5.1 INTRODUCTION

This chapter deals with the theoretical analysis of the newly developed Antennas. The Geometrical Optics field and the diffraction field of the two antenna types are analysed using the image theory and the Geometrical Theory of Diffraction (GTD). The multi-corner structure demands a large number of images to be taken into consideration. Also, the span of illumination of the various image sources are to be taken care of. In the case of the Periodic Strip Attached Corner Reflector (PSACR) Antenna, the strip structures are partially transmitting and in the analysis, the individual strips are considered separately. The strip structure possesses multiple diffracting edges and causes several additional diffraction terms in the theoretical expression for the PSACR Antenna. Typical cases are analysed and are compared with the corresponding experimental patterns.

Various authors have discussed the theory of diffraction and its applications in Electromagnetics. J.B.Keller [42,43] developed the high frequency diffraction theory, the Geometrical Theory of Diffraction (GTD), which combines the classical geometrical optics and the asymptotic diffraction theory, for analysing the optical diffraction phenomenon.
The scattering of linear sources near reflecting bodies of smooth transition was studied using combined GTD-MM (Method of moments) technique by various authors [49-52]. As the antennas studied in this thesis involve abrupt transitions in shape, the image theory backed by GTD is employed for the analysis. Another method of analysis of diffraction, the Integral Equation Formulation (IEF) can be applied to the lower dimensions of our design, but is not seen to be having any advantage over the GTD as the two are in exact agreement in results in these ranges of dimensions [56].

The far fields due to the electromagnetic waves propagated from an aperture antenna consist mainly of two components, viz. the geometrical optics (GO) field and the Geometrical Theory of Diffraction (GTD) field. In the case of corner reflectors, the GO field consists of the direct radiation from the primary feed and the reflection from the sides. These can be computed using image theory. The diffraction fields are due to the radiation from the edges of the reflecting elements of the antenna. In the present analysis, the diffraction fields are derived from the expressions arrived at by E.V.Jull [45]. To get the total field, the sum of the components of the two fields at the far field point \( M(r,\theta, \phi) \) is computed.

The power due to the field at various points \( M(\theta=\pi/2) \) at constant radial distance \( 'r' \), as a function of \( \phi \) will give the H-plane radiation pattern of the corner reflector antenna. The theoretical pattern thus obtained for typical configurations of the antennas are compared with the corresponding experimental ones.

5.2 METHOD OF ANALYSIS

5.2.1 Geometrical Optics Field Analysis (Image Theory)

Kraus [4,5] and Moullin [7,8] have used the image theory for the analysis of the Corner Reflector (CR) antenna. In this theory the rays coming from the reflectors are assumed to be the contributions from the images of the exciting source. They have shown that for a corner angle \( \alpha \), the number of images formed is represented by \( m=180/\alpha \), where \( \alpha \) is the
corner half angle. Primary sources and their images for corner reflectors of angles of 45°, 60°, 90°, and 120° are shown in Fig. 5.1.

![Diagram of corner reflectors and their images for angles of 45°, 60°, 90°, and 120°](image)

*Figure 5.1 Corner reflectors and their images for angles of 45°, 60°, 90°, and 120°.*

The total field of the system can be derived by summing the contributions from the feed and its images. As the feed and its images form a circular array of sources, the array theory can be employed to calculate the total field at an observation point. For a line source feed, for \(|\phi| < \sigma\), the field due to a corner reflector antenna can be represented as

\[
E_2^c = \sum_{n=1}^{\infty} (-1)^n \frac{\exp(-jkr_n)}{kr_n},
\]

where \(n\) is the image number, \(k=2\pi/\lambda\), \(r_n\) is the far field distance of the source and images to the field point and \(\phi\) is the azimuth angle. Here the source \(I_n\) is formed from an even number of reflections if \(n\) is odd and vice versa.
Insertion of $r_n$ in the phase terms of equation (5.1) and replacing $r_n$ by $r$ in the amplitude terms gives the radiation pattern of the geometrical optics field [45].

The primary source and its images are having their own angular span of illumination as shown in Fig. 5.2. The sum of all the angular spans is $360^\circ$, the total span of illumination of the exciting dipole. Considering the effect of all the sources in their corresponding region of illumination, we can get the field of the antenna. All the sources are assumed to be having the same intensity but there will be variation in the directional orientation of the beam and the phase relation.

![Figure 5.2 Span of illumination of various sources in the case of a Square Corner Reflector Antenna. Each source $I_n$ illuminates within the angle between $n$ and $n$.](image)

In the case of multi corner antennas like Extended Triple Corner Reflector (ETCR) and Periodic Strip Attached Corner Reflector (PSACR), the number of images is getting multiplied at every corner. The angular span of illumination of the new images
will be a fraction of the span of illumination of the source from which it is originating. Thus if the design is such that, the images are suitably oriented, the beam width of the antenna will become small with an enhancement of gain. In this case, the sub reflectors must be flaring out, otherwise the rays may get directed back to the source dipole causing an enhancement in VSWR and thereby deteriorating the performance of the antenna.

5.2.2 Diffraction Field Analysis - GTD approach

The metallic structures of the new antennas will cause edge diffraction. The edges parallel to the exciting dipole act as secondary line sources sending out cylindrical wavefront, which will affect the field pattern in the H plane. The resultant field at any point will be the combined effect of the two types of fields - the Geometrical Optics (GO) field and the Diffraction (GTD) field.

The GTD fields are in general very weak in comparison with GO fields. The major contribution to the diffraction field is from the single diffraction from the edges of the conductors due to the excitation by the dipole source. The edges will act as line sources radiating in all directions. The secondary diffraction results from the excitation of the edges by the primary diffracted field. Even though higher order multiple diffraction is there, its contribution to the total diffraction field is negligible in comparison with the total diffracted field.

Each of the images of the exciting dipole can produce a similar set of diffraction field components. The vector sum of all such field components at $M(r, \theta, \phi)$ will constitute the GTD field at that point.

5.2.2.1 Geometrical theory of diffraction for edges

The diffracted field can be considered as an infinite system of rays travelling in straight lines outwards from the edge and originating from the incident ray, which struck the edge. They are diffracted in a direction, which makes the same angle with the edges as the incident ray and are quantitatively related to the field on the incident ray at the point
of diffraction. Thus when a ray is normally incident upon a straight edge, the field on a diffracted ray is

\[ A' D(\theta_0, \theta) \frac{\exp(-jkr)}{\sqrt{r}} \]  

\[ \text{\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\n

\[ \text{\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
5.2.2.2 Diffraction by a slit

In the case of a slit, the singly diffracted field is the sum of the diffracted fields from the two edges. Consider a TE-polarised plane wave

\[ E'_l = \exp[jk(x\cos\theta_0 + y\sin\theta_0)] \]

incident from \( y>0 \) on the slit \(|x|<a\) in a conducting plane in \( y=0 \), as shown in Fig. 5.4.

\[ E'_l = \exp[jk(x\cos\theta_0 + y\sin\theta_0)] \]

incident from \( y>0 \) on the slit \(|x|<a\) in a conducting plane in \( y=0 \), as shown in Fig. 5.4.

Figure 5.3 Top view of the PSACR antenna

Figure 5.4 Edge diffraction due to a slit.
The resulting diffracted field given by equation (5.2) becomes

\[
E_{z}^{\text{Sing. Diff.}} = \exp(jk\cos \theta_0)D(\theta_0, \theta) \frac{\exp(-jkr)}{\sqrt{r}} + \exp(-jka\cos \theta_0)D(\pi + \epsilon_0, \theta) \frac{\exp(-jkr)}{\sqrt{r}}
\] ...........................(5.5)

Considering the far field,

\[
E_{z}^{\text{Sing. Diff.}} = \frac{\exp[-j(kr + \pi/4)]}{\sqrt{r\lambda}} f_s(\phi)
\] ...........................(5.6)

where

\[
f_s(\phi) = \frac{j \sin[ka(Sin\phi + Cos\theta_0)]}{ksin[\frac{1}{2}(\phi - \theta_0 + \pi/2)]} - \frac{\cos[ka(Sin\phi + Cos\theta_0)]}{kCos[\frac{1}{2}(\phi + \theta_0 - \pi/2)]}
\] ...........................(5.7)

gives the diffraction pattern of the singly diffracted fields.

On the shadow boundaries of the edges of the two half-planes \(\theta_1 = \pi + \theta_0, \ \theta_2 = \theta_0\), \(D(\theta_2, \theta)\) is singular and the field from each edge is not defined by equation (5.7). In the far field, however, the singularities of the shadow boundaries cancel and produce the effect of the incident field.

Higher order diffraction fields also will be present. This will add to the field given by the equation (5.6) and the resultant field will be

\[
E_z = \frac{\exp[-j(kr + \pi/4)]}{\sqrt{r\lambda}} [f_s(\phi) + f_m(\phi)]
\] ...........................(5.8)

Where \(f_m(\phi)\) is the pattern of the multiply diffracted fields and for normal incidence \((\theta = \pi/2)\).
Since a slit in a conducting screen is a Babinet compliment of a strip, solutions for high frequency scattering by a strip are given by the expressions in the above sections.

5.3 APPLICATION TO THE NEW ANTENNAS

5.3.1 The ETCR

The ETCR has a symmetric structure as shown in Fig. 5.5. The secondary elements are placed symmetrically on the two sides of the CR at a distance of $L=\lambda/l$ or more from the apex, where 's' is the apex to dipole distance. The number of sources (including images) of the primary gets multiplied at the two sub reflectors. For finite width $W$ of the sub reflectors, the number of sources will be three times that of the original corner. The addition of the secondary elements will affect the GO fields of the antenna at a point $M(r,\theta,\phi)$ in two different ways.

1. It shields a portion of the field from the CR antenna, thereby resetting the shadow boundaries of the GO field.

2. It changes the energy distribution at the point due to the contribution from the resulting sources.

The geometrical optics field of an ETCR antenna with a line source feed is given by the sum of the contributions from the feed and the sources formed by reflection. The field of a single source within its limits of illumination can be represented by

$$E_n^G = (-1)^{n+1} \frac{\exp(-jkr_n)}{\sqrt{kr_n}}.$$
where $r_n$ is the distance of the source 'I_n' to the point $M(r, \theta, \phi)$.

**Theoretical Analysis**

To field point $M$

In the case of an ETCR the field distribution depends on the limits of illumination of the images. A portion of the angular span of illumination of the source and images of the primary corner gets reoriented by the addition of sub reflectors and becomes the span of illumination of the newly formed sources. The total number of sources is decided by the primary corner angle. The GO field $E^{GO}$ is computed by adding the GO field contributions from the different sources at the point 'M'.

For computing the GTD field of the ETCR, we can consider the sub reflector as an extension of the primary. As the diffracting edges are very widely separated the single diffraction terms will be sufficient. The diffraction field can be evaluated using equation (5.4) modified for the corresponding case.

**Fig.5.5 Top view of an ETCR antenna**

In the case of an ETCR the field distribution depends on the limits of illumination of the images. A portion of the angular span of illumination of the source and images of the primary corner gets reoriented by the addition of sub reflectors and becomes the span of illumination of the newly formed sources. The total number of sources is decided by the primary corner angle. The GO field $E^{GO}$ is computed by adding the GO field contributions from the different sources at the point 'M'.

For computing the GTD field of the ETCR, we can consider the sub reflector as an extension of the primary. As the diffracting edges are very widely separated the single diffraction terms will be sufficient. The diffraction field can be evaluated using equation (5.4) modified for the corresponding case.
\[
E_n^d = \frac{\exp(-jkr_n)}{kr_n} \cdot \\
\exp \left[ -jk \left( r - \sqrt{\left( L \sin \sigma + w \sin f \right)^2 + \left( w \cos f \right)^2} \right) \times \cos \left[ \tan^{-1} \left( \frac{L \sin \sigma + w \sin f}{w \cos f} \right) + \phi \right] \right] \\
\times \frac{-\exp(j\pi/4)}{2\sqrt{2}\pi k} \left( \sec \left( \theta - \theta_a \right) - \cos \left( \theta + \theta_a \right) \right)
\]

The total GTD field \( E^{GTD} \) is the sum of the diffraction field components \( E^d \) from the two diffracting edges.

The total field \( E_z \) at any point \( M(r, \theta, \phi) \) is the sum of the GO field and the GTD field at the point.

\[ E_z = E^{GO} + E^{GTD} \]

The square of the resultant electric field at a point will give the power at the point. With the help of 'Mathematica 3.0' software, the total field \( E_z \) and the intensity corresponding to definite values of \( \phi \) at equal spacing in the azimuth plane is computed and is plotted against \( \phi \) in selected configurations. The theoretical and experimental curves are superimposed.

### 5.3.1.1 Case 1.

- Primary corner angle, \( \alpha \) = 180°
- Secondary (passive) corner angle, \( \beta \) = 78°
- Width of primary \( \triangle \) = 1\( \lambda \)
- Width of sub reflector \( W \) = 5\( \lambda \)
- Flare of secondary 'f' = \((\alpha/2) - \beta\) = 12°
- The apex to dipole distance 's' = 0.2\( \lambda \)
Figure 5.6. 180°ETCR

(a) The GO Field

There are six images to be considered in this case. The distance of the point M from each of the sources can be expressed in terms of the distance 'r' from the apex and the dimensions in the Fig. 5.6. The distances are

1. \( r_1 = r - s \cos \phi \)
2. \( r_2 = r + s \cos \phi \)
3. \( r_3 = r + L \sin \phi + (L^2 + s^2)^{1/2} \cos \left( \tan^{-1} \left( \frac{L}{s} \right) - 2f - \phi \right) \)
4. \( r_4 = r + L \sin \phi - (L^2 + s^2)^{1/2} \cos \left( \tan^{-1} \left( \frac{L}{s} \right) - 2f + \phi \right) \)
5. \( r_5 = r - L \sin \phi + (L^2 + s^2)^{1/2} \cos \left( \tan^{-1} \left( \frac{L}{s} \right) - 2f + \phi \right) \)
6. \( r_6 = r - L \sin \phi - (L^2 + s^2)^{1/2} \cos \left( \tan^{-1} \left( \frac{L}{s} \right) - 2f - \phi \right) \)
Also the limits of illuminations can be obtained in terms of the angles and distances in the figure. The limits are,

<table>
<thead>
<tr>
<th>Source number</th>
<th>Lower limit of illumination</th>
<th>Upper limit of illumination</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-a</td>
<td>+a</td>
<td>$a = \tan^{-1}\left(\frac{L + w \sin f}{w \cos f - s}\right)$</td>
</tr>
<tr>
<td>2</td>
<td>-b</td>
<td>+b</td>
<td>$b = \tan^{-1}\left(\frac{L + w \sin f}{w \cos f + s}\right)$</td>
</tr>
<tr>
<td>3</td>
<td>b-2f</td>
<td>$\tan^{-1}(L/s) - 2f$</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>a-2f</td>
<td>$180 - [2f + \tan^{-1}(L/s)]$</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>2f-b</td>
<td>$2f - \tan^{-1}(L/s)$</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>2f-a</td>
<td>$[2f + \tan^{-1}(L/s)] - 180$</td>
<td>--</td>
</tr>
</tbody>
</table>

The field due to each source within its limit of illumination is given by substituting the value of $r_n$ in equation (5.10), which is,

$$E_n^G = (-1)^{n+1} \frac{\exp(-jkr_n)}{\sqrt{kr_n}}$$

and the total GO field $E^G_O$ at the point 'M' is the sum of the individual contributions from these sources.

(b) The GTD field

In this case, the GTD field from one edge due to the original source is given by equation (5.4) modified as below,
Theoretical Analysis

where,

\[ r_0 = [(w \cos \phi - s)^2 + (w \sin \phi + L)^2]^{1/2} \]
\[ \theta = (\pi / 2) - f - \phi \] and
\[ \theta_0 = (\pi / 2) - [\tan^{-1}([L + w \sin (f)])/[w \cos (f) - s]) - f]. \]

The above equations with corresponding modification will provide the diffraction field \( E_i^d \) due to the other edge. The field from the two edges due to the image source, \( E_i^d \) and \( E_i^d \), can also be obtained similarly by substituting \( r_0 \) and \( \theta_0 \) for the corresponding case. The reversal of polarity of this source occurring due to reflection also is taken into consideration. The total diffracted field is given by

\[ E_{GTD} = \sum_{i=1}^{4} E_i^d \] ..............................(5.12)
Figure 5.7 Theoretical and experimental radiation patterns of 180° ETCR with sub reflectors of length 5λ

(c) Total field

The sum of the GO field and the diffraction field gives the total field.

\[ E_z = E_{GO} + E_{GTD} \]

The total field at a fixed distance from the apex is calculated using 'Mathematica 3.0' software for various values of the azimuth angle \( \phi \). The radiated power is calculated from the field, normalised and plotted against the azimuth angle. The theoretical and experimental radiation patterns in the H-plane are compared in Fig. 5.7
5.3.1.2 Case 2

Primary corner angle 'α'
= 120°

Secondary (passive) corner angle, 'β'
= 35°

Width of primary 'L'
= \(\lambda\)

Width of sub reflector W
= 5\(\lambda\)

Flare of secondary, 'f'
= \((\alpha/2) - \beta\)
= 25°

The apex to dipole distance 's'
= 0.3\(\lambda\)

---

(a) The GO Field

In this case, there are nine sources to be considered. Distances of the various sources \(r_n\) to the field point M and their illumination limits are as given below.
1. \( r_1 = r - s \cos \phi \)
2. \( r_2 = r + s \cos(\sigma + \phi) \)
3. \( r'_2 = r + s \cos(\sigma - \phi) \)
4. \( r_3 = r + x \cos(\psi - 2f + e - \phi) - L \cos(\sigma + \phi) \)
5. \( r'_3 = r + x \cos(\psi - 2f + e + \phi) - L \cos(\sigma - \phi) \)
6. \( r_4 = r + x \cos(\psi - 2f - \phi) - L \cos(\sigma + \phi) \)
7. \( r'_4 = r + x \cos(\psi - 2f + \phi) - L \cos(\sigma - \phi) \)
8. \( r_5 = r + (L + s) \cos(\sigma - 2f - \phi) - L \cos(\sigma + \phi) \)
9. \( r'_5 = r + (L + s) \cos(\sigma - 2f + \phi) - L \cos(\sigma - \phi) \)

These values can be substituted in equation (5.10)
$E_n^G = (-1)^{n+1} \frac{\exp(-jkr_n)}{\sqrt{kr_n}}$, to get the GO field of source 'I_n' within the corresponding fields of illumination. The total GO field $E^G$ at the point 'M' is calculated by adding the contribution from each of the individual sources.

(b) The GTD field

The GTD field from one edge due to the original source is given by equation (5.4) modified as below,

$$E_{1t}^d = \frac{\exp(-jkr_0)}{\sqrt{kr_0}} \times \exp\left[ -jk\left( r - \sqrt{(LS\sin \sigma + w\sin f)^2 + (w\cos f)^2} \times \cos \left( \tan^{-1}\left( \frac{LS\sin \sigma + w\sin f}{w\cos f} \right) + \phi \right) \right) \right]$$

$$\times \frac{-\exp(j\pi/4)}{2\sqrt{2}\pi k} \left( \sec \left( \theta - \theta_0 \right) - \csc \left( \theta + \theta_0 \right) \right)$$

where,

$r_0 = [(LC\cos \sigma + w\cos f - s)^2 + (w\sin f + L\sin \sigma)^2]^{1/2}$

$\theta = (\pi/2) - f - \phi$ and

$\theta_0 = (\pi/2) - [\tan^{-1}(LS\sin \sigma + w\sin f)/[LC\cos \sigma + w\cos f - s] - f]$.

The above equations with corresponding modification for $\theta_0$ will provide the diffraction field $E_{2t}^d$ due to the other edge. The field from the two edges due to the image sources, $E_{1t}^d$ to $E_{2t}^d$ is also obtained similarly by substituting the corresponding values of $r_0$ and $\theta_0$. The change of polarity of these sources occurring due to reflection also is taken into consideration. The total diffracted field is given by
Theoretical Analysis

\[ E^{GTD} = \sum_{i=1}^{6} E_i^d \]

(c) Total field

The sum of the GO field and the diffraction field gives the total field.

\[ E_z = E^{GO} + E^{GTD} \]

The total field at a fixed distance from the apex, for various values of the azimuth angle \( \phi \) is calculated using 'Mathematica 3.0' software. The radiated power is calculated from the field, normalised and plotted against the azimuth angle \( \phi \).

The theoretical radiation pattern in the H-plane is presented in Fig. 5.9 along with the experimental pattern for the same configuration.

![Theoretical and experimental radiation patterns of 120° ETCR](image)

*Figure 5.9 Theoretical and experimental radiation patterns of 120° ETCR*
5.3.1.3 Case 3.

- Primary corner angle \( \alpha \) = 90°
- Secondary (passive) corner angle, \( \beta \) = 30°
- Width of primary \( L \) = 1\( \lambda \)
- Width of sub reflector \( W \) = 5\( \lambda \)
- Flare of secondary, \( \theta \) = \((\alpha/2) - \beta\) = 15°
- The apex to dipole distance \( s \) = 0.5\( \lambda \)

\[ \text{Figure 5. 10. ETCR 90°} \]

\( (a) \) The GO Field

The four images of the square corner becomes twelve when it is modified into an ETCR. The distances from the various sources to the field point 'M' are given below.
The illumination limits are as given below:

<table>
<thead>
<tr>
<th>Source number</th>
<th>Lower limit of illumination</th>
<th>Upper limit of illumination</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>$-a$</td>
<td>$+a$</td>
<td>$a = \tan^{-1}\left[\frac{\eta \sin \psi + w \sin f}{\eta \cos \psi + w \cos f}\right]$</td>
</tr>
<tr>
<td>$I_2$</td>
<td>$-b$</td>
<td>$\sigma - \theta$</td>
<td>$b = \tan^{-1}\left[\frac{s + L \sin \sigma + w \sin f}{w \cos f + L \cos \sigma}\right]$</td>
</tr>
<tr>
<td>$I_3$</td>
<td>$-e$</td>
<td>$+e$</td>
<td>$c = \tan^{-1}\left[\frac{s + L \sin \sigma}{L \cos \sigma}\right] - 2f$</td>
</tr>
<tr>
<td>$I_4$</td>
<td>$-(\sigma - \theta)$</td>
<td>$+b$</td>
<td>$e = \tan^{-1}\left[\frac{L \sin \sigma + w \sin f}{s + L \cos \sigma + w \cos f}\right]$</td>
</tr>
<tr>
<td>$I_5$</td>
<td>$-(b - 2f)$</td>
<td>$+c$</td>
<td>$h = \tan^{-1}\left[\frac{L \sin \sigma}{s + L \cos \sigma}\right]$</td>
</tr>
<tr>
<td>$I_6$</td>
<td>$-(\psi - 2f)$</td>
<td>$-(a - 2f)$</td>
<td>$m = 2f - e$</td>
</tr>
<tr>
<td>$I_7$</td>
<td>$-(g - 2f)$</td>
<td>$\psi - 2\sigma$</td>
<td>$\psi = \sigma + \tan^{-1}\left[\frac{\sin \sigma}{L / s - \cos \sigma}\right]$</td>
</tr>
<tr>
<td>$I_8$</td>
<td>$-(h - 2f)$</td>
<td>$+m$</td>
<td>$\eta = (L^2 - s^2 \sin^2 \psi)^{1/2} - \cos \psi$</td>
</tr>
<tr>
<td>$I_9$</td>
<td>$-c$</td>
<td>$b - 2f$</td>
<td>$\theta = \tan^{-1}\left[\frac{\sin \sigma}{L / s - \cos \sigma}\right]$</td>
</tr>
<tr>
<td>$I_{10}$</td>
<td>$-(a - 2f)$</td>
<td>$-(\psi - 2f)$</td>
<td>$g = \tan^{-1}\left[\frac{L \sin \sigma + w \sin f - s}{L \cos \sigma + w \cos f}\right]$</td>
</tr>
<tr>
<td>$I_{11}$</td>
<td>$2\sigma - \psi$</td>
<td>$g - 2f$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>$I_{12}$</td>
<td>$-m$</td>
<td>$h - 2f$</td>
<td>$\psi$</td>
</tr>
</tbody>
</table>

1. $r_1 = r - s \cos \phi$
2. $r_2 = r - s \sin \phi$
3. $r_3 = r + s \cos \phi$
4. $r_4 = r + s \sin \phi$
5. $r_5 = r + x \cos \phi - 2(x - s) \cos \alpha + (x - s) \cos(\alpha - \phi)$
6. $r_6 = r + x \cos \phi - s \cos(\alpha - \phi)$
7. $r_7 = r + s \sin \phi$
8. \( r_8 = r + x\cos\phi - x\cos(\frac{3}{2}\alpha - \phi) \)

9. \( r_9 = r + x\cos\phi - 2x\cos(\frac{3}{2}\alpha) + x\cos(\frac{3}{2}\alpha - \phi) \)

10. \( r_{10} = r - x\cos\phi + s\cos(\alpha - \phi) \)

11. \( r_{11} = r - s\sin\phi \)

12. \( r_{12} = r + x\cos\phi - (x - s)\cos(\alpha - \phi) \)

These values can be substituted in equation (5.10)

\[ E_n^G = (-1)^{n+1} \frac{\exp(-jkr_n)}{\sqrt{kr_n}} \]

to get the GO field of source 'I_n' within its limit of illumination. The total GO field is calculated by adding the contribution from each of the individual sources.

(b) The GTD field

As in the previous case, the GTD field from one edge due to the original source is given by equation (5.4) modified as below,

\[
E_1^G = \frac{\exp(-jkr_0)}{\sqrt{kr_0}} \times \\
\exp \left[ -j k \left( r - \sqrt{(L\sin\sigma + w\sin f)^2 + (w\cos f)^2} \times \cos \left[ \frac{\tan^{-1} \left( \frac{L\sin\sigma + w\sin f}{w\cos f} \right)}{2} + \phi \right] \right) \right] \\
\times \frac{-\exp(j\pi/4)}{2\sqrt{2}nk} \left( \sec \frac{\theta - \theta_0}{2} - \csc \frac{\theta - \theta_0}{2} \right)
\]

where,

\[
r_0 = [(L\cos\sigma + w\cos f - s)^2 + (w\sin f + L\sin\sigma)^2]^{1/2}
\]

\[
\theta = (\pi/2) - f - \phi \quad \text{and}
\]

\[
\theta_0 = (\pi/2) - \left[ \tan^{-1} \left[ \frac{L\sin\sigma + w\sin f}{(L\cos\sigma + w\cos f - s)} \right] - f \right].
\]
The above equations with corresponding modification for \( \theta_0 \) will provide the diffraction field \( E_d \) due to the other edge. The field from the two edges due to the image sources, \( E_1^d \) to \( E_8^d \) can also be obtained similarly by substituting the corresponding values of \( r_0 \) and \( \theta_0 \). Also the polarity reversal occurring during each reflection is accommodated in the corresponding expressions. The total diffracted field is given by

\[
E^{GTD} = \sum_{i=1}^{8} E_i^d
\]

(c) Total field

The sum of the GO field and the diffraction field gives the total field.

\[
E_z = E^{GO} + E^{GTD}
\]

The total field at a fixed distance from the apex, for various values of the azimuth angle \( \phi \) is calculated using 'Mathematica 3.0' software. The radiated power is calculated from the field, normalised and plotted against the azimuth angle.

The theoretical radiation pattern in the H-plane is presented in Fig. 5.11 along with the experimental pattern for the same configuration.
Theoretical Analysis

90 Degree ETCR

![Figure 5.11 Theoretical and Experimental radiation patterns of 90° ETCR](image)

5.3.2 The PSACR

In the case of the PSACR the secondary reflectors are structures with complimentary metal and air gap. Structures with period $\lambda$ and equal width for the strip and gap are analysed here.

The GO field

In this case, as the reflectors are leaky structures, the GO field contribution from the additional sources formed due to reflection on them cannot be computed directly. So, the strips and gaps each of width $p/2$ are denoted as zones, and the zone wise analysis of the GO field is carried out. '$\omega$' is the zone number.
As the alternate zones are transmitting and reflecting, the limits of illumination and direction of radiation are to be determined for each zone. For any source, the strip structure will contribute $2n$ components in the GO field, $n$ each, with opposite polarity for the transmitting and reflecting zones. The odd numbered zones are penetrable. The even numbered zones will reflect the field and within its limit of illumination, the field is taken as the contribution of the corresponding image on the sub reflector.

As the obliquity of the strip structure to the radiation increases towards the farther end, the structure may appear as a continuous sheet beyond a few periods. This aspect also is taken into consideration while executing the zone wise calculation of the GO field.

The gaps being oblique to radiation, they are considered to be penetrable only if the projection of the gap perpendicular to the radiation, $x_0' = (p/2) \sin \theta_{0o}$ in Fig. 5.12 is more than $\lambda/8$, the wire spacing adopted in the grid reflector by Kraus [15].

Here $\theta_{0o} = \psi_0 - f$
When the penetrability ceases i.e., when $x_o \leq \lambda/8$, the strip structure acts as a continuous sheet.

**The diffraction field**

In the diffraction analysis, the strip structure can be considered either as a set of conducting strips or as a set of slots in a conducting sheet. In both the cases the number of diffracting edges and the diffracted field will be the same. Let us consider the strip structure as a set of strips in addition to the edges existing for the primary corner.

The diffraction field is more complex in the case of the PSACR than that of the ETCR. The edges of each of the strips will act as secondary sources of radiation. Thus the number of diffraction sources will increase with the number of strips and also with the number of exciting sources.

The contributions to the diffraction field can be listed as below.

1. **The singly diffracted fields**

   This will consist of the diffracted field from the edges of the primary reflector and the strips. When there are 'n' strips on the strip structure on one side, there will be $2n+1$ edges on one side, resulting in $2(2n+1)$ diffracting edges in all. The exciting sources are the primary source and its images on the primary reflectors. So altogether we will have $2m(2n+1)$ contributions for the diffracted field. Where $m=180/\sigma$ is the number of images for the primary corner. This is calculated using equation (5.4) for each component.

2. **Multiple diffraction fields due to the edges at p/2 separation.**

   The diffracting edges will excite each other and will cause double and higher order diffraction. In this case, only neighbouring edges are considered and there will be diffraction across a strip and also a gap. Thus the total number of pairs of sources will be $2(2n)$ and this can be analysed with the help of equation (5.8) by substituting for $f_m$. 
(3) Multiple diffraction taking place at edges with greater spacing.

(4) Diffraction caused by the images on the sub reflectors

(5) Reflection after diffraction.

(6) Single and multiple diffraction across the axis between the strips on opposite sides.

The contributions from the last four types can be ignored.

The sum of the individual diffraction field components will give the total diffraction field, $E_{\text{GTD}}$.

The sum of the GO field and the diffraction field gives the total field.

$$E_z = E_{\text{GO}} + E_{\text{GTD}}$$

The total field at a fixed distance from the apex is calculated using 'Mathematica 3.0' software for various values of the azimuth angle $\phi$. The radiated power is calculated from the field, normalised and plotted against the azimuth angle $\phi$. The radiation pattern thus obtained is superimposed on the experimental pattern. Typical cases are presented in Fig. 5.14, 5.15, 5.17 and 5.19. The typical cases presented below have representative nature. Corresponding values given to $L$, $n$ and $f(\beta)$ can yield the patterns of the different configurations studied.

5.3.2.1 Case 1. 180 PSACR (Axial single lobe,)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary corner angle 'a'</td>
<td>180°</td>
</tr>
<tr>
<td>Secondary (passive) corner angle, 'β'</td>
<td>78°</td>
</tr>
<tr>
<td>Width of primary 'L'</td>
<td>$L$</td>
</tr>
<tr>
<td>Number of strips on sub reflector 'n'</td>
<td>6</td>
</tr>
<tr>
<td>Number of zones to be analysed</td>
<td>$2n$ = 12</td>
</tr>
<tr>
<td>Flare of secondary, 'f'</td>
<td>$(\alpha/2) - \beta = 12°$</td>
</tr>
<tr>
<td>The apex to dipole distance $s'$</td>
<td>$0.2\lambda$</td>
</tr>
<tr>
<td>Period of the strip structure 'p'</td>
<td>$\lambda$</td>
</tr>
</tbody>
</table>
(a) The GO Field

The image sources and the distances of the different sources to the field point M are the same as in case 1 in section 5.3.1.1. The limits of illumination of sources I₁ and I₂ are also similar. The other sources are contributions of the strip structures.

Referring to Fig. 5.13, the angular span of illumination of any of the zones of the strip structure is given by the equation,

\[ S_{\nu} = \psi_{\nu-1} - \psi_{\nu} \]  

.......(5.13)

where 'u' is the zone number,

\[ \psi_{\nu} = \tan^{-1}\left[ \frac{L + \nu(p/2)\sin[f]}{\nu(p/2)\cos[f] - s} \right], \text{ for illumination from } I_1 \]  

.......(5.14)

and,
\[ \psi_\nu' = \tan^{-1} \left[ \frac{L + \nu(p/2)\sin[f]}{s + \nu(p/2)\cos[f]} \right], \text{ for illumination from } I_2, \quad \ldots \ldots \quad (5.15) \]

The field incident on the odd numbered zones will pass through and contribute to side-lobes in the direction \(-\psi_{\nu-1}\) to \(-\psi_\nu\) and \(\psi_\nu\) to \(\psi_{\nu+1}\) for radiation from source \(I_1\) and \(-\psi_{\nu-1}'\) to \(-\psi_{\nu}'\) and \(\psi_{\nu}'\) to \(\psi_{\nu+1}'\) for radiation from source \(I_2\). But all the odd numbered zones are not penetrable so only a limited number of zones will be forming side-lobes.

When the penetrability ceases, the reflectors appear as a continuous sheet and the limits of illumination are modified accordingly. In this case, the penetrability criterion finds three penetrable gaps each \((\nu=1, 3 \text{ and } 5)\) for the exciting dipole and its image on the primary.

**Table 5.1 Limits of illumination of the reflecting strips for the image sources on the sub reflectors.**

<table>
<thead>
<tr>
<th>Source number</th>
<th>Lower limit of illumination</th>
<th>Upper limit of illumination</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1)</td>
<td>([\tan^{-1}(L/s) - 2f - \psi_0 + \psi_\nu])</td>
<td>([\tan^{-1}(L/s) - 2f - \psi_0 + \psi_{\nu-1}])</td>
</tr>
<tr>
<td>(I_3)</td>
<td>([90 + \tan^{-1}(s/L) - 2f - \psi_0 + \psi_{\nu+1}])</td>
<td>([90 + \tan^{-1}(s/L) - 2f - \psi_0 + \psi_{\nu+1}])</td>
</tr>
<tr>
<td>(I_5)</td>
<td>([-\tan^{-1}(L/s) - 2f - \psi_0 + \psi_{\nu-1}])</td>
<td>([-\tan^{-1}(L/s) - 2f - \psi_0 + \psi_{\nu}])</td>
</tr>
<tr>
<td>(I_7)</td>
<td>([-90 + \tan^{-1}(s/L) - 2f - \psi_0 + \psi_{\nu-1}])</td>
<td>([-90 + \tan^{-1}(s/L) - 2f - \psi_0 + \psi_{\nu}])</td>
</tr>
</tbody>
</table>

The field due to each component, within its limit of illumination is given by substituting the value of \(r_\nu\) from section 5.3.1.1 in equation (5.10), which is,

\[ E^G_n = (-1)^{(n+1)} \frac{\exp(-jk r_\nu)}{\sqrt{kr_\nu}} \]

and the total GO field \(E^G\) at the point 'M' is the sum of the individual components.

(b) The diffraction field.

The single diffraction field can be calculated by using equation (5.4) modified as given below for each of the edges from \(\nu=0\) to \(\nu=2n\)
\[ E^d_v = \frac{\exp(-jkr_v)}{\sqrt{kr_v}} \times \]

\[
\exp \left[ -jk \left( r - \sqrt{\left( L \sin \sigma + \frac{v \sin f}{2} \right)^2 + \left( \frac{v \cos f}{2} \right)^2} \right) \cos \left( \arctan \left( \frac{L \sin \sigma + \frac{v \sin f}{2}}{\frac{v \cos f}{2}} \right) + \phi \right) \right] \\
\times \frac{1}{\sqrt{r}} \exp \left( \frac{j\pi \theta}{4} \right) \left( \sec \left( \frac{\theta - \theta_v}{2} \right) - \csc \left( \frac{\theta + \theta_v}{2} \right) \right)
\]

where,

\[ r_v = \left[ \left( L \cos \sigma + \frac{v \cos f}{2} \right)^2 + \left( \frac{v \sin f}{2} \sin f + L \sin \sigma \right)^2 \right]^{1/2} \]

\[ \theta = (\pi / 2) - f - \phi \]

and

\[ \theta_v = (\pi / 2) - \left[ \arctan \left( \frac{L \sin \sigma + \frac{v \sin f}{2}}{L \cos \sigma + \frac{v \cos f}{2} \cos f - s} \right) - f \right] \]

This is applied for each of the sources separately and the polarity of the exciting source is considered. This will yield \( 4(2n+1) \) components. Thus this case with six strips on the secondary will have 52 components for the single diffraction. The multiple diffraction field calculated using equation (5.8) is insignificantly small and is ignored.

The total diffraction field \( E^{\text{TGD}} \) is the sum of the individual diffraction field components.

(c) **Total field**

The sum of the GO field and the Diffraction field gives the total field.

\[ E^t = E^{\text{GO}} + E^{\text{TGD}} \]

The total field is calculated and the intensity is plotted against the azimuth angle to get the radiation pattern.
The theoretical and experimental patterns are compared in Fig. 5.14.

![Image of PSACR 180 Degrees]

**Figure 5.14 Theoretical and experimental patterns of 180° PSACR.**

5.3.2.2 Case 2. 180° PSACR (Twin lobe radiation pattern)

The above configuration can produce a twin lobed radiation pattern for a different value of β. All the other aspects are common. The diffraction field and the geometrical field are calculated for the corresponding value of the flare angle and the radiation pattern is plotted. A deep null at the centre is the characteristic feature of this configuration. The pattern is compared along with the experimental one in Fig. 5.15.
180 PSACR - Twin lobed radiation pattern

![Graph showing the Twin lobe radiation pattern from 180° PSACR with theoretical and experimental data.]

Figure 5.15 Twin lobe radiation pattern from 180° PSACR

5.3.2.3 Case 3. 120° PSACR

- Primary corner angle 'α' = 120°
- Secondary (passive) corner angle, 'β' = 45°
- Width of primary, 'L' = 1λ
- Number of strips on sub reflector, 'n' = 6
- Flare of secondary, 'f' = (c/2) - β = 15°
- The apex to dipole distance, 's' = 0.3λ
- Period of the strip structure, 'p' = 1λ
This case has the number of images and the distance of all the sources to the field point common with Case 2 of section 5.3.1. 2. The limits of illumination of the first three sources also are the same. But the sub reflectors are to be considered as separate strips and gaps depending on the penetrability of the gaps. In this case, source 1 will find only three penetrable gaps on both the reflectors where as source 2 and 2’ will find 4 gaps in the strip structure on the opposite side and two on that on the same side. Thus 36 terms will represent the GO field due to strip structure reflection if the number of strips ‘n’ is 4 or above. All the spans of illumination are determined separately and the resultant of the 39 field components is taken.
Referring to Fig. 5.16, the angular span of illumination of any of the zones of the strip structure is given, by the equation

\[ S_v = \psi_{v-1} - \psi_v, \]  

where \( v \) is the zone number, and

\[ \psi_v = \tan^{-1}\left[ \frac{L \sin(60°) + v(p/2)\sin(f)}{L \cos(60°) + v(p/2)\cos(f) - s} \right], \]  for illumination from source \( I_1 \), \( \cdots (5.16) \)

\[ \psi'_{v} = \tan^{-1}\left[ \frac{L \sin(60°) + v(p/2)\sin(f) - s \sin(60°)}{L \cos(60°) + v(p/2)\cos(f) + s \cos(60°)} \right], \]  for illumination from source \( I_2 \) and \( I_2' \) on the same side reflector \( \cdots (5.17) \)

and

\[ \psi''_{v} = \tan^{-1}\left[ \frac{L \sin(60°) + v(p/2)\sin(f) + s \sin(60°)}{L \cos(60°) + v(p/2)\cos(f) + s \cos(60°)} \right], \]  for illumination from source \( I_2 \) and \( I_2' \) on the reflector on the opposite side. \( \cdots (5.18) \)

The field incident on the odd numbered zones within the penetration limit, will pass through and contribute to side-lobes in the direction \( -\psi_v \) to \( \psi_v \) and \( \psi_v \) to \( \psi_{v-1} \) for radiation from source \( I_1 \), \( -\psi'_{v-1} \) to \( \psi'_{v-1} \) and \( \psi''_{v-1} \) to \( \psi''_{v-1} \) for radiation from source \( I_2 \) and \( -\psi''_{v-1} \) to \( -\psi'_{v-1} \) and \( \psi'_{v-1} \) to \( \psi''_{v-1} \) for radiation from source \( I_2' \).
Table 5.2 Limits of illumination of the reflecting strips for the image sources on the sub reflectors.

<table>
<thead>
<tr>
<th>Source number</th>
<th>Lower limit of illumination</th>
<th>Upper limit of illumination</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_3$</td>
<td>$2f-e - \psi'_0 + \psi'_v$</td>
<td>$2f - e - \psi'_0 + \psi'_v - 1$</td>
<td>$a = \tan^{-1}\left[ \frac{LS \sin \sigma + ws \sin \varphi}{LC \cos \sigma + w \cos \varphi - s} \right]$</td>
</tr>
<tr>
<td>$I'_3$</td>
<td>$e - 2f + \psi'_0 - \psi'_v$</td>
<td>$e - 2f + \psi'_0 - \psi'_v$</td>
<td>$b = \tan^{-1}\left[ \frac{w \sin \varphi + L \sin \sigma + s \sin 2\sigma}{w \cos \varphi + L \cos \sigma - s \cos 2\sigma} \right]$</td>
</tr>
<tr>
<td>$I_4$</td>
<td>$2f - a - \psi_0 + \psi_v$</td>
<td>$2f - a - \psi_0 + \psi_v - 1$</td>
<td>$c = \tan^{-1}\left[ \frac{w \sin \varphi + L \sin \sigma - s \sin 2\sigma}{w \cos \varphi + L \cos \sigma + s \cos 2\sigma} \right]$</td>
</tr>
<tr>
<td>$I'_4$</td>
<td>$a - 2f + \psi_0 - \psi_v$</td>
<td>$a - 2f + \psi_0 - \psi_v$</td>
<td>$d = \tan^{-1}\left[ \frac{w \sin \varphi - L \sin \sigma + s \sin 2\sigma}{w \cos \varphi - L \cos \sigma - s \cos 2\sigma} \right]$</td>
</tr>
<tr>
<td>$I_5$</td>
<td>$2f - b - \psi''_0 + \psi''_v$</td>
<td>$2f - b - \psi''_0 + \psi''_v - 1$</td>
<td>$e = \tan^{-1}\left[ \frac{w \sin \varphi - L \sin \sigma + s \sin 2\sigma}{w \cos \varphi - L \cos \sigma + s \cos 2\sigma} \right]$</td>
</tr>
<tr>
<td>$I'_5$</td>
<td>$b - 2f + \psi''_0 - \psi''_v$</td>
<td>$b - 2f + \psi''_0 - \psi''_v - 1$</td>
<td>$f = \tan^{-1}\left[ \frac{w \sin \varphi + L \sin \sigma + s \sin 2\sigma}{w \cos \varphi + L \cos \sigma + s \cos 2\sigma} \right]$</td>
</tr>
</tbody>
</table>

The field due to each component, within its limit of illumination is given by substituting the value of $r_0$ from section 5.3.1.2 in equation (5.10), which is,

$$E_\alpha^n = (-1)^{(n+1)} \frac{\exp(-jk_{r_0})}{\sqrt{k_{r_0}}}$$

and the total GO field $E^{GO}$ at the point 'M' is the sum of the individual components.

(b) The diffraction field.

The single diffraction field due to source $I_1$ can be calculated by using equation (5.4) modified as given below for each of the edges from $v=0$ to $v=2n$.
Theoretical Analysis

\[ E_\sigma' = \frac{\exp(-jkr_0)}{kr_0} \times \exp \left[ -jk \left( r - \sqrt{(L\sin\sigma + (\upsilon/2)\sin f)^2 - ((\upsilon/2)\cos f)^2} \right) \times \cos \left( \tan^{-1} \left( \frac{L\sin\sigma + (\upsilon/2)\sin f}{(\upsilon/2)\cos f} \right) + \phi \right) \right] \]

\[ \times \frac{-\exp(j\pi/4)}{2\sqrt{2\pi k}} \left( \sec \left( \frac{\theta - \theta_0}{2} \right) - \csc \left( \frac{\theta + \theta_0}{2} \right) \right) \]

where,

\[ r_0 = \left[ \left( L\cos\sigma + (\upsilon/2)\cos f - s \right)^2 + \left( (\upsilon/2)\sin f + L\sin\sigma \right)^2 \right]^{1/2} \]

\[ \theta = (\pi/2) - f - \phi \text{ and} \]

\[ \theta_0 = (\pi/2) - \left[ \tan^{-1} \left( \frac{L\sin\sigma + (\upsilon/2)\sin f}{L\cos\sigma + (\upsilon/2)\cos f - s} \right) - f \right] \]

This treatment is repeated for the sources \( I_2 \) and \( I_2' \) separately. The equation gets modified and their polarity is opposite. This will yield \( 6(2n+1) \) components. Thus this case with six strips on the secondary will have 78 components for the single diffraction. The multiple diffraction field calculated using equation (5.8) is insignificantly small and is ignored.

The total diffraction field \( E^{\text{GTD}} \) is the sum of the individual diffraction field components.

(c) Total field

The sum of the GO field and the diffraction field gives the total field.

\[ E_z = E^{\text{GO}} + E^{\text{GTD}} \]

The total field is calculated and the intensity at different points in the azimuthal plane is calculated and is plotted against the azimuth angle \( \phi \) to get the radiation pattern.

The theoretical and experimental patterns are compared in Fig. 5.17.
Figure 5.17 Theoretical and experimental radiation patterns of 120° PSACR with 6 strips.

5.3.2.4 Case 4. 90° PSACR

- Primary corner angle, 'α' = 90°
- Secondary (passive) corner angle, 'β' = 30°
- Width of primary, 'L' = 1λ
- Number of strips on secondary reflector, 'n' = 6
- Flare of secondary, 'f' = (α/2) - β = 15°
- The apex to dipole distance, 's' = 0.5λ
- Period of the strip structure, 'p' = 1λ
(a) The GO Field

The GO field will have all the 12 images as in the 90° ETCR. The distances of the various sources to the field point can be represented by exactly the same equations as in section 5.3.1.3. The limits of illumination of the first four sources are also the same. The remaining eight sources have their span of illumination splitted.

In the case of the 90° PSACR, the original source will find three penetrable gaps where as the source I₄ will find only one penetrable gap on the sub reflector. The other two sources will find two gaps on the reflector on the same side and four gaps on the opposite side as penetrable. Thus there will be 40 component fields in addition to that of the four sources formed on the primary corner. Considering the primary source I₁ and the strip structures, the angular span of illumination of the individual zones are given by the general expression

![Diagram of GO Field and PSACR antenna](image-url)
\[ S_o = \psi_{u-1} - \psi_u \] .............(5.13)

Where \( \psi_u = \tan^{-1} \left[ \frac{LS\sin \sigma + \nu(p/2)\sin f}{LC\cos \sigma - s + \nu(p/2)\cos f} \right] \), .............(5.19)

Here \( \nu \) stands for the number of zone.

Similarly for the source \( I_3 \) and the sub reflectors the expression for \( \psi_u \) becomes

\[ \psi'_u = \tan^{-1} \left[ \frac{LS\sin \sigma + \nu(p/2)\sin f}{LC\cos \sigma + s + \nu(p/2)\cos f} \right] \] .............(5.20)

The images \( I_2 \) and \( I_4 \) will have different values of \( \psi_u \) for the strip structures on opposite sides according to the equations

\[ \psi''_u = \tan^{-1} \left[ \frac{LS\sin \sigma + \nu(p/2)\sin f - s}{LC\cos \sigma + \nu(p/2)\cos f} \right] \] .............(5.21)

and

\[ \psi'''_u = \tan^{-1} \left[ \frac{LS\sin \sigma + \nu(p/2)\sin f + s}{LC\cos \sigma + \nu(p/2)\cos f} \right], \] .............(5.22)

depending whether the strip is on the same side or the other.

The odd numbered zones will cause side-lobes if they are penetrable in directions given by \(-\psi_{u-1}\) to \(-\psi_u\) and \(\psi_u\) to \(\psi_{u-1}\). All other zones will be reflecting.

The field due to each component, within its limit of illumination is given by substituting the value of \( r_a \) from section 5.3.1.3 in equation (5.10), which is,
The total GO field $E^G$ at the point 'M' is the sum of the individual components existing there:

$$E^G = (-1)^{(n+1)} \frac{\exp(-jkx)}{kr}$$

Table 5.3 Limits of illumination of the reflecting strips for the image sources on the sub reflectors.

<table>
<thead>
<tr>
<th>Source</th>
<th>Lower limit of illumination</th>
<th>Upper limit of illumination</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_5$</td>
<td>$-(b-2f) - \psi''_0 + \psi''$</td>
<td>$-(b-2f) - \psi''_0 + \psi''_0 + 1$</td>
<td>$a = Tan^{-1}\left[ \frac{LSin\sigma + wSinf}{LCos\sigma + wCosf - s} \right]$</td>
</tr>
<tr>
<td>$I_6$</td>
<td>$(a-2f) - \psi_0 + \psi_0$</td>
<td>$(a-2f) - \psi_0 + \psi_0 + 1$</td>
<td>$b = Tan^{-1}\left[ \frac{wSinf + LSin\sigma + sSin2\sigma}{wCosf + LCos\sigma - sCos2\sigma} \right]$</td>
</tr>
<tr>
<td>$I_7$</td>
<td>$-(g-2f) - \psi''_0 + \psi''_0$</td>
<td>$-(g-2f) - \psi''_0 + \psi''_0 + 1$</td>
<td>$e = Tan^{-1}\left[ \frac{wSinf + LSin\sigma - sSin2\sigma}{wCosf + LCos\sigma - sCos2\sigma} \right]$</td>
</tr>
<tr>
<td>$I_8$</td>
<td>$-(h-2f) - \psi_0 + \psi_0$</td>
<td>$(h-2f) - \psi_0 + \psi_0 + 1$</td>
<td>$g = Tan^{-1}\left[ \frac{LSin\sigma + wSinf - s}{LCos\sigma + wCosf} \right]$</td>
</tr>
<tr>
<td>$I_9$</td>
<td>$b-2f + \psi''_0 - \psi''_0 - 1$</td>
<td>$b-2f + \psi''_0 - \psi''_0 - 1$</td>
<td>$h = Tan^{-1}\left[ \frac{LSin\sigma}{s + LCos\sigma} \right]$</td>
</tr>
<tr>
<td>$I_{10}$</td>
<td>$a-2f + \psi_0 - \psi_0$</td>
<td>$(a-2f) + \psi_0 - \psi_0$</td>
<td></td>
</tr>
<tr>
<td>$I_{11}$</td>
<td>$g-2f + \psi''_0 - \psi''_0$</td>
<td>$g-2f + \psi''_0 - \psi''_0$</td>
<td></td>
</tr>
<tr>
<td>$I_{12}$</td>
<td>$h-2f + \psi_0 - \psi_0$</td>
<td>$h-2f + \psi_0 - \psi_0$</td>
<td></td>
</tr>
</tbody>
</table>

(b) The diffraction field

The GTD field of this configuration contains $8(2n+1)$ components due to the illumination from the source and the images on the primary reflectors alone. The fields due to illumination from other sources are ignored.
\[ E_o^d = \frac{\exp(-jkr_o)}{\sqrt{kr_o}} \times \]

\[ \exp\left[ -jk\left( r - \sqrt{(L \sin \sigma + (up/2) \sin f)^2 + ((up/2) \cos f)^2} \right) \right. \]

\[ \left. \times \exp\left( j \frac{\pi}{4} \right) \frac{\sec (\theta - \theta_o)}{2} - \csc (\theta + \theta_o) \right] \]

where,

\[ r_o = \sqrt{\left[ L \cos \sigma + (up/2) \cos f - s \right]^2 + \left( (up/2) \sin f + L \sin \sigma \right)^2} \]

\[ \theta = \left( \frac{\pi}{2} - f - \phi \right) \text{ and} \]

\[ \theta_o = \left( \frac{\pi}{2} - \left[ \tan^{-1}\left( L \sin \sigma + (up/2) \sin f \right) \right] / \left[ L \cos \sigma + (up/2) \cos f - s \right] - f \right] \]

This equation is applied to all the edges \( \nu = 0 \) to \( \nu = 2n \) to get the individual diffraction fields due to all four sources. In this case with 6 strips, will have 104 components for the single diffraction field. The polarity of the exciting sources is taken into consideration and the sum of the individual components of the single diffraction field is calculated as \( E^{GTD} \). The resultant multiple diffraction field from equation (5.8) is seen to be negligibly small.

(c) Total field

The total field is calculated using the equation

\[ E_z = E^{GO} + E^{GTD} \]

and the intensity is plotted against the azimuth angle \( \phi \). The radiation pattern is compared with the experimental pattern in Fig. 5.19.
Figure 5.19 Theoretical and experimental radiation patterns of 90° PSACR Antenna