as composed of collections of nodes in turn, and in particular, that the start up nodes can be viewed as names for sub-networks of relations. This result is ensured if the constructed system exhibits self-organized criticality (SOC), which enforces a fractal structure on the system.

The SOC requirement is intrinsically linked with a new processing notion of time in this system. This is because the relational fractal structure is generated by a noisy non-linear
iterative map displaying SOC behaviour. Thus, in attempting to construct a model that
does not postulate a priori fundamental objects, we find the need to introduce a time-like
process. In contrast, standard physics with its use of a priori objects is linked with the
standard geometrical model of time and the associated problems mentioned above.

It is assumed that the $B$ matrix representing this system of nodes updates discretely
according to the following iterative process:

$$B_{ij} \rightarrow B_{ij} - \alpha \left( B + B^{-1} \right)_{ij} + \omega_{ij} \quad (16)$$

where $i, j = 1, 2 \ldots N$ and $N \rightarrow \infty$. The constant $\alpha$ is an arbitrary tuning parameter, while
the term $\omega_{ij}$ represents an additive noise term, which provides a sense of openness to the
system.

At each iteration, the action of the noise term leads to the creation of new $B_{ij}$ links,
incorporating a sense of innovation and contingency into the system. The noise term,
when used iteratively in Equation 16 is responsible for the notion of time that arises in
the model. The dynamics are irreversible, with one particular past, which can be recorded
as a history, but not relived. Future states of the system cannot be known. However
certain sets of ensemble predictions can be made. In this sense a dynamical notion of
time is captured by the system, with a markedly different ontology from the static four
dimensional spacetime of standard physics. This leads to the identification of this system
and its associated modelling techniques under the broad categorisation of Process
Physics. This modelling of time is far more appropriate in the modelling of living
systems, providing a sense of contingency and dynamism that is generally lacking from
more standard approaches.

The nonlinear matrix inversion term also performs a critical role in the system. It causes
separate structures brought into existence by the noise term to link up, modelling a
process of self-assembly. It is interesting to examine the dynamics of this process in detail.

The system can be started with $B \approx 0$ which represents the absence of any significant relational information. Under successive iterations of Equation 16, the $B$ matrix assumes a sparse structure that can be organised into a block diagonal form. This suggests that there is a tendency for nodes to organise into some sort of modular structure. If the nodes are seen as themselves made up of structures of connected nodes, then clearly some sort of modular hierarchical system has been created.

8. MATLAB Simulation of iterator equation

A simulation was carried out in MATLAB for the iterator Equation 16. The steps were:

1. A 200 x 200 ($N \times N$) $B$ Matrix was initialized and its diagonal elements were set to 0.
2. Another 1000 elements in $B$ matrix were set to small numbers between -0.1 and 0.1 so that $B_{ij} = B_{ji}$ (antisymmetry holds).
3. The values for the circuit in Figure 7 were entered into the $B$ matrix. This was the initial $B$ matrix, called as $B_{initial}$.
4. In $B_{initial}$ matrix, those elements whose absolute value was above 0.5 (a threshold for substantial degree of connectedness) were counted and these are listed in table below. In all cases, the value is 11 as represented by circuit in Figure 7. In the initial circuit, these are the only connections with substantial connectedness.

5. Iterative equation was applied $P = 10, 100$ and 1000 and 10,000 times with the following steps in each iteration:
   a) The inverse of $B$ matrix was generated and $B_{ij} - \alpha \left( B + B^{-1} \right)_{ij}$ was computed.
b) The additive noise term $\omega_{ij}$ was generated randomly within range [-0.1, 0.1] and added to the result in step 5(a).

6. In the final $B$ matrix $B_{\text{final}}$, the elements whose absolute value was above 0.5 (a threshold for substantial degree of connectedness) were again counted and these are listed in Table 1 for all the three iteration counts.

The value of $\alpha$ was taken as 0.1.

**Table 1** Results of MATLAB simulation of iterative equation

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{initial}}$</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{final}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11</td>
<td>11697</td>
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<tr>
<td>100</td>
<td>11</td>
<td>15678</td>
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<tr>
<td>1000</td>
<td>11</td>
<td>29176</td>
</tr>
<tr>
<td>10000</td>
<td>11</td>
<td>25942</td>
</tr>
</tbody>
</table>

($N = 200$ in all cases)

We have maximum connectedness for 1000 iterations. A possible explanation could be that it is of the order of non-zero elements initially assumed in the sparse B matrix (Step 2).

Splitting the large $B$ matrix into its constituent independent submatrices $B_{\text{sub}}$, we realize that each is almost singular ($\det(B_{\text{tree}}) \approx 0$) but that the noise term ensures extra $B_{ij}$ terms which lead to a small valued determinant. This small valued determinant implies that the next iterative step of the system will lead to new large valued $B_{ij}$ entries upon inversion of the $B$ matrix. Thus, under the influence of the iterative Equation 16, the system can be seen to grow with a steady increase in relational structure. The nonlinear term in the iterator equation is self-referencing; all elements of $B$ that arise in a previous
iterative step are required in the computation of the next value of each $B_{ij}$ element. Thus, this term can be seen to incorporate a weak notion of internal self-observation into the system. In particular, any node has the capacity to profoundly affect the rest of the system if it randomly receives a large $\omega_{ij}$ value at the next iteration of the map. The fact that one node is not close to another in a particular $S^3$ embedding does not mean that it cannot be affected strongly by it. This leads to a very strong form of contextual behaviour in the system, as no one element can be considered as isolated, in fact, at the next iterative step it may become strongly linked to a node which was previously not considered important to its dynamics.

Through a proper treatment of context, we can dispense with much of the confusion that often surrounds an examination of the Universe; an observer of the Universe can exist within that Universe. However, it may be necessary to adopt more than one model of the Universe in order to explain its complex emergent behaviour.

9. Simulation of Iterator equation for various stages of Quantum Teleportation

MATLAB simulation was carried out for iterator Equation 16 for stage 0 of teleportation with 2 nodes (node 1 and ground node 2).

![Figure 8](Stage 0 of teleportation (2 nodes))
Stage 0 (Composite Form)

\[
\begin{align*}
\text{Nodes} & \quad 1 & 2 \\
B & = \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}
\end{align*}
\]

The iteration equation (16) is

\[
B_{ij} \rightarrow B_{ij} - \alpha \left( B + B^{-1} \right)_{ij} + \omega_{ij}
\]

where \( i, j = 1, 2 \ldots N \) and \( N \rightarrow \infty \).

The value of \( \alpha \) was taken as 0.1 and the noise term \( \omega_{ij} \) was generated randomly as a random number between 0 and 0.1. The results are shown in Table 2. A high degree of connectedness is formed even with as low as 10 iterations.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Non-zero elements with absolute value &gt; 0.5 in</th>
<th>Non-zero elements with absolute value &gt; 0.5 in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( B_{\text{initial}} )</td>
<td>( B_{\text{final}} )</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>19178</td>
</tr>
<tr>
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<td>2</td>
<td>12711</td>
</tr>
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<td>2</td>
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</tr>
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<td>1,000,000</td>
<td>2</td>
<td>25414</td>
</tr>
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</table>

MATLAB simulation was also carried out for iteration equation for stage 0 of teleportation with 5 nodes (1, 2, 3, 4 and ground node 5).
Stage 0 (Composite Form)

**Figure 9**  Stage 0 of teleportation (5 nodes)

Nodes  1     2    3    4      5  
| 1 0 0 0 0 | -1 
| 2 0 0 0 0 | -1 
| B = 3 0 0 0 0 | -1 
| 4 0 0 0 0 | -1 
| 5 1 1 1 1 | 0 |

The value of $\alpha$ was taken as 0.1 and the noise term $\omega_j$ was generated randomly as a random number between 0 and 0.1. The results are shown in Table 3. A high degree of connectedness is formed even with as low as 10 iterations.

**Table 3**  Results of simulation of iteration equation for Figure 9

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{initial}}$</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{final}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
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<tr>
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<td>1000</td>
<td>8</td>
<td>38800</td>
</tr>
<tr>
<td>10000</td>
<td>8</td>
<td>29174</td>
</tr>
</tbody>
</table>

MATLAB simulation was carried out for iterator equation for stage 3 of teleportation shown in Figure 10.
Figure 10  Stage 3 of teleportation (9 nodes)

The results for top left matrix are shown in Table 4.

<table>
<thead>
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<th>3</th>
<th>4</th>
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<th>9</th>
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<tbody>
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<td>0</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B = 5)</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
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</tr>
</tbody>
</table>

Table 4
Table 4  Results of simulation of iterator equation for top left matrix

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{initial}}$</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{final}}$</th>
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</thead>
<tbody>
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<td>1,000</td>
<td>12</td>
<td>3984</td>
</tr>
<tr>
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<td>12</td>
<td>17404</td>
</tr>
<tr>
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<td>19763</td>
</tr>
<tr>
<td>1,00,000</td>
<td>12</td>
<td>13353</td>
</tr>
</tbody>
</table>

The results for bottom right matrix are shown in Table 5.

$$
\begin{bmatrix}
5 & 0 & 1 & 1 & 1 \\
6 & -1 & 0 & 0 & 0 \\
7 & -1 & 0 & 0 & 0 \\
8 & -1 & 0 & 0 & 0 \\
9 & -1 & 0 & 0 & 0 \\
\end{bmatrix}
$$

Table 5  Results of simulation of iterator equation for bottom right matrix

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{initial}}$</th>
<th>Non-zero elements with absolute value &gt; 0.5 in $B_{\text{final}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8</td>
<td>14931</td>
</tr>
<tr>
<td>100</td>
<td>8</td>
<td>1348</td>
</tr>
<tr>
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<td>8</td>
<td>38731</td>
</tr>
<tr>
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<td>8</td>
<td>31169</td>
</tr>
<tr>
<td>50,000</td>
<td>8</td>
<td>25964</td>
</tr>
<tr>
<td>1,00,000</td>
<td>8</td>
<td>16592</td>
</tr>
</tbody>
</table>