CHAPTER 3
ELECTROMAGNETIC SCATTERING FROM FERRITE CYLINDERS

In this Chapter, an analysis of electromagnetic scattering from finite array of ferrite cylinders positioned parallel to z-coordinate in free space is presented for a generalized case [41, 42]. All cylinders are considered to be of infinite length, same radius and magnetized axially by an externally applied constant DC magnetization, $H_0$ (see Fig.3.1). The analysis is carried out through calculation of unknown expansion coefficients of the scattered field for each cylinder separately, by matching tangential field components at the surface of each cylinder. A generalized solution for the fields inside a single ferrite cylinder is available in the literature [19, 22]. Therefore, a boundary value type solution for electromagnetic scattering from a finite array of ferrite cylinders can be effectively obtained through the matching of tangential fields at the surface of each cylinder. Okamoto has given a generalized solution for the electromagnetic scattering from a finite periodic array of gyrotropic cylinders [26]. In that analysis, angle of incidence in azimuthal plane is assumed to be $0^\circ$ (i.e., $\phi_0=0^\circ$). In other words, the transformation of incident field from global coordinate system to local coordinate system is not included in the analysis. The analysis presented in this Chapter has effectively removed this limitation by including the transformation of incident field from global coordinates to the local coordinates of microwire under consideration. The solution is obtained through the calculation of unknown expansion coefficients of the scattered field for each cylinder in a similar manner as done in the previous Chapter. Graf’s Addition theorem is utilized for translating the scattered field from global coordinate system to local coordinates of the microwire under consideration. A linear $1 \times 6$ and a planar $2 \times 3$ array of ferromagnetic microwires are considered for obtaining numerical results through MATLAB. The numerical results are
obtained for $E_z$ and $H_z$ components of the scattered field in near field region and scattering cross section (SCS) $\sigma_{2D}$, for $TM_z$ and $TE_z$ polarized uniform plane wave incident at angle $\theta_0 = 45^\circ$. Simulation results are also obtained by the method given in [26] for a linear $1 \times 6$ array for $E_z$ and $H_z$ components of the scattered field in near field region and scattering cross section (SCS) $\sigma_{2D}$, at $\phi_0 = 0$. The simulation results of analysis are matched with the results obtained through the method given in [26] for a linear $1 \times 6$ array at $\phi_0 = 0$ and thus validated. A comparison for the numerical results is also shown for a linear $1 \times 6$ and a planar $2 \times 3$ array consisting of ferromagnetic microwires. The results obtained in this Chapter specialized to the case of $\theta_0 = 90^\circ$ and single microwire (i.e., $M = 1$) for $TM_z$ polarization are shown to match with the results available in [21] for single microwire with the same specialization. Thus, generalization of the solution is shown in terms of number of microwires.

![Figure 3.1: Schematic representation of the cross section of ‘$M$’ number of ferrite cylinders.](image)

3.1 Mathematical Formulation

Let, ‘$M$’ number of ferrite cylinders having, infinite length, radius $a$, with applied magnetization $H_0$ along their axis be placed with their axis parallel to the $z$-axis (see Fig.3.1).
A uniform plane wave is impinged upon each cylinder having a polarization angle, $\alpha_0$. Permeability of ferrite medium tends to become a tensor on account of the interaction of applied magnetization $\mathbf{H}_0$ with the magnetic field component of wave which lie in a plane normal to $\mathbf{H}_0$ [40]. Such a permeability tensor is represented by [34, 38, 40] (Appendix D.2)

$$
\vec{\mu} = \begin{bmatrix}
\mu & j\kappa & 0 \\
- j\kappa & \mu & 0 \\
0 & 0 & \mu_0
\end{bmatrix},
$$

where

$$
\mu = \mu_0 (1 + \chi_p - j\chi_s),
$$

$$
\kappa = \mu_0 (K_p - jK_s),
$$

$$
\chi_p = \frac{\omega_0\omega_m (\omega_0^2 - \omega^2) + \omega_0\omega_m\omega^2\alpha^2}{[\omega_0^2 - \omega^2 (1 + \alpha^2)]^2 + 4\omega_0^2\omega^2\alpha^2},
$$

$$
\chi_s = \frac{\omega_0\omega_m \omega^2 \alpha}{[\omega_0^2 - \omega^2 (1 + \alpha^2)]^2 + 4\omega_0^2\omega^2\alpha^2},
$$

$$
K_p = \frac{\omega_0\omega_m \omega^2 \alpha}{[\omega_0^2 - \omega^2 (1 + \alpha^2)]^2 + 4\omega_0^2\omega^2\alpha^2},
$$

$$
K_s = \frac{2\omega_0\omega_m \omega^2 \alpha}{[\omega_0^2 - \omega^2 (1 + \alpha^2)]^2 + 4\omega_0^2\omega^2\alpha^2}.
$$

Here, each symbol has the usual meaning as defined in the previous chapter. For the case of $TM_z$ polarization, an extraordinary wave propagates inside the ferrite medium. The effective permeability and complex permittivity for this medium are respectively given by

$$
\mu_e = \frac{\mu^2 - \kappa^2}{\mu},
$$

$$
\epsilon_c = \epsilon_0 - j\frac{\sigma}{\omega}.
$$

In cylindrical coordinates, the $z$-components of the incident and scattered fields for the $i^{th}$ cylinder are given by [25, 34, 36] (Appendix C.1)

$$
E_{z,i}^{inc} = E_0 \sin \theta_0 e^{j\beta_0 z \cos \theta_0} e^{j\beta_0 \rho_i \sin \theta_0 \cos(\phi_i - \phi_0)} e^{j\beta_0 \rho_i \sin \theta_0 \cos(\phi_i - \phi_0)},
$$

$$
= E_0' e^{j\beta_0 z \cos \theta_0} e^{j\beta_0 \rho_i \sin \theta_0 \cos(\phi_i - \phi_0)} \sum_{n=-\infty}^{+\infty} j^n J_n (\beta_0 \rho_i) e^{-j\beta_0 z} e^{-jn(\phi_i - \phi_0)}. 
$$
We can rewrite (3.11) as

\[ E_{zi}^{inc} = E_0' P_i \cos \alpha_0 \sum_{n=-\infty}^{+\infty} j^n J_n (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)}, \]  

(3.12)

\[ H_{zi}^{inc} = \frac{E_0'}{\eta_0} \sin \alpha_0 \sum_{n=-\infty}^{+\infty} j^n J_n (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)}, \]  

(3.13)

\[ E_{zi}^s = E_0' \sum_{n=-\infty}^{+\infty} C_n H_n^{(2)} (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)}, \]  

(3.14)

\[ H_{zi}^s = \frac{E_0'}{\eta_0} \sum_{n=-\infty}^{+\infty} D_n H_n^{(2)} (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)}. \]  

(3.15)

Here \( E_0' = E_0 \sin \theta_0 \), \( P_i = e^{-j\beta_{p_0} r_i \cos(\phi'_0 - \phi_0)} \) is the transformation factor which transforms incident field from local coordinates into global coordinates, \( \beta_p = \beta_0 \sin \theta_0, \beta_z = \beta_0 \cos \theta_0 \), \( \beta_0 = \omega \sqrt{\mu_0 \varepsilon_0} \) and \( \eta_0 = \sqrt{\mu_0 \varepsilon_0} \) are the propagation constant and intrinsic impedance for free space, respectively. The superscripts \( i \) and \( s \) denote the incident and scattered field, respectively. \( J_n \) is the Bessel’s function of the first kind and \( n^{th} \) order, \( H_n^{(2)} \) is the Hankel’s function of the second kind and \( n^{th} \) order, \( C_n \) and \( D_n \) are the unknown expansion coefficients of the scattered field. In the method given in [26], the transformation factor \( P_i = e^{-j\beta_{p_0} r_i \cos(\phi'_0 - \phi_0)} \) is not taken into consideration. Due to which the method proposed in [26] is consistent with the proposed method only for \( (\phi'_0 - \phi_0) = \frac{\pi}{2} \) (i.e., a linear array with incident wave in x-z plane) such that, \( P_i \) becomes equal to 1 [41,42].

The \( \phi \)-components for the incident and scattered fields for the \( i^{th} \) cylinder are deduced from Maxwell’s equations to be

\[ E_{\phi i}^{inc} = -E_0 P_i \frac{n \cos \theta_0 \cos \alpha_0}{\beta_0 r_i \sin \theta_0} \sum_{n=-\infty}^{+\infty} j^n J_n (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)} \]

\[ + j E_0 \sin \alpha_0 \sum_{n=-\infty}^{+\infty} j^n J_n' (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)}, \]  

(3.16)

\[ H_{\phi i}^{inc} = -\frac{j E_0 P_i}{\eta_0} \sin \alpha_0 \sum_{n=-\infty}^{+\infty} j^n J_n' (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)} \]

\[ - \frac{E_0 n \cos \theta_0 \cos \alpha_0}{\beta_0 r_i \sin \theta_0} \sum_{n=-\infty}^{+\infty} j^n J_n (\beta_{p_0} r_i) e^{-j\beta z} e^{-jn(\phi_i - \phi_0)}, \]  

(3.17)
where, \( \frac{\partial}{\partial z} \) denotes the first derivative with respect to argument.

The \( z \)-component of the inside fields for a ferrite cylinder in cylindrical coordinates are represented as given in [19,22]

\[
E_{zi}^d = E_0 \sum_{n=-\infty}^{+\infty} [A_n J_n(\gamma \rho_i \rho_i) + B_n J_n(\gamma \rho_i \rho_i)] e^{-j\beta z z} e^{-jn(\phi_i - \phi_0)}, \tag{3.20}
\]

\[
H_{zi}^d = E_0 \sum_{n=-\infty}^{+\infty} [\eta_1 A_n J_n(\gamma \rho_i \rho_i) + \eta_2 B_n J_n(\gamma \rho_i \rho_i)] e^{-j\beta z z} e^{-jn(\phi_i - \phi_0)}, \tag{3.21}
\]

\[
E_{\phi i}^d = E_0 \sum_{n=-\infty}^{+\infty} [A_n X_{1n}(\rho_i) + B_n X_{2n}(\rho_i)] e^{-j\beta z z} e^{-jn(\phi_i - \phi_0)}, \tag{3.22}
\]

\[
H_{\phi i}^d = E_0 \sum_{n=-\infty}^{+\infty} [A_n \Lambda_{1n}(\rho_i) + B_n \Lambda_{2n}(\rho_i)] e^{-j\beta z z} e^{-jn(\phi_i - \phi_0)}, \tag{3.23}
\]

where, \( X_{in}(\rho_i) \) and \( \Lambda_{in}(\rho_i) \) are defined as

\[
X_{in}(\rho_i) = \frac{1}{D} \left( \frac{d n \gamma \rho_i}{\beta_z} - b \gamma \rho_i \eta \right) J_n' (\gamma \rho_i \rho_i) - j \frac{n}{D \rho_i} \left( a \eta + \frac{b \omega \epsilon_c}{\beta_z} \right) J_n (\gamma \rho_i \rho_i), \tag{3.24}
\]

\[
\Lambda_{in}(\rho_i) = \frac{1}{D} \left( \frac{b \gamma \rho_i}{a \eta} - b \gamma \rho_i \eta \right) J_n' (\gamma \rho_i \rho_i) - j \frac{n}{D \rho_i} \left( a \eta + \frac{b \omega \epsilon_c}{\beta_z} \right) J_n (\gamma \rho_i \rho_i), \tag{3.25}
\]

where

\[
a = j \beta z \beta_p^2, \tag{3.26}
\]

\[
b = \omega^2 \kappa \beta z \epsilon_c, \tag{3.27}
\]

\[
c_1 = \left( \frac{\beta_p^2 - \omega^2 \kappa^2 \epsilon_c}{\mu} \right), \tag{3.28}
\]

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\[ d = -j\omega\mu c_1, \quad \text{(3.29)} \]
\[ e = \omega\kappa\beta_z^2, \quad \text{(3.30)} \]
\[ D = (\omega^2\kappa\varepsilon_c)^2 - \beta^4, \quad \text{(3.31)} \]
\[ \eta_i = \frac{-jg_1}{(\gamma_{\rho_i}^2 - f_1)}, \quad \text{(3.32)} \]
\[ g_1 = \frac{\omega\kappa\beta_z\varepsilon_c}{\mu}, \quad \text{(3.33)} \]
\[ f_1 = \frac{\mu_0\beta^2}{\mu}, \quad \text{(3.34)} \]
\[ \gamma_{\rho_i} = \sqrt{\frac{1}{2} \left( (f_1 + c_1) \pm \sqrt{(f_1 - c_1)^2 + 4d_1g_1} \right)}, \quad \text{(3.35)} \]
\[ d_1 = \frac{\mu_0\omega\kappa\beta_z}{\mu}, \quad \text{(3.36)} \]
\[ \beta_{\rho} = \sqrt{(\omega^2\mu\varepsilon_c - \beta_z^2)}. \quad \text{(3.37)} \]

Here, \( i \) denotes the suffix 1 or 2 depending upon the \( \pm \) sign chosen inside the square root in (3.35), respectively.

Graf’s addition theorem is applied suitably for transformation of the components of scattered field from global coordinates into the local coordinates of the cylinder under consideration [35] (Appendix C.2). Thus, the contributions of scattered field from other cylinders at the surface of reference cylinder is calculated. For example, the scattered field from \( j^{th} \) cylinder transformed in terms of the coordinates of \( i^{th} \) cylinder is represented by:

\[
H_n^{(2)}(\beta_{\rho_0}\rho_j)e^{j\phi_{ij}} = \sum_{m=-\infty}^{+\infty} J_m(\beta_{\rho_0}\rho_i) H_{m-n}^{(2)}(\beta_{\rho_0}d_{ij}) e^{j\phi_i} e^{j(m-n)\phi_{ij}}, \quad \text{(3.38)}
\]
\[
d_{ij} = \rho_i' + \rho_j' - 2\rho_i'\rho_j'\cos(\phi_i' - \phi_j'), \quad \text{(3.39)}
\]
\[
\phi_{ij} = \begin{cases} 
\cos^{-1} \left[ \frac{\rho_i'\cos(\phi_i') - \rho_j'\cos(\phi_j')}{d_{ij}} \right], & \text{for } \rho_i'\sin(\phi_i') \geq \rho_j'\sin(\phi_j')' \\
-\cos^{-1} \left[ \frac{\rho_i'\cos(\phi_i') - \rho_j'\cos(\phi_j')}{d_{ij}} \right], & \text{for } \rho_i'\sin(\phi_i') < \rho_j'\sin(\phi_j').
\end{cases} \quad \text{(3.40)}
\]
The continuity of tangential field components at the surface of the \(i^{th}\) cylinder translates to

\[
E_{z_i}^{\text{inc}} + \sum_{l=1}^{M} E_{z_i}^{s} = E_{z_i}^{d}, \tag{3.41}
\]

\[
H_{z_i}^{\text{inc}} + \sum_{l=1}^{M} H_{z_i}^{s} = H_{z_i}^{d}, \tag{3.42}
\]

\[
E_{\phi_i}^{\text{inc}} + \sum_{l=1}^{M} E_{\phi_i}^{s} = E_{\phi_i}^{d}, \tag{3.43}
\]

\[
H_{\phi_i}^{\text{inc}} + \sum_{l=1}^{M} H_{\phi_i}^{s} = H_{\phi_i}^{d}. \tag{3.44}
\]

Substituting the values of field components from (3.12)-(3.23) in (3.41)-(3.44) and solving further yields the following equations.

\[
\sum_{l=1}^{M} C_{ln}\Theta_{ln} - \sum_{l=1}^{M} D_{ln}\Phi_{ln} = \Psi_{ln} \tag{3.45}
\]

\[
\sum_{l=1}^{M} C_{ln}\Delta_{ln} - \sum_{l=1}^{M} D_{ln}\Omega_{ln} = \Upsilon_{ln} \tag{3.46}
\]

where, \(l = 1, 2, \ldots M\) is the index number of the cylinders and

\[
\Theta_{ln} = \left\{ \eta_0\eta_2 \sin \theta_0 \left( \frac{\chi_1(a)}{U_{1i}} + \frac{\chi_2(a)}{U_{2i}} \right) + \frac{n \cos \theta_0}{\beta_p a} \right\} H_{ln}, \tag{3.47}
\]

\[
\Phi_{ln} = \left\{ \sin \theta_0 \left( \frac{\chi_1(a)}{U_{1i}} + \frac{\chi_2(a)}{U_{2i}} \right) H_{ln} + jH'_{ln} \right\}, \tag{3.48}
\]

\[
\Psi_{ln} = j^n P_i \left\{ \sin \alpha J_n'(\beta_p a) - \frac{\cos \alpha \cos \theta_0 J_n(\beta_p a)}{\beta_p a} \right\}
- \left( \frac{V_{1i}\chi_1(a)}{U_{1i}} + \frac{V_{2i}\chi_2(a)}{U_{2i}} \right), \tag{3.49}
\]

\[
\Delta_{ln} = \left\{ \eta_0 \sin \theta_0 \left( \frac{\eta_2 A_1(a)}{U_{1i}} + \frac{\eta_1 A_2(a)}{U_{2i}} \right) H_{ln} + \frac{jH'_n}{\eta_0} \right\}, \tag{3.50}
\]

\[
\Omega_{ln} = \left\{ \sin \theta_0 \left( \frac{A_1(a)}{U_{1i}} + \frac{A_2(a)}{U_{2i}} \right) - \frac{n \cos \theta_0}{\eta_0 \beta_p a} \right\} H_{ln}, \tag{3.51}
\]
\[
\Upsilon_{in} = \frac{j^{n+1} P_i}{\eta_0} \left\{ \cos \alpha J'_n(\beta_{\rho_0} a) + \frac{n \sin \alpha \cos \theta_0 J_n(\beta_{\rho_0} a)}{\beta_{\rho_0} a} \right\} \\
- \left( \frac{V_{1i} \Lambda_{1n}(a)}{U_{1i}} + \frac{V_{2i} \Lambda_{2n}(a)}{U_{2i}} \right), \quad (3.52)
\]

\[
U_{1i} = \eta_0 (\eta_1 + \eta_2) J_n(\gamma_{\rho_1} a), \quad (3.53)
\]

\[
U_{2i} = \eta_0 (\eta_1 + \eta_2) J_n(\gamma_{\rho_2} a), \quad (3.54)
\]

\[
V_{1i} = j^n P_i (\eta_0 \eta_2 \cos \alpha - \sin \alpha) J_n(\beta_{\rho_0} a), \quad (3.55)
\]

\[
V_{2i} = j^n P_i (\eta_0 \eta_1 \cos \alpha - \sin \alpha) J_n(\beta_{\rho_0} a). \quad (3.56)
\]

\eta_1 and \eta_2 are given by (3.32), and

\[
H_{ln} = \begin{cases} 
H_n^{(2)}(\beta_{\rho_0} \rho), & \text{for } l = i, m = n \\
\sum_{m=-\infty}^{+\infty} J_m(\beta_{\rho_0} \rho_1) H_{m-n}^{(2)}(\beta_{\rho_0} d_{ij}), & \text{for } l \neq i, m = n \\
0, & \text{for } m \neq n 
\end{cases}, \quad (3.57)
\]

Where, \( m, n = 0, \pm 1, \pm 2, \ldots, \pm N_i \) and the value of \( N_i \) is related to the radius of \( i^{th} \) cylinder \( a_i \) by \( N_i \approx (1 + 2k_i a_i) \). Here, \( k_i \) is wavenumber of the medium inside ferrite cylinder [24]. Similarly, all unknown scattering field coefficients are calculated by applying the tangential boundary conditions at the surface of \( 'M' \) cylinders one by one. All of these equations thus obtained are represented in the form of a matrix as

\[
\begin{bmatrix} \Theta & \Phi \\ \Delta & \Omega \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} \Psi \\ \Upsilon \end{bmatrix}, \quad (3.58)
\]

The solutions of this matrix given in (3.58) yields the unknown expansion coefficients of the scattered field \( C_m \) and \( D_m \) after suitable truncation. The Scattering cross section (SCS) can be defined as given in [34]

\[
\sigma_{2D}^{TM} = 10 \log \left[ \lim_{\rho \to \infty} 2\pi \rho \frac{|E_z^s|}{|E_z^{inc}|^2} \right], \quad (3.59)
\]

\[
\sigma_{2D}^{TE} = 10 \log \left[ \lim_{\rho \to \infty} 2\pi \rho \frac{|H_z^s|}{|H_z^{inc}|^2} \right]. \quad (3.60)
\]
Figure 3.2: Schematic representation of a $1 \times 6$ linear array.

Figure 3.3: Schematic representation of the cross section of a $2 \times 3$ planar array.
3.2 Numerical results

A linear $1 \times 6$ and a planar $2 \times 3$ arrays of the ferromagnetic microwires are considered for obtaining numerical results as shown in Fig.3.2 and Fig.3.3, respectively. Since, the maximum radius considered is only $50 \mu m$ and maximum operating frequency is $(15 \text{ GHz})$, radius-to-wavelength ratio is found to be only $2.5 \times 10^{-3}$. Therefore, the terms $n = 0 \text{ and } 1$ make any significant contribution in expansions for the scattered and inside fields. The numerical results are obtained for the Cobalt based microwires with following specifications [27]: radius $a = 1 \mu m$, spacing $d = 3 \text{ mm}$, conductivity $\sigma = 6.7 \times 10^5 \text{ S/m}$, gyromagnetic ratio $\gamma = 2 \times 10^{11} \text{ T}^{-1}\text{s}^{-1}$, saturation magnetization $\mu_0 M_s = 0.55 \text{ T}$, magnetic loss factor $\alpha = 0.02$, internal magnetization $H_0 = 113.45 \text{ kA/m}$ along the $z$-axis and the frequency band of $5 - 15 \text{ GHz}$ is considered. Simulation results are obtained for $E_z$ and $H_z$ components of the scattered field in near field region and scattering cross section (SCS) $\sigma_{2D}$, plotted against the operating frequency, radius of microwires and spacing between the microwires for $TM_z$ and $TE_z$ polarizations (i.e., polarization angles $\alpha_0 = 0^\circ \text{ and } 90^\circ$ respectively).

3.2.1 Near Field Components

Fig. 3.4 and Fig. 3.5 depicts plots for $E_z$ and $H_z$ components of the scattered field in near field region, calculated at $\rho = a$ and $\phi_0 = 0^\circ$ for $TM_z$ and $TE_z$ polarizations, respectively (i.e., $\alpha_0 = 0^\circ \text{ and } \alpha_0 = 90^\circ$). These results are plotted against the considered frequency band for a $2 \times 3$ planar array consisting of six ferromagnetic microwires of radius $1 \mu m$ each (see Fig. 3.3) at three different angle of incidence $\theta_0 = 30^\circ, 60^\circ \text{ and } 90^\circ$, respectively. In order to explain the scattering response of microwire grid, scattering response of single microwire is discussed here as given in [21]. According to Liberal et al. [21], the frequency band of $5 - 15 \text{ GHz}$ is divided into two halves containing frequencies below and above FMR frequency (i.e., $10 \text{ GHz}$). For frequencies below the FMR frequency, $\text{Re}[\mu_e] > 0$ (see Fig. 1.13) therefore, the medium inside the microwire behaves like a lossy dielectric which results in weak scattering. However, for frequencies above the FMR, $\text{Re}[\mu_e] < 0$ therefore, the imaginary part of propagation constant for medium inside the microwire becomes negative. Therefore, the microwire supports only evanescent wave inside the medium and it essentially behaves like a plasma medium giving rise in the scattering.
Therefore, the scattering response of single microwire for frequencies on either side of FMR shows a remarkable difference. A much similar scattering response is observed with increased magnitude in case of the ferromagnetic microwire array considered here. The magnitude of scattered field is increased due to superposition of the scattered fields from all microwires in the array. As the value of tangential field components is very small at small angle of incidence (say $\theta_0 \rightarrow 30^\circ$), the magnitude of near field components turns out to be very small.

In case of $TE_z$ polarization, applied magnetization $H_0$ does not interact with the $H$ vector of incident wave as the plane containing $H$ vector is parallel to applied magnetization. Therefore, ferromagnetic resonance (FMR) does not take place and an ordinary wave propagates inside the medium of microwire. The medium acts like a lossy dielectric and thus, scattering is very weak in case of $TE_z$ polarization. Moreover, the magnitude of scattered field increases with an increment in frequency (see Fig. 3.5). This is due to the decrement in skin depth with increase in frequency which results in a decrement in the transmitted field. In order to compensated this decrement in the transmitted field, magnitude of scattered field is increased.

Figure 3.4: $E_z$ component of the scattered field in near field region at $\rho = a$ for a $2 \times 3$ array for $TM_z$ polarization.
Figure 3.5: $H_z$ component of the scattered field in near field region at $\rho = a$ for a $2 \times 3$ array for $TE_z$ polarization.

3.2.2 Scattering Cross section

Fig. 3.6 and Fig. 3.7 depicts the curves of scattering cross section (SCS) plotted against spacing among the microwires $d/\lambda$, at $\phi = 0^o$, $45^o$ and $90^o$ for $TM_z$ and $TE_z$ polarizations, respectively. Each microwire is considered to be of $1\mu m$ radius. In case of $TM_z$ polarization, the magnitude of SCS varies periodically in relation to spacing $d/\lambda$ (see Fig. 3.6). This is due to successive change in phase angle of scattered field from each microwire along with the spacing among microwires with respect to the observation point. Therefore, the magnitude of scattered field varies according to the constructive or destructive interference. The SCS varies periodically with a period of about $1.5\lambda$ and having a maximum magnitude at $d = \lambda$ for $\phi = 90^o$. However, SCS peaks for the microwire spacings $d = 1.4\lambda$ and $1.6\lambda$ at $\phi = 0^o$ and $45^o$, respectively and varies with the period of about $\lambda$. Moreover, the plot corresponding to $\phi = 90^o$ (i.e., $y$-axis) has the maximum magnitude which shows that the microwire array behaves like an End-fire array (i.e., maximum magnitude of scattering is along the axis of array). Fig. 3.7 depicts the plot of SCS vs $d/\lambda$ for $TE_z$ polarization. In contrast to $TM_z$ case, SCS shows a minimum and maximum magnitude at $\phi = 90^o$ and $\phi = 0^o$, respectively. The curve of SCS corresponding to $\phi = 90^o$ also depicts
that the variation in spacing among the microwires \(d\), does not affect magnitude of SCS along the axis of array (i.e., y-axis). However, SCS curves corresponding to \(\phi = 0^\circ\) and \(45^\circ\) shows that the magnitude of SCS varies periodically w.r.t spacing with a period of about \(\lambda\). Moreover, maximum scattering is observed in a direction perpendicular to axis of the array (at \(\phi = 0^\circ\) or x-axis) which means that the microwire array behaves like a Broadside array.

Fig. 3.8 and Fig. 3.9 depicts the curve of scattering cross section (SCS) for a \(2 \times 3\) array plotted against the radius of the microwires \(a\), at an operating frequency \(f = 15 \text{ GHz}\) for \(TM_z\) and \(TE_z\) polarizations, respectively. The range of radius \(a\) is considered to be \(1 \mu m - 50 \mu m\). The magnitude of SCS increases with the radius of microwires in both cases of \(TM_z\) and \(TE_z\) polarizations (see in Fig. 3.8 and Fig. 3.9). This is due to the fact that the magnitude of transmitted field through the grid is decreased with an increment in the thickness of microwires. Consequently, the magnitude of scattered field is increased. Moreover, there is sudden rise in magnitude of SCS within the range of radius from \(1 \mu m - 5 \mu m\) (see Fig. 3.8). However, the magnitude of SCS is almost steady within the range of radius from \(5 \mu m - 50 \mu m\).
Figure 3.7: Scattering cross section (SCS) of a $2 \times 3$ array plotted against the spacing $d'$ among the microwires at $\theta_0 = 45^\circ$, $\phi_0 = 0^\circ$ and $f = 15\, \text{GHz}$ for $TE_z$ polarization.

### 3.3 Comparison of the Numerical results

A comparison of scattering cross section (SCS) is depicted in Fig. 3.10 and Fig. 3.11 for three different type of linear arrays consisting of 9, 6 and 3 microwires for $TM_z$ and $TE_z$ polarizations, receptively. The radius of each microwire is considered to be $1\, \mu m$ with an incidence angle $\theta_0 = 45^\circ$ and $\phi_0 = 0^\circ$. It is to be noticed that the magnitude of SCS increases as the number of microwires in array increases, due to contribution of all microwire to the total scattered field (see Fig. 3.10 and Fig. 3.11 ). However, the rise in magnitude of SCS is comparatively larger in case of 3 to 6 microwires than 6 to 9 microwires. However, this relative difference becomes minute at higher frequency end ($15\, \text{GHz}$). This is due to the fact that the ferrite medium behaves like a plasma beyond FMR frequency and hence, the scattering is increased.

A comparison of numerical results is shown in Fig. 3.12 and Fig. 3.13 for $E_z$ and $H_z$ components of the scattered field in near field region at $\rho = a$ for a linear $1 \times 6$ and planar $2 \times 3$ array for $TM_z$ and $TE_z$ polarizations, respectively. The linear $1 \times 6$ array has larger magnitude of the near field component of scattered field except at $\phi = 0^\circ$ and $360^\circ$ in case of $TM_z$ polarization (see Fig. 3.12). It shows that the scattering along the
Figure 3.8: Scattering cross section (SCS) of a $2 \times 3$ array plotted against the radius of microwires with a range $1 - 50 \, \mu m$, at $\theta_0 = 45^\circ$, $\phi_0 = 0^\circ$ and $f = 15 \, GHz$ for $TM_z$ polarization.

direction perpendicular to the plane of array is slightly more for $2 \times 3$ array in comparison to the $1 \times 6$ array. However, the magnitude of scattered field are almost same in case of $TE_z$ polarization (see Fig. 3.13) for both types of arrays. Moreover, the scattered field for both types of array has minimum magnitude at $\phi = 90^\circ$ and $270^\circ$ and equal peaks at $\phi = 0^\circ, 180^\circ$ and $360^\circ$ (see Fig. 3.12). Therefore, both $1 \times 6$ and $2 \times 3$ arrays produce a Broadside response with equal scattering in forward and backward directions for $TE_z$ polarization.

A comparison of the numerical results for scattering cross section (SCS) of a $1 \times 6$ and $2 \times 3$ array is depicted in Fig. 3.14 and Fig. 3.15 for $TM_z$ and $TE_z$ polarizations, respectively. Fig. 3.14 shows a larger magnitude of SCS for $1 \times 6$ array in comparison to $2 \times 3$ array for $TM_z$ polarization. However, the difference in the magnitude of SCS for both types of array is not so significant for $TE_z$ polarization. The points corresponding to minimum and maximum magnitude on both plots coincide with each other (see Fig. 3.15). However, the curve of SCS for $1 \times 6$ array has steeper slopes at crest and trough which represents abrupt variation in SCS in the x-y plane for $TE_z$ polarization.

A comparison of numerical results for $E_z$ and $H_z$ components of the scattered field in near field region calculated at $\rho = a$, is depicted in Fig. 3.16, Fig. 3.17, Fig. 3.18, Fig. 3.19
Figure 3.9: Scattering cross section (SCS) of a $2 \times 3$ array plotted against the radius of microwires with a range $1 - 50 \ \mu m$, at $\theta_0 = 45^\circ$, $\phi_0 = 0^\circ$ and $f = 15 \ \text{GHz}$ for $TE_z$ polarization.

depicts the comparison of the numerical results for Scattering cross section (SCS), with the results obtained by the method proposed in [26] at $\rho = a$ (i.e., surface of the reference microwire) and $\phi_0 = 0^\circ$ for a $1 \times 6$ array for $TM_z$ and $TE_z$ polarizations, respectively. Simulation results are obtained at three different angles of incidence i.e., $\theta_0 = 30^\circ$, $60^\circ$ and $90^\circ$, respectively. It can be noticed that all results are found to be matched perfectly. Therefore, it is shown that the analysis given in [26] is a specialized case of the analysis presented in this Chapter.

Fig. 3.20 shows the comparison of simulation results obtained by the analysis presented in this Chapter with result available in [21]. These numerical results are specialized to the case of $\theta_0 = 90^\circ$ and $M = 1'$ (i.e., single microwire) for $TM_z$ polarization. The following parameters are considered in this case as given in [21]: radius $a = 45 \ \mu m$, conductivity $\sigma = 6.5 \times 10^4 \ \Omega^{-1} m^{-1}$, gyromagnetic ratio $\gamma = 1.33 \times 10^{11} \ T^{-1} s^{-1}$, saturation magnetization $\mu_0 M_s = 0.55 \ T$, magnetic loss factor $\alpha = 0.02$, internal magnetization $H_0 = 213 \ \text{kA/m}$ along the $z$-coordinate. An operating frequency band of $8 - 12.5 \ \text{GHz}$ is considered for obtaining simulation results. It can be noticed clearly from the Fig. 3.20 that results obtained in this Chapter reduces to results given in [21] for the specialized case.
Figure 3.10: Comparison of Scattering cross section (SCS) for three different linear arrays consisting of 9, 6 and 3 ferromagnetic microwires for $TM_z$ polarization at $\theta_0 = 45^\circ$ and $\phi_0 = 0^\circ$.

Figure 3.11: Comparison of Scattering cross section (SCS) for three different linear arrays consisting of 9, 6 and 3 ferromagnetic microwires for $TE_z$ polarization at $\theta_0 = 45^\circ$ and $\phi_0 = 0^\circ$. 
Figure 3.12: Comparison of the $E_z$ component of the scattered field in near field region at $\rho = a$ for a $2 \times 3$ and $1 \times 6$ array at $\theta_0 = 45^\circ, \phi_0 = 0^\circ$ and $f = 15 \text{GHz}$ for $TM_z$ polarization.

Figure 3.13: Comparison of the $H_z$ component of the scattered field in near field region at $\rho = a$ for a $2 \times 3$ and $1 \times 6$ array at $\theta_0 = 45^\circ, \phi_0 = 0^\circ$ and $f = 15 \text{GHz}$ for $TE_z$ polarization.
Figure 3.14: Comparison of the SCS of a $2 \times 3$ and $1 \times 6$ array at $\theta_0 = 45^\circ$, $\phi_0 = 0^\circ$ and $f = 15 \text{ GHz}$ for $TM_z$ polarization.

Figure 3.15: Comparison of the SCS of a $2 \times 3$ and $1 \times 6$ array at $\theta_0 = 45^\circ$, $\phi_0 = 0^\circ$ and $f = 15 \text{ GHz}$ for $TE_z$ polarization.
Figure 3.16: Comparison of the $E_z$ component of the scattered field of a linear $1 \times 6$ array in near field region at $\rho = a$ for $TM_z$ polarization and $\phi_0 = 0^\circ$ with the method proposed by Okamoto in [26].

Figure 3.17: Comparison of the $E_z$ component of the scattered field of a linear $1 \times 6$ array in near field region at $\rho = a$ for $TE_z$ polarization and $\phi_0 = 0^\circ$ with the method proposed by Okamoto in [26].
Figure 3.18: Comparison of the scattering cross section (SCS) of a linear $1 \times 6$ array for $TM_z$ polarization and $\phi_0 = 0^\circ$ with the method proposed by Okamoto in [26].

Figure 3.19: Comparison of the scattering cross section (SCS) of a linear $1 \times 6$ array for $TE_z$ polarization and $\phi_0 = 0^\circ$ with the method proposed by Okamoto in [26].
Figure 3.20: Comparison of the numerical results specialized to the case of $\theta_0 = 90^\circ$, radius $a = 45\mu m$ and $'M=1'$ for TM$_z$ polarization with the numerical result available in [21] for the specialized case.

3.4 Continuity of the Fields

Continuity of the tangential $E_z$ and $H_z$ field components is depicted Fig. 3.21 and Fig. 3.22 for a $1 \times 6$ array. The $z$-components of the incident and scattered electric fields and field inside the microwire (i.e., $|E_s^z| + |E_i^z|$) and $|E_d^z|$, respectively) are shown to be matched perfectly at the surface of reference microwire for TM$_z$ polarization (see Fig. 3.21). Similarly, the matching of $z$-components of the incident and scattered magnetic fields and magnetic field inside inside the microwire (i.e., $|H_s^z| + |H_i^z|$ and $|H_d^z|$, respectively) is depicted in Fig. 3.22 for $TE_z$ polarization. Thus, the validity is shown for the boundary value type solution obtained in this chapter.
Figure 3.21: Continuity of the tangential $E_z$ components at the surface of the reference microwire.

Figure 3.22: Continuity of the tangential $H_z$ components at the surface of the reference microwire.
3.5 Conclusion

The analysis for generalized case of scattering from an array consisting of finite number of ferrite cylinders is presented in this Chapter. A solution is obtained through the matching of tangential boundary conditions at the surface of each cylinder one by one and the unknown expansion coefficients of the scattered field are thus calculated. In the literature, analysis of the scattering from a finite periodic array of gyrotropic cylinders is given using reciprocity theorem. In that analysis, angle of incidence in azimuthal plane is assumed to be $0^\circ$ (i.e., $\phi_0=0^\circ$). In other words, the transformation of incident field from global coordinate system to local coordinate system is not included in the analysis. The analysis done in this Chapter has effectively removed this limitation by including the transformation of incident field from global coordinates to the local coordinates of microwire under consideration. Effect of the radius of microwires and relative spacing among the microwires upon scattering behavior of an array is discussed in detail. Numerical results are obtained for the near field components and scattering cross section (SCS) for $TM_z$ and $TE_z$ polarizations, respectively. Simulation results of the analysis obtained in this Chapter are compared with the results derived by a method available in literature and applied to a numerical example considered in this Chapter. Both results are found to be perfectly matched and thus validated. Numerical results specialized to the case of electromagnetic scattering from single ferromagnetic microwire available in the literature are also compared to match perfectly with the results available in the literature. Hence, the solution for electromagnetic scattering from $'M'$ number of ferrite cylinders is shown to be the most generalized solution for any arbitrary number of cylinders. Authors expect that the analysis presented in this chapter may find various applications e.g., wire based metamaterials and beam steering.