CHAPTER – 3

Methodology of the Study
3.0 Research

Research is a systematic attempt to obtain answers to meaningful questions about phenomena or events through the application of scientific procedures. (Lokesh Kaul, 1984, pp. 10).

Educators undertake studies in all phases of education; they investigate phenomena in the area of curriculum, administration, guidance, methods and teacher preparation; and they probe fundamental problems concerning the nature of learning and of child development.

The methodology of research comprises of the selection of an appropriate research design, sample, operations to be performed and tools and statistical techniques to be utilized. Generally, three methods are employed by the educators to solve the problems and they are as follows:

1. Historical Research
2. Descriptive Research
3. Experimental Research

3.1.0 Experimental Research

In experimental research, the educator does not merely chronicle past events, determine the status of something or observe and describe what exists. Through manipulating and experimental variable under highly controlled conditions, he strives to ascertain how and why a particular condition or event occurs. (Van Dalen, pp. 241)
The experimental research starts with identification and rigorous analysis of the problem. The issues involved are further sharpened by formulating the hypotheses and deducing the consequences that are implied logically by them. A research design is devised to discover whether the consequences that are said to occur are observable and of the hypotheses are to be confirmed or rejected.

3.2.0 Research Design

A research design is to the experimenter what a blueprint is to an architect. A well-developed design provides the structure and strategy that controls the investigation and extract dependable answers to the questions raised by the problem hypotheses.

Mostly the researcher utilizes a design that provides full experimental control through randomization. But if it is difficult to achieve this ideal, he may consider a design that exerts a maximum control under existing conditions. However, he must carefully consider the likelihood that these variables rather than the experimental treatment may account for the desired results. An appropriate statistical technique can be utilized for data analysis so as to attain more experimental validity.

In the present study, the researcher has used ‘Non-randomized Control-group Pre test-Post test Design’ of quasi-experimental type (Design with partial control).
3.2.1.0 Non-randomized Control Group Pre-test Post-test Design

Employing randomization procedures is not difficult, but upsetting class schedules, getting scattered subjects to participate, and obtaining a sufficiently large sample to ensure that the laws of chance will operate cannot always be done due to administrative difficulties in the school. Under some circumstances, therefore an experimenter (E) may have to use pre-assembled groups such as paradigm for the design:

<table>
<thead>
<tr>
<th></th>
<th>Pre-test</th>
<th>Treatment</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental group</td>
<td>( T_1E )</td>
<td>( X )</td>
<td>( T_2E )</td>
</tr>
<tr>
<td>Control group</td>
<td>( T_1C )</td>
<td>( T_2C )</td>
<td></td>
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</tbody>
</table>

3.2.2.0 Internal Validity

The presence of control group enables the E to assume that the main effects of history, pre-testing, maturation and instrumentation will not be mistaken for the effect of \( X \), for both the experimental and control groups will experience these effects.

The interaction of selection and maturation can be controlled as the intact groups of the subjects studying in the same classes for the academic year are selected.

Statistical regression is a source of invalidity that can be avoided while analyzing the data carefully.
The analysis of co-variance, which is done after $T_2$ is given, achieves the same results as matching without discarding or shifting any subjects ($S_a$). The $E$ selects two intact groups, administers the experimental treatment, and then adjusts $T_2$ means to compensate for the lack of equivalency between the two groups. When the assumptions underlying analysis of co-variance can be met, this is the most desirable tool to employ for the present design.

3.2.3.0 External Validity

An interaction of selection and $X$ can be controlled as the intact classes are used for both experimental and control groups.

The reactive effects of experimental treatment on the subjects may not be seen when an experiment is conducted without the subjects being aware of it.

3.2.4.0 Sample

<table>
<thead>
<tr>
<th>No.</th>
<th>VII-A (Expt. Group)</th>
<th>47</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VII-B (Control Group)</td>
<td>47</td>
</tr>
<tr>
<td>Sex</td>
<td>Both Girls and Boys</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>13 years</td>
<td></td>
</tr>
</tbody>
</table>

3.3.0 Methodology: Model of Systematic Instruction Design
(Hughes, 1977)

1. ANALYSIS PHASE  2. DESIGN PHASE

5. EVALUATION PHASE  3. PREPARATORY PHASE

4. IMPLEMENTATION PHASE

3.3.1.0 Analysis Phase

It comprises of two major components viz. content analysis and learner analysis. The researcher analysed the content of Mathematics syllabus of Std.VII as prescribed by Maharashtra State Text Book Bureau of (1995). Learner analysis was meant for understanding the students' general capabilities so that appropriate instruction and activities could be planned for them.

3.3.1.1 Content Analysis

This is the part of analysis phase. The content of Mathematics syllabus of Std. VII was thoroughly analysed.
Unit 1.0 Rational Numbers

1.1. Rational numbers include integers as well as fractions.

1.2. The rational number with greater numerator is greater if the denominators of given rational numbers are equal.

1.3. The rational number with smaller denominator is greater if the numerators of given rational numbers are equal.

1.4 Equivalent rationals can be obtained by dividing or multiplying the numerator and denominator by the same number.

1.5 Rational numbers can be written in the form of decimal fractions by dividing the numerator by the denominator.

Unit 2.0 Indices

2.1 If 'a' is any positive integer and m and n are non-zero positive integers, then \( a^m \times a^n = a^{m+n} \)

2.2 If a is any positive integer and m and n are non-zero positive integers then, \( a^m \div a^n = a^{m-n} \)

If \( m < n \), then \( a^m \div a^n = 1/a^{n-m} \)

2.3 If a and b are positive integers and m is any non-zero positive integer then \((a \times b)^m = a^m \times b^m\)

2.4 \((a \div b)^m = a^m \div b^m\)

2.5 \((a)^0 = 1\)

2.6 \(a^{-1} = 1/a\)
Unit 3.0 Congruence

3.1 The figures that coincide with each other are called congruent figures.

3.2 Segments having equal length are called congruent segments.

3.3 Angles having equal measures called congruent angles.

3.4 Circles with equal radii are called congruent circles.

3.5 With respect to given correspondence between vertices of two triangles, there are three pairs of corresponding sides and three pairs of corresponding angles.

3.6 For two triangles to be congruent, at least three pairs of corresponding sides and angles should be congruent.

3.7 Figures having equal areas are called equi-areal figures. But equi-areal figures are not necessarily congruent. (ex. equi-areal rectangles)

Unit 4.0 Congruence of Triangles

4.1 SSS (side-side-side) test: When three sides of a triangle are congruent to the corresponding sides of other triangle, then the triangles are said to be congruent by SSS test.

4.2 SAS (side-angle-side) test: When two sides and angle included by them in one triangle are congruent to the corresponding two sides and angle included by them in other triangle, then two triangles are said to be congruent by SAS test.
4.3 ASA (angle-side-angle) : When two angles and side included by them in one triangle are congruent to the corresponding two angles and side included by them in another triangle, then two triangles are said to be congruent by ASA test.

4.4 Hypotenuse-side test : When the hypotenuse and one side of a right angled triangle are congruent to the hypotenuse and corresponding side of other right-angled triangle, the triangles are said to be congruent by hypotenuse-side-test.

Unit 5.0 Operations on rational numbers

5.1 Addition of rational numbers follows the commutative property.

5.2 If the sum of two numbers is zero, then the numbers are said to be opposite numbers.

5.3 Subtraction of two rational numbers means addition of a rational number and the opposite of the second number.

5.4 Multiplication of two rational numbers follows the commutative property.
   (i.e. the product remains same even after changing the order of two numbers.)

5.5 If the product of two numbers is one, then the numbers are said to be reciprocals or multiplicative inverse of each other.

5.6 Division of one rational number by the other rational number means multiplication of one rational number by the reciprocal of the other rational number.
5.7 When mixed operations on rational numbers are given, BODMAS rule is used.

Unit 6.0 Equations

6.1 To denote the unknown quantity in a given statement, a letter of alphabet (x, y etc.) is used which is known as variable.

6.2 To maintain the relation of equality between left hand side and right hand side of an equation, the same operations involving the same numbers are performed on both sides.

Unit 7.0 Variation

7.1 In direct variation between two quantities, as one-quantity changes, the other quantity changes in the same direction.

7.2 In inverse variation between two quantities, as one quantity changes the other quantity changes inversely.

Unit 8.0 Time, Distance and Speed

8.1 Distance can be calculated by taking the product of time and speed i.e. distance = speed x time.

Unit 9.0 Time and Work

9.1 There is direct variation between work and time required to complete the work.

9.2 There is inverse variation between number of labourers and time required to complete the work
Unit 10.0 Quadrilateral

10.1. Figure (convex) having four sides is called a quadrilateral.

10.2. Properties of rectangle: Opposite sides are congruent
   Each angle is of $90^\circ$
   Diagonals are congruent.

10.3. Properties of square: All four sides are congruent
   Each angle is of 90
   Diagonals bisect each other.

10.4. Properties of parallelogram:
   **Opposite sides are parallel and congruent**
   Opposite angles are congruent.

10.5 Properties of rhombus: All four sides are congruent
   Diagonals bisect each other at right angles.

10.6 Trapezium: Only one pair of opposite sides is parallel and the other
   pair is non-parallel.

Unit 11.0 Concurrence in a Triangle

11.1. A perpendicular segment drawn from a vertex of a triangle to its
   opposite side is called an altitude.

11.2. The point in which three attitudes intersect each other is called point of
   concurrence of altitudes or orthocentre.

11.3. Orthocentre of altitudes in acute angled triangle lies in its interior, in
   obtuse angled triangle lies in its exterior and in right angled triangle lies
   on the vertex of a triangle.
11.4. The point of concurrence of perpendicular bisectors of sides of a triangle is called circumcentre.

11.5. Circumcentre lies in the interior of acute angled triangle, in the exterior of obtuse angled triangle and on the side of a right-angled triangle.

11.6. Taking circumcentre as a centre and distance between circumcentre and any vertex of a triangle as radius, a circle can be drawn. This circle passing through all the vertices of a triangle is known as circumcircle.

11.7. The point of concurrence of bisectors of angles of a triangle is known as in circle.

11.8. A circle that touches all three sides of a circle internally is called in circle.

**Unit 12.0 Percentage**

12.1 The value calculated per hundred taken as standard value is percentage.

12.2 We can express marks, amount of a constituent in a substance, literacy etc. using percentage.

**Unit 13.0 Profit and Loss**

13.1 When selling price (S.P.) is greater than cost price (C.P.), there is profit, Profit = S.P. - C.P.

13.2 When cost price is greater than selling price, there is loss.
13.3. Profit percent can be calculated using formula, \( \frac{\text{Profit}}{\text{C-P}} \times 100 \)

13.4. Loss percent can be calculated using formula, \( \frac{\text{Loss}}{\text{C-P}} \times 100 \)

Unit 14.0 Square and Square roots

14.1. The product of a given number and itself is called square.

14.2. The numbers having 2, 3, 7, 8 in unit's place are not perfect squares.

14.3. A square-root of a perfect square can be obtained by factorizing a given number into its prime factors.

14.4. Square of a number having 5 in unit's place can be calculated verbally, ex. \((15)^2 = ?\)

   The digit in ten's place is 1 and the integer following it is 2
   Therefore, \(1 \times 2 = 2\)
   Write 25 after 2 so that we get 225 i.e. a square of 15

14.5. Square root of a number having last two digits forming a number 25 can be obtained verbally. e.g. \(\sqrt{625}\)

   The digit in hundred's place is 6.
   6 can be factorize as \(2 \times 3\)
   The smaller factor is 2
   Write 5 after 2 so as to get the number 25,
   \(\sqrt{625} = 25\)
Unit 15.0  Theorem of Pythagoras

15.1. According to theorem of Pythagoras, the area of a square built on the hypotenuse of a right-angled triangle is equal to the sum of the areas of squares built on each of the remaining two sides.

15.2. \[(\text{Hypotenuse})^2 = (\text{one side})^2 + (\text{other side})^2\]
\[(\text{Other side})^2 = (\text{Hypotenuse})^2 - (\text{one side})^2\]

15.3. If the square of the largest number out of three given numbers is equal to the sum of the squares of remaining two numbers then the group of given three numbers is called Pythagorean triplet.

Unit 16.0  Circle (Properties)

16.1. A perpendicular drawn from the centre of the circle to the chord bisects the chord.

16.2. Congruent chords of the same circle are equidistant from the centre of the circle.

16.3. Central angles subtended by the congruent chords are congruent.

16.4. Angles subtended by the same arc in a circle are congruent.

16.5. Angles inscribed in the same are congruent.

16.6. Angle inscribed in a semicircle is a right angle.

16.7. Quadrilateral having all its vertices on the circle is called a cyclic quadrilateral.

16.8. Opposite angles of cyclic quadrilateral are supplementary.
Unit 17.0 Product of Algebraic Expressions

17.1. While taking the product of two monomials, the product of co-efficients and variables is taken together.

17.2. While taking the product of a monomial and a binomial, each term of the binomial is multiplied by the monomial.

17.3. While taking the product of two binomials, each term of the second binomial is multiplied by each of the terms of the first binomial. In this, the like terms thus obtained are added or subtracted following the sign rules.

17.4. Product of algebraic expressions can be verified for the given values of the variables involved in the expressions.

Unit 18.0 Identities

18.1. Identities are the algebraic expressions that can be verified for all the possible values of the variables used in them

18.2. If a and b are two terms in a binomial, then the square of the addition of two terms can be obtained using the formula \((a + b)^2 = a^2 + 2ab + b^2\)

18.3. The square of the difference between two terms a and b can be obtained using formula-
\[(a - b)^2 = a^2 - 2ab + b^2\]

18.4. The product of the addition of two terms a and b and the difference between them can be obtained using formula-
\[(a + b) (a - b) = a^2 - b^2\]
Unit 19.0 Factors of Algebraic Expressions

19.1. For factorizing a monomial, the coefficient is factorized into prime factors and the variables are written in the form of multiplication as per their powers.

19.2. For factorization of binomials, both the terms are divided by a common factor so as to obtain the other factor.

19.3. For factorization of polynomials, terms are paired or grouped in such a way that they have a common factor and then the other factors are obtained.

19.4. For the factorization of a trinomial, it is essential to know whether it is a perfect square. If twice product of the square roots of the first and the last term is equal to the middle term, then the given trinomial is a perfect square.

19.5. If the given trinomial is a perfect square and all its terms are positive, then the given trinomial is factorized by using the formula,

\[ a^2 + 2ab + b^2 = (a + b)^2 \]

19.6. If the middle term of a perfect square trinomial is negative, then it can be factorized by using the formula,

\[ a^2 - 2ab + b^2 = (a - b)^2 \]

19.7. If the given algebraic expression is the difference between two perfect squares then it can be factorized by the formula

\[ a^2 - b^2 = (a + b)(a - b) \]
Unit 20.0 Simple Interest

20.1 Simple interest can be calculated using formula,

\[ I = \frac{P \times N \times R}{100} \]

Any of the four quantities I, P, N, R can be calculated using the same formula: \( I = \frac{PNR}{100} \)

Unit 21.0 Area

21.1. Formula for the area of parallelogram can be derived by converting the given parallelogram into a rectangle and then using the formula for the area of rectangle. (Paper folding technique)

21.2. Formula for the area of triangle can be derived using the formula for that of parallelogram. (Paper folding technique).

21.3. Formula for the area of rhombus can be derived using the formula for that of the rectangle. (Paper folding technique).

21.4. For deriving the formula of area of trapezium, the formula of area of triangle and rectangle can be used.

Unit 22.0 Discount, Commission and Rebate

22.1. Discount expressed in terms of percentage.

To calculate the net discount we should know the list price of the article and percent discount offered on it.

\[ \text{i.e. Discount (Rs.)} = \text{list price} \times \frac{\text{discount}}{100} \]
22.2. Commission is charged on the cost price of the commodity and it is expressed in terms of percentage.

22.3. Commission in rupees can be calculated if the rate of commission and total price of the commodity are given.

22.4. Rebate is offered so as to promote the sale of some goods, e.g. Khadi fabrics, handicrafts etc. Rebate in rupees can be calculated, if the total price of the goods and rebate percentage are given.

**Unit 23.0 Average**

23.1. Average is a figure which is representative of given data and around which most of the values of a given quantity cluster e.g. average marks, average temperature etc.

23.2. Average of the given data can be calculated by dividing the sum of the given values by the total number of the values i.e.

\[
\text{Average marks} = \frac{\text{total marks in all subjects}}{\text{number of subjects}}
\]

23.3. The sum total of the given values can be obtained, if average value and the number of values are given.

**Unit 24.0 Bar Graph**

24.1. Graph has three main components viz. X-axis, Y-axis and the origin i.e. 'O'

24.2. On X-axis independent variables such as names of students, years etc. are shown whereas on the Y-axis the respective values i.e. dependent variables are shown.
24.3. The scale on X-axis and Y-axis is determined according to the range of the given values and it is shown on the right hand side of the graph.

24.4. The value of each quantity on X-axis is indicated by a point in such a way that it co-ordinates the corresponding number on the Y-axis.

24.5. Columns of appropriate width are drawn corresponding to each plotted point on the graph.

3.3.1.2 Objectives and Specifications

1.0 Rational numbers

1.1.1. Knowledge : Student tells natural numbers, whole numbers and integers.

1.1.2. Application : Student classifies the given numbers as rational numbers and non-rational numbers.

1.2.1. Application : Student writes the order relation between the given rational numbers with equal numerators.

1.3.1. Application : Student writes the order relation between the given rational numbers with equal denominators.

1.4.1. Application : Student obtains the equivalent rational to the given rational number by taking LCM of the denominator.

1.5.1. Application : Student writes the given rational number in the form of decimal fraction.
2.0 Indices

2.1.1. Knowledge : Student tells the first law of indices i.e. \( a^m \times a^n = a^{m+n} \)

2.1.2. Application : Student solves the example based on First law of indices

2.2.1. Knowledge : Student tells the second law of indices i.e. \( a^m - a^n = a^{m-n} \)

\[(\text{when } m > n \text{ and when } m < n \)]

2.2.2. Application : Student solves the example using second law of indices

2.3.1. Knowledge : Student tells the third law of indices

\[ (a \times b)^m = a^m \times b^n \]

2.3.2. Application : Student solves the examples using third law.

2.4.1. Knowledge : Student tells the fourth law of indices i.e. \( (a^m)^n = a^{mn} \)

2.4.2. Application : Student solves the examples based on fourth law.

2.5.1. Knowledge : Student tells the fifth law of indices i.e. \( (a/b)^m = (a^m / b^m) \)

2.5.2. Application : Student solves the examples based on fifth law

2.6.1. Knowledge : Student tells the sixth law of indices i.e. \( a^0 = 1 \)

2.6.2. Application : Student solves the example based on sixth law.

2.7.1. Knowledge : Student tells the seventh law of indices i.e. \( a^{-1} = 1/a \)

2.7.2. Application : Student solves the examples based on seventh law.

3.0 Congruence

3.1.1. Knowledge : Student defines 'congruence'.

3.1.2. Application : Student shows congruent objects in surroundings.

3.1.3. Skill : Student draws congruent figures.

3.2.1. Knowledge : Student defines congruent segments
3.2.2. Application : Student pairs the congruent segments from the given list of segments and their lengths.

3.2.3. Skills : Student draws segments congruent to each other.

3.3.1. Knowledge : Student defines congruence of angles.

3.3.2. Application : Student identifies congruent angles from the given list.

3.3.3. Skills : Student draws the angles congruent to each other using scale and protractor.

3.4.1. Knowledge : Student defines congruence of circles.

3.4.2. Application : Student identifies congruent circles by measuring their radii.

3.4.3. Skills : Student draw congruent circles using pairs of compasses.

3.5.1. Knowledge : Student tells what is one-to-one correspondence between vertices of triangles.

3.5.2. Comprehension : Student tells the correspondence between vertices of two triangles in possible ways.

3.5.3. Application : Student shows the correspondence between vertices of two triangles.

3.6.1. Knowledge : Student tells the condition to be satisfied for the triangles to be congruent.

3.7.1. Knowledge : Student defines equi-areal figures

   Process : Given the area, student identifies equi-areal figures (such as circle square, rectangle etc.) from a pair, student draws them to confirm if they are congruent or otherwise.
Unit 4.0 Congruence of triangles


4.1.2. Comprehension: By observing the given figures, student tells the test of congruence.

4.1.3. Application: Student tells the necessary conditions so that SSS test of congruence of triangles, could be applied for the given pair of triangles to be congruent.

4.1.4. Skill: Student constructs the triangle, given the lengths of its three sides.

4.2.1. Knowledge: Student tells the statement of side-angle-side test of congruence of triangles.

4.2.2. Comprehension: Student identifies the test of congruence observing given pair of triangles, student tells necessary conditions so that the triangles are congruent by SAS test.

4.2.4. Skill: Student draws a triangle when given the lengths of its two sides and measure of the angle included by them.

4.3.1. Knowledge: Student tells the statement of angle-side-angle test of congruence of triangles.

4.3.2. Comprehension: Student tells the test of congruence of triangles, by observing the given figures.

4.3.3. Application: Given a pair of triangles, student tells the congruent pairs of angles and sides so that they are said to be congruent by ASA test.
4.3.4. Skill: Student draws a triangle, when given the measures of two angles and the length of the side included by them.

4.4.1. Knowledge: Student tells the statement of hypotenuse-side test of right angled triangle.

4.4.2. Comprehension: Student identifies the test of congruence by which given pair of right-angled triangles is congruent.

4.4.3. Application: For the given triangles to be congruent by hypotenuse-side test, student tells the necessary conditions.

4.4.4. Skill: Student draws a right angled triangle when given the lengths of hypotenuse and one side.

Unit 5.0 Operations on rational numbers

5.5.1. Knowledge: Student tells the properties of addition of rational numbers.

5.1.2. Application: Student adds the given rational numbers and finds their sum.

5.1.3. Process: Student observes and compares the properties of addition of positive rationals and negative rationals.

5.2.1. Knowledge: Student defines opposite numbers.

5.2.2. Application: Student tells the opposite number of a given number.

5.3.1. Knowledge: Student tells the rules of substraction of rational numbers

5.3.2. Application: Student finds the difference between given rational numbers
5.4.1. Knowledge: Student tells the commutative property of multiplication.

Student tells sign rules followed while carrying out multiplication.

5.4.2. Application: Student finds the product of given rational numbers.

5.5.1. Knowledge: Student defines the term 'reciprocals'

5.5.2. Application: Student tells the reciprocal of a given rational number.

5.6.1. Knowledge: Student tells the rules of division of rational numbers.

5.6.2. Application: Student writes the division of rational number in the form of multiplication. Student carries out the division of given rational numbers.

5.7.1. Knowledge: Student tells the BODMAS rule for carrying out mixed operations on numbers.

5.7.2. Application: Student solves the mixed operations on rational numbers using BODMAS rule.

Unit 6.0 Equations

6.1.1. Knowledge: Student defines the term 'variable' and tells its use in solving the equation.

6.1.2. Application: Student writes the given statement in the form of equation using variable for the unknown quantity.

6.2.1. Knowledge: Student tells what is the relation of equality in a mathematical expression.

6.2.2. Application: Student tells the right operation using a right number to be carried out so as to find the value of the variable.
6.2.3. Process: Student solves the equation so as to obtain the value of the unknown quantity.

Unit 7.0 Variation

7.1.1. Knowledge: Student tells the types of variation.

Student defines direct variation.

7.1.2. Application: Student tells the type of variation existing between two quantities.

Student solves the problems based on direct variation.

7.2.1. Knowledge: Student defines inverse variation.

7.2.2. Student solves the problems based on inverse variation

Unit 8.0 Time, distance and speed

8.1.1. Knowledge: Student tells the formula to calculate distance when speed and time are given.

8.1.2. Application: Given any of the two quantities out of distance, speed and time, student calculates the third quantity.

8.1.3. Process: Student compares the units of the given quantities and converts them into appropriate units and then carry out the further calculations.

Unit 9.0 Time and work

9.1.1. Comprehension: Student tells the type of variation between amount of work and time required to complete it.

9.1.2. Application: Student solves the problems based on time and work.
9.2.1. Comprehension: Student tells the type of variation between number of labourers and time taken by them to complete the given work.

9.2.2. Knowledge: Student solves the problems based on time, work and number of labourers.

Unit 10.0 Quadrilateral

10.1.1. Knowledge: Student defines the term 'quadrilateral'.
   Student tells the type of quadrilateral.
   Student tells the properties of each type of quadrilateral.

10.2.1. Knowledge: Student tells the general properties of rectangle.

10.2.2. Application: Student solves the problems based on properties of rectangle.

10.2.3. Process: Student finds the properties of rectangle related to its diagonals by studying the diagrams.

10.3.1. Knowledge: Student tells the general properties of square.

10.3.2. Application: Student solves the problems based on the properties of square.

10.3.3. Process: Student observes and derives the properties of square related to its diagonals.

10.4.1. Knowledge: Student tells the general properties of parallelogram.

10.4.2. Application: Student solves the problems based on the properties of parallelogram.

10.5.1. Knowledge: Student tells the general properties rhombus.
10.5.2. Application: Student solves the problems based on the properties of a rhombus.

10.5.3. Process: Student studies and derives the properties of rhombus related to its diagonals and angles.

10.6.1. Knowledge: Student defines 'trapezium'.

10.6.2. Application: Student solves the problems based on the properties of trapezium.

Unit 11.0 Concurrence in triangle

11.1.1. Knowledge: Student defines the term 'altitude'.

11.1.2. Skill: Student draws attitudes on each side of a triangle using set square.

11.2.1. Knowledge: Student defines the term 'orthocentre'.

11.2.2. Student names the point of concurrence of altitudes i.e. orthocentre in the diagram.

11.3.1. Skill: Student draws each type of triangle (i.e. acute, obtuse and right angled triangle) and draws their altitudes.

11.3.2. Process: Student compares the position of orthocentre in each type of triangle.

11.4.1. Knowledge: Student defines 'perpendicular bisector'.

11.4.2. Skill: Student draws the perpendicular bisector on each side of the triangle and marks the point of concurrence i.e. the circumcentre.

11.5.1. Process: Student draws each type of triangle (Acute, obtuse and right angled).
Student draws perpendicular bisector of each side of triangle of each type.
Student marks the position of point concurrence of perpendicular bisectors i.e. the circumcentre in each type of triangle and compares.

11.6.1. Knowledge: Student defines circumcircle.
11.6.2. Skill: Student draws circumcircle.

11.7.1. Knowledge: Student defines the term 'angle bisectors'.
11.7.2. Applications: Student draws angle bisector of each angle of a triangle.
11.7.3. Process: Student draws acute angled triangle, obtuse angled triangle and right angled triangle and draws the angle bisector of each angle of a triangle of each type.
Student marks the position of point of concurrence of angle bisectors in each type of triangle.

11.8.1. Knowledge: Student defines 'incircle'.
11.8.2. Skill: Student draws incircle.

**Unit 12.0 Percentage**

12.1.1. Knowledge: Student tells the meaning of 'percentage'.
12.2.1. Knowledge: Student tells its usage in practice.
12.3.1. Application: Student solves the problems based on percentage.
Unit 13.0  Profit and loss

13.1.1. Knowledge: Student tells the formula to calculate profit in Rs.
13.2.1. Knowledge: Student tells the formula to calculate loss in Rs.
13.2.2. Application: Student calculates loss in Rs.
13.3.1. Knowledge: Student tells the formula to calculate profit percent.
13.3.2. Application: Student solves the problems based on profit percent.
13.4.1. Knowledge: Student tells the formula to calculate loss percent.
13.4.2. Application: Student solves the problems based on loss percent.

Unit 14.0  Square and square root

14.1.1. Knowledge: Student tells the meaning of 'square' of a number.
14.1.2. Application: Student finds the square of given numbers.
14.2.1. Process: Student observe squares of numbers from 1 to 30 and notes the digits that never appear in the units place of any perfect square.
14.3.1. Process: Student factorizes a given perfect square in order to find its square root.
14.4.1. Process: Student finds verbally the square of a number ending with 5.
14.5.1. Process: Student finds verbally the square root of a number ending with 25.

Unit 15.0 The theorem of Pythagoras

15.1.1. Knowledge: Student tells the statement of the theorem of Pythagoras.

15.1.2. Process: Student verifies the theorem of Pythagoras geometrically.

15.2.1. Knowledge: Student tells the formula to find the length of the hypotenuse when the lengths of the other two sides are given.

15.3.1. Knowledge: Student tells what is a 'Pythagorean triplet'

15.3.2. Application: Student verifies mathematically whether the given numbers form a 'Pythagorean triplet'.

Unit 16.0 Circle

16.1.1. Knowledge: Student tells the property of a perpendicular drawn from centre of a circle to the chord.

16.1.2. Application: Given the length of the chord, student tells the length of each intercept on a chord and vice versa.

16.2.1. Knowledge: Student tells the 2nd property of the circle.

16.2.2. Application: Given the lengths of chords and the distance between the centre of the circle and any one chord, student tells the length of another perpendicular.

16.3.1. Knowledge: Student tells the property of central angles subtended by congruent chords.
16.3.2. Application: Student finds the measure of central angle by using the property 3.

16.4.1. Knowledge: Student tells the property of angles subtended by same arcs.

16.4.2. Application: Student applies the above property and solves the problems.

16.5.1. Knowledge: Student tells the property of angles inscribed in the same arc.

16.5.2. Application: Student applies the above property and solves the problems.

16.6.1. Knowledge: Student tells the property of angle inscribed in a semicircle.

16.6.2. Application: Student applies the property and solves the problems.

16.7.1. Knowledge: Student defines 'cyclic quadrilateral'.

16.8.1. Knowledge: Student tells the property of opposite angles of cyclic quadrilateral.

16.8.2. Application: Student applies the property and solves the examples.
Unit 17.0  The product of algebraic expressions

17.1.1.  Application: Student finds the product of given monomials.

17.2.1.  Application: Given a monomial and a binomial, student finds their product.

17.3.1.  Application: Given two binomials, student finds their product.

17.4.1.  Process: Given the values of the variables involved in the given algebraic expressions to be multiplied, student verifies the equation.

Unit 18.0  Identities

18.1.1.  Knowledge: Student tells what is an 'identity' in algebra.

18.1.2.  Process: Student verifies whether the given algebraic expressions are identities.

18.2.1.  Process: Student derives the formula to square a binomial in which two terms viz. 'a' and 'b' are added.

18.3.1.  Process: Student derives the formula to square the difference between two terms of a binomial.

18.4.1.  Process: Student derives a formula to find the product of sum of two terms and the difference between two terms.

Unit 19.0  Factors of algebraic expressions

19.1.1.  Application: Student factorizes a given monomial.

19.2.1.  Application: Student factorizes a given binomial.

19.3.1.  Application: Student factorizes a given polynomial by grouping the terms having common factor.
19.4.1. Comprehension: Student identifies whether the given trinomial is a perfect square.

19.5.1. Application: Student factorizes the given trinomial having all terms positive.

19.6.1. Application: Student factorizes the given trinomial having at the middle term negative.

19.7.1. Application: Student factorizes the given binomial consisting of difference between two terms which are the perfect squares.

**Unit 20.0 Simple interest**

20.1.1. Knowledge: Student tells the formula to calculate simple interest.

20.1.2. Application: Student solves the problems based on simple interest.

**Unit 21.0 Area**

21.1.1. Process: Given a piece of paper shaped into a parallelogram, student converts it into a rectangle, applies the formula to find area of rectangle and derives the formula to find area of parallelogram.

21.2.1. Process: Given a piece of card-paper shaped into a triangle, student converts it into a parallelogram, applies the formula to find area of parallelogram and derives the formula to find the area of rectangle.
21.3.1. Process: Given a card-paper shaped into rhombus student converts it into a rectangle and derives the formula to find the area of rhombus

21.4.1. Process: Student derives the formula to find the area of trapezium using the formulae to find the area of rectangle and that of triangle.

Unit 22.0 Discount, commission and rebate

22.1.1. Knowledge: Student tells the meaning of discount and gives examples in day-to-day life.

22.1.2. Application: Given the list price of the article and discount percent.

22.2.1. Knowledge: Student tells the purpose for which the commission is charged.

22.2.2. Application: Student solves the problems based on commission.

22.3.1. Knowledge: Student explains what is rebate.

22.3.2. Application: Given the cost of the articles and percent rebate, student calculates the selling price with rebate.

Unit 23.0 Average

23.1.1. Knowledge: Student tells the meaning of the term 'average'.

23.2.1. Application: Student calculates the average of given data.
23.3.1. Application: Student calculates the sum total of the values when average value and the number of values are given.

Unit 24.0 Bar graph

24.1.1. Knowledge: Student tells the main components of graph.

24.1.2. Application: Student draws X-axis, Y-axis and marks the origin.

24.2.1. Application: Student shows independent variables on X-axis and dependent variables on Y-axis.

24.3.1. Application: Student takes appropriate scale on Y-axis and writes it on the right hand side of the graph paper.

24.4.1. Application: Student plots the point on a graph paper so that it co-ordinates the correct value on the Y-axis.

24.5.1. Application: Student draws the columns of appropriate width corresponding to the points plotted on the graph.

3.3.2.0 Design Phase

Instruction was designed so as to meet the needs of the present study. Control group was taught by the traditional approach (i.e. lecture-discussion, chalk board) whereas Constructivist lessons were prepared for the Experimental group. Constructivist lesson plans are based on Constructivist design which defines the Constructivist teacher’s approach to the teaching-learning process in the classroom.
3.3.2.1 Constructivist Design

The following schematic diagram represents the Constructivist teacher's approach to teaching-learning process in the classroom.

![Constructivist Design Diagram]

- **Constructivist Teacher**
  - Is response correct? → NO → Unusual Response
  - YES → Build meaning
    - Probe into thinking pattern
      - Does it coincide with your thinking pattern? → NO → Understand their constructs & probe errors
      - YES → Let the students decide on adequacy of their constructs → Conscious Reflection
    - Adjust task difficulty according to needs
The Constructivist Design can be summarized as follows:

i) Teacher rejects the traditional assumption that information can be passed on to the learners and understanding will take place.

ii) Teacher approaches an unusual response with genuine interest in learning its character, origin and implication.

iii) Construction has integrity and sensibility within its framework. If erroneous, it has to be restructured. The important aspect of this is error analysis. For this teacher needs to probe deep into thinking pattern of the child and instead of merely suggesting the corrections the teacher has to find out where, why and how the students' perception had gone wrong and how it has led to a wrong construction.

The lesson plans provide a scope for the teacher to probe into thinking pattern of the students by means of various 'talk through' sessions. This will be discussed later in this chapter.

3.3.2.2 Strategies for implementing Constructivist design

The following strategies are suggested by Yager (1991) for implementing a Constructivist lesson plan (Hanley S.,1999)

1. Starting the lesson
   - Observe surroundings for point to question.
   - Ask questions.
   - Consider possible responses to questions.
   - Note unexpected phenomena.
   - Identify situations where student perceptions vary.
2. Continuing the lesson

Engage in focused play.
Brainstorm possible alternatives.
Look for information.
Experiment with materials.
Observe a specific phenomenon.
Design a model.
Collect and organize data.
Employ problem-solving strategies.
Select appropriate resources.
Students discuss solutions with others.
Students design and conduct experiments.
Students evaluate and debate choices.
Students identify risks and consequences.
Define parameters of an investigation.

3. Proposing explanations and solutions

Communicate information and ideas.
Construct and explain a model.
Construct a new explanation.
Review and criticize solutions.
Utilize peer evaluation
Assemble appropriate closure.
It integrates a solution with existing knowledge and experiences.

4. Taking actions

Make decisions.
Apply knowledge and skills
Share information and ideas.
Ask new questions.
Develop products and promote new ideas.
Use models and ideas to illicit discussions and acceptance by others.

Yager (1991, p.56) also offers a checklist for a teacher to determine degree of Constructivist teacher in the classroom Versus a more objectivist approach.
### Table 3.1
Objectivist and Constructivist Teachers

<table>
<thead>
<tr>
<th>More Objectivist</th>
<th>More Constructivist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher</td>
<td>Identifies the issue/topic</td>
</tr>
<tr>
<td>No</td>
<td>Issue is seen as relevant</td>
</tr>
<tr>
<td>Teacher</td>
<td>Asks the questions</td>
</tr>
<tr>
<td>Teacher</td>
<td>Identifies written and human resources</td>
</tr>
<tr>
<td>Teacher</td>
<td>Locates written resources</td>
</tr>
<tr>
<td>Teacher</td>
<td>Contacts needed human resources</td>
</tr>
<tr>
<td>Teacher</td>
<td>Plans investigation and activities</td>
</tr>
<tr>
<td>No</td>
<td>Varied evaluation techniques used</td>
</tr>
<tr>
<td>No</td>
<td>Students practice self-evaluation</td>
</tr>
<tr>
<td>No</td>
<td>Concepts and skills applied to new situations</td>
</tr>
<tr>
<td>No</td>
<td>Students take action(s)</td>
</tr>
<tr>
<td></td>
<td>Science concepts and principles</td>
</tr>
<tr>
<td>No</td>
<td>emerge because they're needed</td>
</tr>
<tr>
<td></td>
<td>Extension of learning outside the school in evidence.</td>
</tr>
</tbody>
</table>
The strategies and checklist given by Yager served as guidelines during the planning and implementation of the present study. However, Yager's model was employed for science education and the present study deals with Mathematics teaching. Hence, appropriate modifications were made so as to fulfill the requirements of the present study.

3.3.3.0 Preparatory Phase

It includes the components such as getting the physical environment, collecting the materials and resources and selecting the groups of subjects required for the study.

During the preparatory phase, planning and preparation for the pilot study and the actual study were done. Constructivist lesson plans based on each unit of Mathematics of Std. VII, formative tests, post test etc. were already designed. The other supplementary material such as printed worksheets, tests, extracts, pictures, statistics from reference books and newspapers etc. was gathered and organized.

The main task was to get the sample for the implementation of the study. For the pilot study, VIIIth standard classes comprising of 90 students were chosen from S.P.M. English School, Pune 30. The actual study was planned for the different batch of Std. VII students of the academic year 2002-03 from the same school.

The details of Pilot Study have been given in Chapter 4.
3.3.4.0 Implementation Phase

This Chapter includes implementation of Constructivist lesson plans, administration of formative tests and the post-test. The Researchers made due modifications in the Constructivist Teaching Programme, after realizing the difficulties encountered during the pilot study. The details of the sample, research design, validity of the design, tools, techniques etc. are given in module 3.2.0 and 3.3.0.

Summary

1. Researcher selected pre-assembled classes of Std. VII from S.P.M. English School, Pune.

2. The classes were randomly assigned to experimental and control groups.

3. The experimental group was taught by Constructivist method whereas the control group was taught by conventional method (i.e. lecture, narration-explanation etc.).

4. There were total 24 lessons to be covered by the researcher using Constructivist approach. The Constructivist lesson plans are given at the end of this chapter.

5. After the completion of each unit, a formative test of 10 marks was administered to the students of two groups.

6. The errors made by the students of experimental group were noted in order to study the error patterns.
7. The researcher had "talk through" sessions with the students of experimental group in order to probe the thinking pattern and investigate the erroneous constructs.

8. After the completion of entire portion, the post-test of 80 marks was administered to both the groups.

9. Data was collected and analyzed statistically.

10. Achievement on each learning objective, specified in terms of knowledge, application, skill, process etc. was assessed on the basis of percentage of correctly answered questions by the students of both the groups.

3.3.4.1 Main features of the Study

1. Formative Evaluation

   Formative testing is done to monitor student progress over a period of time. The test results indicate how the students attain the instructional objectives (Wiersma, Jurs, 1990, p.18).

   Formative test based on each unit was administered to both the experimental and control groups after the completion of each unit. The main purpose of the test was to build meaning of students' responses in order to study the construction of their thought leading to a particular response. These tests helped to identify the error patterns and investigate the constructs leading to unusual response.
Formative test was criterion-referenced. It was designed to study the students' construction of each concept contained by a particular unit. There was a test item on each domain of a learning objective set by the researcher, e.g.

**Unit 1: Rational Numbers**

Domain: Given the rational numbers \( \frac{a}{b} \) or \( \frac{p}{q} \) the student can find the order relation between them, by taking their cross-products.

Sample Item: Which one is greater of \( \frac{12}{17} \) and \( \frac{18}{33} \) ?

**Unit 2: Indices**

Domain: Given in the form \( a^m \times a^n \), the student finds the product of two exponents where 'a' is any rational number and \( m \) and \( n \) are positive integers.

Sample item: \( 3^4 \times 3^5 = 3 \)  

**Unit 5: Operation on Rational Numbers**

Domain: Given the division of two rational numbers viz. \( \frac{a}{b} \) and \( \frac{p}{q} \), student finds the quotient.

Sample Item: \( \frac{4}{5} - \frac{8}{15} = \)  

Each item in the formative test had four choices out of which one was correct. The student had to tick off against the correct option.
2. **Student “talk through’ sessions**

The search for error patterns can be facilitated by having the student “talk through” the process used to form an answer to the question (Wiersma, Jurs, 1990). The teachers can then see where the process went astray. During these sessions, student was asked the question in order to probe the organization of constructs in his mind. Sometimes the student was told to show the step-wise solution on the paper while working out the problem, he/she was told to “think aloud” thus making his thought process “apparent”. These strategies helped the researchers to investigate the ‘key-step’ in the student’s construction leading to unexpected response.

Some examples of students “talk through” have been presented below.

**Examples**

**Unit 1: Rational Numbers**

Problem: The table below shows the temperature (Minimum and Maximum of four cities in the country on 28th June.

<table>
<thead>
<tr>
<th>City</th>
<th>Minimum (°C)</th>
<th>Maximum (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mumbai</td>
<td>29.5</td>
<td>35.5</td>
</tr>
<tr>
<td>Kolkata</td>
<td>28.5</td>
<td>34.5</td>
</tr>
<tr>
<td>Delhi</td>
<td>31.5</td>
<td>37.5</td>
</tr>
<tr>
<td>Chennai</td>
<td>31.5</td>
<td>35.5</td>
</tr>
</tbody>
</table>
Which number system includes the numbers in the given table?

Sample answer (of one student): Whole number

Discussion with the student:

Which numbers are represented by "whole number"

Student: 0, 1, 2, 3, ....

What type of number is 29.5?

Student: Decimal fraction

Does the system of whole numbers include decimal fractions?

Student: NO

The system that includes both integers and fractions is called the system of rational numbers. Can there be negative integers?

Student: Yes

What are they?

Student: -1, -2, -3, ....

Teacher shows a strip of cadburry

How many squares are there in this cadburry?

Student: 8

Teacher gives him one square. How much part of cadburry you've taken? Out of how many parts?
Student: 1 part out of 8

How will you write it in the numerical form?

Student: 1/8

Tell some more fractions.

Student: ¼, ¾ etc.

How will you write ¼ in the form of decimal fractions?

Student: Shows division, Ans. 0.25.

In each of the above step of interrogation, student was given time to think and to actually work out the answer. Interestingly, the student himself could determine the limits rather the adequacy of his own construction. Accordingly, he adjusted the task difficulty himself.

Unit 7: Rational Numbers

Problem: Soham secured 32 ½ marks in English. Represent it on the number line.

Student's response: Incorrect.

Discussion with the student:

What is the integer part in 32 ½?

Student: 32

Which one is bigger? - 32 or 32 and ½

Student: 32 and ½
Which integer immediately follows 32?

Student: 33

Write 32, 33 and 32 and \( \frac{1}{2} \) in ascending order.

Student: 32, 32 \( \frac{1}{2}, \) 33

Between which two integers 32 and \( \frac{1}{2} \) will lie?

Student: 32 and 33

After which number on the number line you will show 32 \( \frac{1}{2} \)?

Student: 32

What is the fraction part in 32 and \( \frac{1}{2} \)?

Student: \( \frac{1}{2} \)

What is the denominator?

Student: \( \frac{1}{2} \)

\( \frac{1}{2} \) means how many parts out of 2?

Student: 1

Into how many parts you will divide the distance between 32 and 33 on the number line to get \( \frac{1}{2} \) of it?

Student: 2. Shows division of distance into 2 parts.

Mark the first part out of it.

Student: Shows the position of 32 and \( \frac{1}{2} \) on the number line.
Problem: Out of 50 students in a class, 25 students secured first class in the annual examination, what is the percentage of students securing first class?

Students response: 60 p.c.

How will you write 25 out of 50?

Student: 25/50

What will you do to find percentage?

Student: unable to answer.

Multiply 25/50 by 100. Show the calculation.

Student: Simplifies 25/50 x 100. Ans. 50 p.c.

In the fraction 25/50, what does number 25 indicate?

Student: Number of students securing first class.

What does the number 50 indicate?

Student: Total number of students in the class.

How will you write the formula to calculate percentage in this example:

\[
\frac{\text{No. of students securing 1st class}}{\text{No. of students in the class}} \times 100
\]

How will you represent 504 marks out of 600?

Student: 504/ 600
How will you calculate percent marks?

Student: \[
\frac{504}{600} \times 100
\]

What is the percent of marks?

Student: 84 percent

What is the formula used in the above example?

Student: \[
\frac{\text{Marks obtained}}{\text{Total marks}} \times 100
\]

Unit 22: Discount, Commission and Rebate

Problem: The labelled price of a saree is Rs. 500. There is a discount of 20 p.c. What is the selling price of the saree?

Student's sample answer: Rs. 100.

What do you mean by labelled price?

Student: The price printed on the price tag.

What do you mean by 20 p.c. discount?

Student: Rs. 20 less on the price Rs. 100.

How will you write 20 percent in the form p/q?

Student: 20/100
How will you calculate 20/100 of 500?

Student: unable to answer

'of means' multiplication (x sign)

Student: 20/100 x 500

Show the calculation and find the answer:

Student: 20 is the rate of discount, Rs. 500 indicates labelled price.

So, how will you calculate discount in Rs.?

Rate of discount

\[
\frac{\text{Rate of discount}}{100} \times \text{labelled price}
\]

How will you find the selling price?

Student: Subtract discount in Rs. from the labelled price.

The examples sited above are only a few and with a few number of students. Actually, there were many such sessions conducted with the students. It was interesting to make the student's thinking process observable and study the construction or building up of an answer. During the discussion with the teacher, the student himself used to realize the inadequacy of his constructions and he/she used to seek help from the teacher. This process also revealed improper constructions of basic skills such as multiplication, division, mathematical expressions etc. When such barriers were removed, the student went ahead smoothly to solve the problem.
3.3.5.0 Evaluation Phase

3.3.5.1 Tools:
1. Formative tests
2. Post test
3. Opinionnaire

Two types of tests were designed to evaluate the progress of the students on the specific learning objectives viz. Knowledge and comprehension, application, analysis-synthesis (process) and skill.

1. Formative Tests

(Refer to Appendix No. B-1 to B-24)

24 tests were designed and each test was administered to the students of both the experimental and control group after the completion of respective unit. Details of formative tests have been discussed earlier in module 3.3.4.2 under the title “Main Features of the Study”.

Each formative test was of 10 marks and each question had four choices. Each correct answer was given one mark. The scores on the formative tests administered to both the groups were collected and analysed statistically. The achievement of students of the control group and experimental group was measured in terms of percentage of correctly answered questions based on each learning objective viz. knowledge and comprehension, application, process and skill.

Objective-wise raw scores of the students of two groups are given in Appendix No. D-1 to D-24. The detailed analysis and results are given in Chapter V.
2. Post-test

Post-test was of total 80 marks: Objective-wise distribution of marks is given below.

<table>
<thead>
<tr>
<th>Objective</th>
<th>No. of Items</th>
<th>Total Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge and comprehension</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Application</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Process</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Skill</td>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

Post-test was criterion-referenced. The details of CRT are given in module 3.3.4.2 under the title formative evaluation.

Post-test was administered to the students of two groups viz. experimental and control. The scores of the students on each objective and analyzed statistically. The difference between pre-test, post-test scores of two groups was measured employing the statistical technique called ANOCOVA.

3. Opinionnaire

(Refer to Appendix E-1)

Opinionnaire was administered to the students of experimental group taught by Constructivist Approach. The items in the scale were designed so as to know the students' opinion about Constructivist Approach.
There were total 20 items. Each item had two choices viz. Yes or No. The students were also allowed to express their opinions in detail, if any.

The following scores were given for each response.

YES: 2  
NO: 1

Following scales were used to measure the degree of liking.

Table 3.3
Scale for Opinionnaire

<table>
<thead>
<tr>
<th>Score Range</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>81-100</td>
<td>Strongly liked</td>
</tr>
<tr>
<td>61-80</td>
<td>Liked</td>
</tr>
<tr>
<td>41-60</td>
<td>Disliked</td>
</tr>
<tr>
<td>21-40</td>
<td>Strongly disliked</td>
</tr>
<tr>
<td>0-20</td>
<td>Can't say</td>
</tr>
</tbody>
</table>

3.3.5.1 Statistical Techniques

1. ANOCOVA (Analysis of Co-variance)

Analysis of co-variance represents an extension of analysis of variance to allow for the correlation between initial and final scores. Covariance analysis is especially useful to experimental psychologists when for various reasons it is impossible or quite difficult to equate control and experimental groups at the start, a situation which often obtains in actual experiments. Through covariance analysis one is able to affect adjustments in final or
terminal scores which will allow for differences in some initial variable.
(Garrett H., 1985, pp. 295).

In the present study, researcher used non-randomized, pre-test, post-test, control group design. The groups were not equated initially due to administrative difficulties of the school. Thus, ANOCOVA proved to be useful in order to compensate the lack of equivalency between two groups.

2. Percentage Proficiency Level on Learning Objectives

Formative tests (24 in number) and post-test had questions so designed as to test the students’ achievement on each of the learning objectives viz. knowledge, comprehension, application, process and skill. Researchers calculated the percentage of correctly answered questions by two groups on each objective of the formative tests and post-test. Results were tabulated and graphically represented in order to indicate the difference between proficiency levels obtained by the students of two groups. Data and the related graphs are given in Chapter 5.
(Refer to Appendix No. C-3 and C-4 for objective-wise analysis of post-test scores).
Constructivist Lesson Plans 1-24
CONSTRUCTIVIST LESSON PLANS

Note: The organization of units and sub-units included in each chapter has been given in Section 3.3.1.1. The objectives and specifications for each unit have been given in Section 3.1.2.

CHAPTERS 1 TO 24

CHAPTER LIST

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Title</th>
<th>Approximate Number of Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Rational Numbers</td>
<td>10</td>
</tr>
<tr>
<td>2.</td>
<td>Indices</td>
<td>10</td>
</tr>
<tr>
<td>3.</td>
<td>Congruence</td>
<td>6</td>
</tr>
<tr>
<td>4.</td>
<td>Congruence of triangles</td>
<td>10</td>
</tr>
<tr>
<td>5.</td>
<td>Operations on rational numbers</td>
<td>8</td>
</tr>
<tr>
<td>6.</td>
<td>Equations</td>
<td>8</td>
</tr>
<tr>
<td>7.</td>
<td>Variation</td>
<td>4</td>
</tr>
<tr>
<td>8.</td>
<td>Time, Distance and speed</td>
<td>4</td>
</tr>
<tr>
<td>9.</td>
<td>Time and work</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>Quadrilaterals</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>Concurrence in a triangle</td>
<td>10</td>
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<td>12</td>
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<td>6</td>
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<td>Profit &amp; Loss</td>
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<td>15</td>
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<td>16</td>
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<td>21</td>
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<td>10</td>
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<tr>
<td>22</td>
<td>Discount, Commission Rebate</td>
<td>8</td>
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<tr>
<td>23</td>
<td>Average</td>
<td>4</td>
</tr>
<tr>
<td>24</td>
<td>Bar Graph</td>
<td>4</td>
</tr>
</tbody>
</table>
1. Rational Numbers

Activity: Collect the various types of numerical data such as minimum and maximum temperatures of different cities, list of grocery, scores of your favourite cricket star etc.

1. Temperature of some cities on 10\textsuperscript{th} January

<table>
<thead>
<tr>
<th>City</th>
<th>Min. (° C.)</th>
<th>Max. (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pune</td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>Mumbai</td>
<td>12</td>
<td>23.5</td>
</tr>
<tr>
<td>Nagpur</td>
<td>13.5</td>
<td>24</td>
</tr>
<tr>
<td>Goa</td>
<td>13.2</td>
<td>23.5</td>
</tr>
</tbody>
</table>

2. List of Grocery

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity (kg.)</th>
<th>Price / kg. (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sugar</td>
<td>1 ½</td>
<td>13.50</td>
</tr>
<tr>
<td>Rice</td>
<td>10</td>
<td>18.75</td>
</tr>
<tr>
<td>Wheat</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>Semolina</td>
<td>1 ¼</td>
<td>12</td>
</tr>
<tr>
<td>Tea</td>
<td>½</td>
<td>36</td>
</tr>
</tbody>
</table>

3. B. P. of Oxygen | -96 ° C.  
B. P. of Nitrogen | -84 ° C.  
B.P. of helium | -269 ° C.  
Freezing pt. of water | 0 ° C.  

1. From the above data, circle different types of numbers and classify them as positive integers, negative integers, decimal fractions, vulgar fractions etc.
**Mathematical Note**: All these types of numbers taken together are called Rational numbers. Rational number can be expressed in the form $\frac{p}{q}$. ex.: $6 = \frac{12}{2}$ OR $\frac{6}{1} = \frac{18}{-3}$ etc.

2. Draw a number line showing 0, positive integers and negative integers. Mark exactly half part of every unit distance between two consecutive integers. Write number indicating each point. Also divide each unit distance into 3 equal parts and write the numbers that are represented by these points.

**Mathematical Note**: Rational numbers can be represented on a number line. Denominator indicates the number of equal parts to be made between two successive integers whereas the numerator indicates the position of the given number from the preceding integer.

3. Discuss and write in how many different forms the numbers 2, -16, $\frac{4}{5}$ etc can be written

   e.g. $-\frac{3}{4} = -\frac{3}{4} = -\frac{12}{-16} = \frac{18}{24}$ etc.

**Mathematical Note**: The rational numbers having equal values are called equivalent rational numbers. Equivalent rationales can be obtained either by multiplying or dividing both numerator and denominator by the same number.

**Observe and Understand**

Example 1: Without changing the values of a rational number, convert the following rational number into equivalent rational number.
a. Convert denominator of 3/5 into 15
   Multiply N and D by 3
   \[ \therefore \frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15} \]

b. Convert numerator of 4/-5 into -48
   Multiply both N & D by -12
   \[ \therefore \frac{4}{-5} = \frac{4 \times (-12)}{-5 \times (-12)} = \frac{-48}{60} \]

c. Convert numerator of 28/36 into 7
   Divide both N & D by 4
   \[ \therefore \frac{28}{36} \div 4 = \frac{7}{9} \]

Assignment:

Obtain the equivalent rational in each of the following:
1. Convert denominator of 9/11 into 33
2. Convert numerator of 7/-16 into -28
3. Convert numerator of -4/-5 into 20
4. Convert denominator of 5/12 into 60

Observe and Understand

Example 2: Reduce the following rational numbers to lowest terms.

a. \[ \frac{48}{80} = \frac{48 \div 16}{80 \div 16} = \frac{3}{5} \]
   (Divide N & D by their GCD 16)

b. \[ \frac{-64}{128} = \frac{-64 \div 64}{128 \div 64} = \frac{-1}{2} \]
   (GCD of 64 and 128 = 64)
Assignment: Reduce to lowest terms:

(1) 24/32  (2) 48/28  (3) 48/144  (4) 65/91  (5) 180/450

**Observe and Understand**

Example 3: Determine the greater or smaller rational.

a) \(\frac{3}{5}, \frac{4}{5}\)

Denominators are equal but the numerators \(3 < 4\)

\[ \therefore \frac{3}{5} < \frac{4}{5} \]

b) \(\frac{3}{2}, \frac{-9}{4}\)

First cross product = \(3 \times 4 = 12\)

Second cross product = \(-9 \times 2 = -18\)

\[ \therefore 12 > -18 \]

\[ \therefore \frac{3}{2} > \frac{-9}{4} \]

c) \(\frac{5}{7}, \frac{-5}{9}, \frac{2}{3}\)

Taking LCM of Denominators

LCM of 7, 9 and 3 = 63

\[
\begin{align*}
\frac{5}{7} \times \frac{9}{9} &= \frac{45}{63} \\
\frac{-4}{9} \times \frac{7}{7} &= \frac{-28}{63} \\
\frac{2}{3} \times \frac{21}{21} &= \frac{42}{63}
\end{align*}
\]

Ascending order:

\[\frac{-28}{63}, \frac{42}{63}, \frac{45}{63}\]

\[\therefore \frac{-4}{9}, \frac{2}{3}, \frac{5}{7}\]
Assignment:

1) Use cross product method to decide smaller, greater or equal.
   a) \(\frac{4}{9}, \frac{5}{9}\),  (b) \(\frac{9}{12}, \frac{7}{2}\)  (c) \(\frac{5}{4}, \frac{6}{9}\)  (d) \(-\frac{4}{7}, \frac{6}{5}\)

2) Write the following rational numbers in ascending order:
   a) \(\frac{7}{2}, \frac{2}{5}, \frac{3}{7}\)  (b) \(-\frac{3}{4}, \frac{5}{7}, \frac{3}{5}\)  (c) \(\frac{3}{5}, \frac{4}{11}, \frac{5}{8}\)
2. Indices

Read the following information about micro-organisms:

Micro-organisms are extremely small in size. They are not visible to the naked eyes. Size of the micro-organisms is measured in smaller units such as micro-meter nano-meter etc.

\[
1 \text{ micro-meter} = \frac{1}{1000} \text{ milli-meter} = 10^{-3} \text{ millimeter} \\
1 \text{ nano-meter} = 1 \text{ micro-meter} = 10^{-6} \text{ micro-meter}
\]

Viruses are so small that they cannot be seen under compound microscope. They can be seen only under electron microscope which gives the magnification of nearly 1,00,000 i.e. \(10^5\) times.

Mathematical Note: Multiples of 10 such as 1000, 10,000, 1,00,000 etc. are expressed in the concise form in the above information.

Thus, when 10 is multiplied by itself for 3 times, it is written as \(10^3\). Similarly, when any other number say 'x' is multiplied by itself 'n' number of times, it is written as \(x^n\) where x is called base.

\[
e.g. \ 3 \times 3 \times 3 \times 3 = 3^4
\]

Observe and Understand

Example 1: Fill in the boxes with proper numbers. \(5^4 \times 5^1 = 5\)

\[
5^4 = 5 \times 5 \times 5 \times 5 \quad \text{and} \quad \therefore 5^4 \times 5^1 = 5 \times 5 \times 5 \times 5 \times 5 = 5
\]

Thus the basic principle: \(a^m \times a^n = a^{m+n}\).

Where, \(a\) is any rational number and \(m\) and \(n\) are positive integers.
Assignment:

Fill in the boxes with proper numbers:

1) \(3^4 \times 3^3 = 3\)
2) \((1/5)^4 \times (1/5)^5 = (1/5)^9\)
3) \(8^c \times 8^d = 8^{c+d}\)
4) \((3/4)^8 \times (3/4)^7 = (3/4)^{15}\)

Observe and Understand

Example 2: Fill in the boxes with proper numbers.

a) \(2^3 + 2^3 = 2^6\)
   \[\therefore 2^{3+3} = 2\]
   Basic Principle: \(a^m + a^n = a^{m+n}\) if \(m > n\)

b) \(5^4 + 5^4 = 1/5 = 5\)
   \[\therefore 5^1 + 5^4 = 1/5^{4-1} = 1/5^3\]
   \[\therefore 1/5^3 = 5^3\]
   \[\therefore 5^1 + 5^4 = 1/5^3 = 5^3\]
   Basic Principle: \(a^m + a^n = 1 / a^{n-m}\) if \(n > m\)
   Also, \(1/a^m = a^{-m}\)

C) \(2^3 + 2^3 = 2\)
   \[2^3 + 2^3 = 2^{3+3} = 2^0 \quad (a^m + a^n = a^{m+n}\) if \(m = n)\]
   but \(2^0 = 1\)
   Basic Principle: \(a^0 = 1\)
Assignment:

Fill in the boxes with proper numbers:

1. $8^5 + 8^3 = 8$
2. $2^4 + 2^6 = 1/2^4 = 2$
3. $(3/5)^7 \div (3/5)^4 = 3/5$
4. $1 / 10^3 = 10$
5. $1 / 2^4 = 2$
6. $2^5 + 2^5 = 2$

Mathematical Note:

If $a$ is a rational number and $m$ is negative integer then $a^{-m} = 1/a^m$. Here negative index power is changed to positive.

$a^m = a / a^m$. Here positive index power is changed to negative.

This can be verified mathematically.

e.g. $3^3 + 5^3 = 3$

By actual division we write,

\[
\frac{3^3}{3^5} = \frac{3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = \frac{1}{3^2}
\]

By using the law of indices, $a^m \div a^n = a^{m-n}$ we write,

\[
3^3 \div 5^3 = 3^{3-5} = 3^{-2}
\]

\[
\therefore \frac{1}{3^2} = 3^{-2}
\]

Observe and Understand

Example 3: Fill in the boxes with proper numbers:

a) $(3 \times 5)^2 = 3 \times 5$

$(3 \times 5)^2 = (3 \times 5) \times (3 \times 5)$
= 3 \times 3 \times 5 \times 5 \\
= 3^2 \times 5^2 \\
Basic principle: \((a \times b)^m = a^n \times b^m\)

b) \(\frac{2}{3}^4 = 2^4 \div 3^4\)

\[
\left(\frac{2}{3}\right)^4 = 1/3 \times 2/3 \times 2/3 \times 2/3 \\
= 2 \times 2 \times 2 \div 3 \times 3 \times 3 \times 3 \\
= 2^4 \div 3^4 \\
\]
Basic principle: \((a / b)^m = a^m / b^m\)

Assignment:

1. Complete the following:
   
   (a) \(a^m \times a^n = a\)
   
   (b) \(a^m \div a^n = a\) if \(m > n\)
   
   (c) \(a^m \div a^n = 1/a\) if \(n > m\)
   
   (d) \((a \times b)^m = a \times b\)
   
   (e) \((a \times b)^m = a \div b\)
   
   (f) \(1 / a^m = a\)
   
   (g) \(a^0 = 1\)

2. State the base and index in each of the following:
   
   \(2^0, \ 10^1, \ 4^3, \ (1.1)^2, \ (2/3)^4\)

3. Evaluate:
   
   \(2^5, \ 3^4, \ 11^2, \ 12^2\)
4. Write with positive index.

\[ 8^{-4}, \ 10^{-3}, \ \frac{1}{2^{-5}}, \ 9^{-6}, \ p^3, \ (u + v)^{-5}, \ \frac{1}{4^{-5}} \]

5. Write with negative index:

\[ 2, \ (a + b), \ \frac{1}{(x + y)} \ (2 + x) \]

6. Write the reciprocals:

\[ 3, \ 4^3, \ 2^5, \ 3^{-6}, \ (3 / 2)^6 \]

7. Express the following using powers of 10:

(a) 3,429  (b) 81.75  (c) 7.217  (d) 67.274

8. Fill in the boxes with proper numbers.

(a) \( \left(\frac{1}{3}\right)^2 \times \left(\frac{1}{3}\right)^3 = \left(\frac{1}{3}\right)^5 \)

(b) \( 5^7 \times 5^3 = 5^{10} \)

(c) \( (-5)^3 \times (-5)^4 = (-5)^7 \)

(d) \( 4^0 + 4^5 = 1 / 5 \)

(e) \( (2^2)^2 = 2 \)

(f) \( (7 \times 3)^5 = 7 \times 3 \)

(g) \( (\frac{3}{11})^7 = 3 / 11 \)

(h) \( (x + y)^0 + 2 = \)
3. Congruence

Activity: Let's tidy up our classroom (work in a group of 10 students in each).
Discuss the strategies to make your work easier. E.g.

1. Collect all the newspapers lying in the corner, on the cupboard etc. and pile up.
2. Arrange the rows of desks and chairs with proper alignment.
3. Pick up the note-books, text-books and work-books according to their size and subject and arrange them properly in the cupboard.

Discussion: Add some more strategies to above list. What can you say about the size of English note books in a pile? What can you say about the size of Maths workbooks piled together? All English note-books are of equal size. Similarly all Maths work-books are of same size. The objects that are of equal size are said to be congruent. If arranged in a pile they perfectly fit on each other. Their edges coincide. Now, mix up the books in two piles together. Books do not fit on each other. Their edges do not coincide. Thus they are not congruent.

Mathematical Note:
Plane geometrical figures are rectangle, square, triangle, square etc. Two rectangles are said to be congruent of their lengths and breadths are congruent. Two squares are said to be congruent if, the length of each side of two squares are congruent. Two circles are said to be congruent if their radii are equal.
Assignment:

1. Make cut-outs of rectangle, square, triangle and circle on card paper. How will you obtain congruent figure to each of these? Work with your partners.

2. What different combinations of lengths and breadths can be obtained if the area of rectangle is given as 48 cm.sq.

Draw different diagrams of rectangles for every pair of length and breadth. Remember, area of rectangle must be 48 sq. cm. Are these rectangles congruent?

**Mathematical Note:**
Equi-areal figures are the figures having equal area. But they’re not necessarily congruent e.g. Equi-areal rectangles are not always congruent.
4. Congruence of Triangles

Activity: Construct the following triangles on a card paper

1) Length of each side of triangle is 6 cm.
2) △ DEF in which side DE = 5 cm, m∠DEF = 60° and side EF = 6 cm.
3) △ LMN in which m∠m = 70°, m∠N = 50° and side MN = 7 cm.
4) △ RST in which m∠RST = 90°, side ST = 6 cm and side RT = 8 cm.

Mathematical Note: Triangle has 3 sides and 3 angles. But it is possible to draw a triangle if any 3 of there elements are given. These constructions are named as side-side-side (SSS), side-angle-side (SAS), angle-side-angle (ASA) and hypotenuse-side (in case of right triangle) depending upon the elements given.

Now, cut out each figures from the paper. Keep it on another card paper to trace its outline. Cut it along the outline. Thus you get total 4 pairs of congruent triangles. When two triangles from each pair are kept on each other, their sides and angles coincide with each other. Thus, all three sides and all three angles of one triangle are congruent to corresponding three sides and angles of other triangle in each pair of congruent triangles. However, we mention only those sides and / or sides and angles which are given for construction.

Observe and Understand

Construction of triangle when all three sides are given (SSS):

In △ ABC, l(AB) = l(BC) = l(AC) = 6 cm.

Draw another triangle, △ PQR with the same measurements when △ PQR is kept on △ ABC, both triangle coincide with each other.
Vertex P coincides with vertex A i.e. P —► A

Also, Q —► B and R —► C

\[ \therefore \quad \text{Side } AB \equiv \text{side } PQ \]
\[ \text{Side } BC \equiv \text{side } PR \]
\[ \text{Side } AC \equiv \text{side } PR \]
\[ \text{Side } AC \equiv \text{side } PR \]

\[ \therefore \quad \triangle ABC \equiv \triangle PQR \text{ for the correspondence } ABC \leftrightarrow PQR \]

Side-Side-Side-Test: When three sides of one triangle are congruent to the corresponding three sides of the other triangle, the two triangles are congruent.

**Observe and Understand**

Draw \( \triangle XYZ \) in which \( m \angle y = 90^0 \), hypotenuse \( XZ = 5 \text{ cm.} \)
and \( l (YZ) = 4 \text{ cm.} \)

Also, construct \( \triangle ABC \) in which hypotenuse \( AC = 5 \text{ cm.} \)
and \( l (BC) = 4 \text{ cm.} \)

When we keep \( \triangle ABC \) on \( \triangle XYZ \), we find that hypotenuse
\[ \text{AC} \equiv \text{hypotenuse } XZ. \]
\[ \text{side } BC \equiv \text{side } YZ \]

\[ \therefore \quad \triangle ABC \equiv \triangle XYZ \text{ by hypotenuse – side test.} \]

Hypotenuse side Test: When the hypotenuse and a side of one right triangle are congruent to the hypotenuse and corresponding side of another right triangle two right triangle are said to be congruent by hypotenuse side test.
Assignment:

1) Construct the following,
   i) $\triangle ABC$, $\overline{AB} = 3.8 \text{ cm}$, $\overline{BC} = 5 \text{ cm}$, $\overline{AC} = 42 \text{ cm}$.
   ii) $\triangle PQR$, $\overline{QR} = 5 \text{ cm}$, $\overline{PQ} = 3.8 \text{ cm}$, $\angle R = 45^\circ$
   iii) $\triangle RTP$, $\angle T = 58^\circ$, $\angle R = 35^\circ$, $\overline{RT} = 5.5 \text{ cm}$
   iv) $\triangle LMN$, hypotenuse $LN = 13 \text{ cm}$, and $\overline{MN} = 5 \text{ cm}$.

2) In the figures below the congruent parts of triangles are indicated by like markings. Write the test of congruence.

Figures:
5. Operations on Rational Numbers

Observe and understand

Example 1:

a. \(10 + 12 = 22\)
b. \(-6 + 6 = 0\)

The sum of rational numbers is also a rational number.

Observe and understand

Example 2:

\(\frac{6}{5} + \frac{2}{5} = \frac{11}{11}\) and
\(\frac{5}{5} + \frac{6}{5} = \frac{11}{11}\)

And

\(\frac{2}{3} + \frac{5}{6} = \frac{2 \times 6 + 5 \times 3}{3 \times 6} = \frac{12 + 15}{18} = \frac{27}{18} = \frac{3}{2}\)

\(\frac{5}{6} + \frac{2}{3} = \frac{5 \times 3 + 2 \times 6}{6 \times 3} = \frac{15 + 12}{18} = \frac{27}{18} = \frac{3}{2}\)

Thus, the sum of two rational numbers does not change if the order of addition is changed. This is commutative law of addition.

Observe and understand

Example 3:

\(\frac{2}{3} + \frac{1}{2} + \frac{3}{5} = \frac{(2 \times 2 \times 5) + (3 \times 3 \times 5) + (1 \times 2 \times 5)}{3 \times 2 \times 5} = \frac{20 + 45 + 10}{30} = \frac{75}{30} = \frac{5}{2}\)
Thus, the sum of three rational numbers does not change if the order of addition is changed.

\[ i.e. \ (A + B) + C = A + (B + C) \]

**Observe and Understand**

**Example 4:**

a) \( 3 + 0 = 3 \)

b) \( -\frac{7}{9} + 0 = -\frac{7}{9} \)

c) \( -6 + 0 = -6 \)

Thus, the sum of a rational number and zero is the same rational number.

**Assignment:**

Verify with example whether each of the properties of addition of rational numbers is applicable in case of multiplication of rational number or not.
Observe and Understand

Study the following example:

\[
\frac{2}{9} = 0.222\ldots = 0.2\overline{2}
\]

\[
\frac{23}{99} = 0.2323 = 0.23
\]

\[
\frac{105}{999} = 0.105
\]

Thus, a decimal in which one digit or a group of digits repeats without end is called a non-terminating or recurring decimal. A dot or a bar indicates the digit or a group of digits recurring.

Assignment:

1. Convert the following rational numbers into decimal fractions
   a) \(\frac{3}{4}\)  (b) \(-\frac{11}{4}\)  (c) \(-\frac{13}{2}\)  (d) \(\frac{5}{8}\)

2. Convert into vulgar fractions
   a) 4.2  (b) 0.731  (c) 0.28  (d) 0.73

3. Solve:
   a) \[
       \frac{1}{3} + \frac{2}{3} \times \frac{5}{3}
   \]
   b) \[
       12 \times \frac{1}{6} + \frac{4}{5} \times \frac{20}{7}
   \]
   c) \[
       \frac{8}{3} - \frac{2}{9} - \frac{1}{5}
   \]
6. Equations

Activity: What is on your mind? Play with your partner. Think of any number in your mind. Your partner will tell you to add or to subtract some other number from the number, which you've thought of. You carry out the operation in your mind and tell the answer within 20 seconds. Each one of you take two trials with 10 different numbers. The one who identifies the number in the partner's mind more number of times is the winner.

You can put a tougher challenge for your partner by asking him to carry out operations such as multiplication or division with negative numbers or fractions. Both the partners may take 20 seconds each to carry out the given operations and also to identify the unknown number.

**Mathematical Note:** Relation of equality:

Equation has two sides separated by '=' sign. The relation of equality is maintained between two sides. If any one side of the equation contains an unknown quantity denoted by a variable such as x, y etc., you've to solve the equation to find the unknown quantity. In order to maintain the relation of equality you must carry out the same operation with the same number on both sides of equation:

Ex. : \( x + 20 = 40 \) \( \therefore x + 20 - 20 = 40 - 20 \) \( \therefore x = 20 \)
\[ 3y/5 = 20, 3y / 5 \times 5 = 20 \times 5 \therefore 3y = 100 \therefore y = 100/3 \]

Assignment:

1. Frame the equations and find the value of variable.
   
   (a) Number of boys is 4 more than number of girls in two class. The total number of students in the class is 48.
   
   (b) Number of lady teachers in the school is 12 more than the number of male teachers in the school. Total number of teacher in the school is 25.
Observe and Understand

Example: A mother's age is three times that of her son but 12 years hence it will be only twice as much. Find their present ages.

Solution: Let the present age of son be \( x \) years

\[ \therefore \text{Mother's present age} = 3x \]

12 years later,

\[ \text{Son's age} = x + 12 \]

\[ \therefore \text{Mother's age} = 2(x + 12) \]

i.e. \[ 3x + 12 = 2(x + 12) \]

\[ 3x + 12 = 2x + 24 \]

\[ 3x - 2x + 12 = 2x - 2x + 24 \]

\[ x + 12 = 24 \]

\[ x + 12 - 12 = 24 - 12 \]

\[ x = 12 \]

\[ \therefore \text{Son's present age} = 12 \text{ years} \]

\[ \therefore \text{Mother's present age} = 36 \text{ years} \]

Assignment:

1) A person distributed Rs.400 among 72 students. He gave Rs. 6 to each boy and Rs.5/- to each girl. Find the number of girls and boys.

2) If 9 is added to thrice a number, the result is 51. Find the number.

3) The sum of the ages of Sohan and Mohan is 52 years. Sohan is older than Mohan by 8 years. What are their ages?

4) The perimeter of an equilateral triangle is 78 cm. Determine the length of each side.

5) Five times a natural number and 10 is equal to three times the same number and 30. Find the number.
7. Variation

Activity: “Climb the ladder”. Play this game in the class. It requires five students representing five rows one from each. There are five pictures of ladder displayed on the board. A number is written at the bottom of each ladder. A positive or negative integer is written outside each rung of the ladder. Each the five students have to start from the bottom and go to the next rung adding the number written outside each rung. Thus one who climbs the ladder fast is the winner of the game. But remember time limit is 30 seconds and there are 2 points for climbing each rung of the ladder.

<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
<th>Player 4</th>
<th>Player 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of rungs climbed</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Points gained</td>
<td>6</td>
<td>10</td>
<td>8</td>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

Discussion: The one who climbs more rungs he gains more points. Thus, as the number of rungs climbed increases, the points gained also increase. One quantity increases and the other related quantity increases in the same proportion. Think of some other quantities that change in the same proportion in the same direction. Are there examples in which increase in one quantity brings about decrease in other associated quantity and vice versa e.g. temperature of air increases, the density of air decreases and air pressure also decreases?
Mathematical Note: Variation means change. When two quantities say $x$ and $y$, change in the same direction, it is called direct variation. In such case, the ratio $x/y$ remains constant when two quantities, $x$ & $y$ change in opposite direction, it is called inverse variation. It means that as $x$ increases $y$ decreases, product $xy$ remains constant.

Assignment:

Study the following data and identify type of variation.

<table>
<thead>
<tr>
<th></th>
<th>Distance (Km)</th>
<th>45</th>
<th>90</th>
<th>112.5</th>
<th>__</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Petrol (lit)</td>
<td>1</td>
<td>2</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>2.</td>
<td>Distance (Km)</td>
<td>30</td>
<td>15</td>
<td>45</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Time (hrs)</td>
<td>1</td>
<td>½</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>3.</td>
<td>No of Workers</td>
<td>8</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Time taken to Complete the job (hrs)</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Speed (Km/hr)</td>
<td>20</td>
<td>10</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Time (hrs)</td>
<td>1</td>
<td>___</td>
<td>½</td>
<td>¼</td>
</tr>
</tbody>
</table>
8. Time, Distance and Speed

Activity: See the following data from 'Auto India' 1998

<table>
<thead>
<tr>
<th>Motor cycle</th>
<th>Average (Km/lit) *</th>
<th>Speed (Kmph) *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hero Honda</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>Bajaj Caliber</td>
<td>75</td>
<td>95</td>
</tr>
<tr>
<td>Splender</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Yamaha</td>
<td>85</td>
<td>95</td>
</tr>
</tbody>
</table>

(* Under test conditions )

Exploration: What is meant by "Average" of a vehicle?

The claims made by manufactures are proved to be true only under standard test conditions such as smooth roads etc. You can talk to the bike owners and find out whether the vehicle gives the same average as claimed by the company. The unit of measurement of speed is Km/hr or m/s.

Do you know that subsonic and supersonic jet aircrafts travel with the speed equal to that of sound in air? The speed of these aircrafts is expressed as 'Mach Number'. Try to collect more information about these fast paced aircrafts.

Mathematical Note:

Distance covered by an object is calculated as the product of speed of the object and time taken by the object to cover the given distance. Distance = speed * time. Distance is measured in Km or Meter or Cm whereas speed is expressed in Km/hr or m/s.

Also Speed = Distance / Time
Time = Distance / Speed
Example 1: An aeroplane takes 4 hours to reach Aden from Mumbai at a speed of 670 kms per hour, find the distance between Aden and Mumbai.

Solution: To find distance = ?

Given Speed of Aeroplane = 670 Km/h
Time = 4 Hrs
Distance = Speed X Time
= 670 X 4 = 2680 Km

Distance between Mumbai and Aden = 2680 km

Example: A train of length of 300 m. crosses a signal pole in 24 seconds. Find the speed in Kms per hour.

Solution: To find: speed of the train = ?

Given length of the train = 300 m = \frac{300}{1000} = \frac{3}{10} m

Time = 24 Seconds = \frac{24}{3600} = \frac{1}{150} hours

Speed = \frac{distance}{time}

= \frac{300}{1000} \times \frac{3600}{24} = \frac{300}{1000} \times 150 = 45 km/h

Speed of the train = 45 Km/h
Assignment:

Solve

1. A plane started at 14.10 reached its destination at 18.10, covered a distance of 3648 Km. Find its speed.

2. A train 300 m long crossed a pole in 20 seconds. Find the speed of the train: a) in m/s  b) in Km/h

3. A gun was fired at a distance of 2.40 km and the sound of firing was heard after 6 seconds. Find the speed of sound in meters per second.

4. The distance between Aurangabad and Jalana is 60 km. It takes 12 hours and 30 minutes to cover the same distance on a bicycle. Find the speed of bicycle in Km/hr.

5. An aeroplane takes 50 minutes to reach Goa from Mumbai at a speed of 650 kms per hour. Find the distance between Goa and Mumbai.
9. Time and Work

Problem 1: Razia takes 8 hrs to cane some chairs. Raja takes 6 hrs to complete the same work. How much time will they takes to complete the work if they both work together?

Exploration: Plan a strategy to arrive at the answer with the help of following questions.

1. How much part of work will Razia complete in one hour?
2. How much part of work will Raja complete in one hour?
3. How much part of work will they both complete together in one hour?
4. How much time they take to complete the work together?

Discuss these questions among the peers in your group and find the answers.

Problem 2: 8 masons construct a room in 7 days if they work every day for 5 hrs. How many days will be required for the construction of room if 10 mesons work for 4 hrs. every day?

Exploration: Discuss the given problem with the help of following questions:

1. For total how many hrs 8 masons work in 7 days?
2. For total how many hrs 10 masons work in the second case?
3. What type of variations exists between two quantities – no of masons and time required to complete the work?
4. Tabulate the entries as follows:

<table>
<thead>
<tr>
<th>No of Masons (X)</th>
<th>Daily hrs (Y)</th>
<th>No of days (Z)</th>
<th>Product (xyz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>5</td>
<td>280</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>4</td>
<td>?</td>
</tr>
</tbody>
</table>

Equate two products to get the answers.
Mathematical Note:

There is direct variation between work and time required to complete the work. Thus the ratio time and work remains constant.

There is inverse variation between number of workers and time taken to complete the work. In this case, the product of these two quantities remains constant.

Thus \( \frac{\text{Time}}{\text{work}} = \text{constant} \) and \( \text{No. workers} \times \text{time} = \text{constant} \)

Assignment:

Solve

1. A former ploughed his field in 7 days. What part of field will be ploughed by him in 4 days time?

2. Neha can do a piece of work in 10 days and Jay can in 15 days. In how many days will they do it together?

3. Two workers require 9 hours to complete the given work. In how may hovers 3 workers will complete the same work?

4. 24 workers work every day 8 hours sow the field in 14 days. Then find the number of workers required to complete the same work by working 8 hrs every for 8 days?

5. 40 workers do a piece of work in 6 months. It the work is to be completed in 4 months, how many more workers have to be employed?
10. Quadrilaterals

Activity: Chart Display

<table>
<thead>
<tr>
<th>Type</th>
<th>Slides</th>
<th>Angles</th>
<th>Diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Square</td>
<td>All congruent</td>
<td>Each $90^\circ$</td>
<td>Congruent</td>
</tr>
<tr>
<td>2. Rectangle</td>
<td>Opposite Sides congruent</td>
<td>Each $90^\circ$</td>
<td>Congruent</td>
</tr>
<tr>
<td>3. Parallelogram</td>
<td>Opposite Side congruent, parallel</td>
<td>Opposite angles congruent</td>
<td>Bisect each other</td>
</tr>
<tr>
<td>4. Rhombus</td>
<td>All congruent</td>
<td>Opposite angles congruent</td>
<td>Bisect each other at right angles</td>
</tr>
<tr>
<td>5. Trapezium</td>
<td>One pair of opposite sides parallel</td>
<td>Adjacent angles are supplementary</td>
<td>---</td>
</tr>
</tbody>
</table>

Exploration: Confirm the above properties using paper folding techniques.

Discussion: Discuss the following in your groups and note down the points.

1. Every square is a rectangle.
2. Every rectangle is a parallelogram.
3. Rhombus is not a square.
4. Parallelogram is not a rhombus.

Assignment:

1. Draw the diagrams with the help of given information and identify the type of quadrilateral.
   
a. In ABCD, each side is 6 cm in length and each angle is of $90^\circ$.

b. In PQRS, side PQ = side SR = 6 cm and side PS = QR = 4 cm.
   Each angle is $90^\circ$.
c. In DEFG, each side is 5 cm. Diagonal DF = 8 cm, diagonal EG = 6 cm. Diagonals bisect each other at right angle.
d. In STUV, side ST is parallel to side VU, side SV is parallel to side TU.
Side ST = side VU = 7 cm and side SV = side TU = 5 cm

2. Make cut-outs of different types of quadrilateral from a card–paper and stick them in your notebook.

3. Practice geometrical constructions of different types of quadrilateral.
11. Concurrence in a Triangle

Activity: Carry out the following geometrical constructions.

1. Draw two lines intersecting each other at $90^\circ$
2. Divide a line segment of 8 cm into two equal parts.
3. Divide an angle of $60^\circ$ into two equal parts

**Mathematical Note:**

1. Perpendicular: A line that intersects the given line or line segment at $90^\circ$ is called a perpendicular to the given line or line segment.
2. Perpendicular bisector: A line that divides the given line segment into two equal parts making an angle of $90^\circ$ is called perpendicular bisector.
3. Angle bisector: A ray that divides the given angle into two equal parts is called an angle bisector.

Learn and practice the constructions of perpendicular lines, perpendicular bisector of a line segment and angle bisector.

Activity:

1. Draw a line perpendicular to a given line from a point not lying on the given line.

2. Draw an acute angled triangle. Draw a perpendicular to each side from the respective opposite vertices. Carry out the same construction in case of obtuse angled and right angled triangles. In each case, note down the position of intersection of the perpendiculars drawn to these sides of triangle.
**Mathematical Note:**

A perpendicular drawn to a side of triangle from its opposite vertex is called an altitude.

A point in which the altitudes meet together is called a point of concurrence of altitudes. It is also called orthocenter.

In acute angled triangle, orthocentre lies in the interior of a triangle. In obtuse angled triangle, the orthocentre lies in the exterior of a triangle. In right-angled triangle the orthocentre is nothing but the vertex of a triangle.

Activity:

Draw perpendicular bisectors of any two sides of an acute angled triangle. Mark the point of concurrence of two perpendicular bisectors. Taking this point as a centre and distance between this point and any of the vertices of a triangle, draw a circle. Check whether the circle you have drawn passes through all three vertices of a triangle.

**Mathematical Note:**

A point of concurrence of perpendicular bisectors of sides of triangles is called a circumcentre. The circle that passes through all three vertices of a triangle is called circumcircle.

Assignment:

Draw circumcircle to obtuse angled triangle and right-angled triangle. Note down the position of circumcenters in each of the two triangles.
Activity:

1. Take a paper cut-out of acute angled triangle. Fold it along every corner so that each angle is divided into two equal parts. Mark the point in which three folds meet together.

2. Carry out geometrical construction of angle bisectors of angles of a triangle. You can bisect any two angles. Mark the position of point of concurrence of angle bisectors in different types of triangles.

3. From the point of concurrence of angle bisectors draw a perpendicular to any side of the triangle. Taking this perpendicular as a radius and point of concurrence of angle bisector as a centre, draw a circle that touches all three sides of a triangle internally.

**Mathematical Note:**

A point of concurrence of angle bisectors of a triangle is called incentre.

A circle that touches all three sides of a triangle internally called incircle.

Activity:

Take a sufficiently large paper cut-out of an equilateral triangle. Fold it along each side such that the midpoint of each side is obtained. Mark the midpoints of three sides with the pencil. Now with the protractor measure the angle formed at every midpoint. Also measure each of the two angles formed at every vertex due to folding of the paper. What do you observe? See if your observations match with those of your group-mates.
Mathematical Note:

A segment joining a vertex of a triangle and the midpoint of its opposite side is called a median. The point of concurrence of medians is called centroid. In case of an equilateral triangle, median of each side of a triangle is nothing but its altitude, perpendicular bisector and also the bisector of the opposite angle.

Assignment:

Draw the medians of isosceles and scalene triangles. Do they coincide with altitudes, perpendicular bisectors and angle bisectors? Check with the help of paper folding and also geometrically.
12. Percentage

Introduction:

Following table shows the marks obtained by first five toppers in Class VII.

<table>
<thead>
<tr>
<th>Name</th>
<th>Marks obtained</th>
<th>Total Marks</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ajinkya</td>
<td>68.5</td>
<td>700</td>
<td>95%</td>
</tr>
<tr>
<td>Rohan</td>
<td>65.8</td>
<td>700</td>
<td>94%</td>
</tr>
<tr>
<td>Sayali</td>
<td>64.4</td>
<td>700</td>
<td>92%</td>
</tr>
<tr>
<td>Sourabh</td>
<td>63.0</td>
<td>700</td>
<td>90%</td>
</tr>
<tr>
<td>Neha</td>
<td>62.3</td>
<td>700</td>
<td>89%</td>
</tr>
</tbody>
</table>

Figures in the last column show the percent marks of the students.

Percentage of marks is calculated by a simple formula:

\[
\text{Percentage} = \left( \frac{\text{Marks obtained}}{\text{Total marks}} \right) \times 100
\]

Assignment:

Gather the marks obtained by your friends in all subjects in sem-I examination. Calculate the percentage subject-wise as well as overall.

Mathematical Note:

Percent means per hundred. Percentage denotes the value per hundred. Percentage is useful for giving a precise statistics for various purposes. Percentage reduces large numbers to small figures that are compatible and easy to note. For instance, the number of literate people in India and total population of India are huge numbers. But if percentage of literacy is calculated, it is easy to express. See the following table Percentage of Literacy in some Indian states: (1997 census)
<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
<th>Literate Population %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maharashtra</td>
<td>7,87,06,719</td>
<td>75</td>
</tr>
<tr>
<td>Gujrath</td>
<td>4,11,74,060</td>
<td>71</td>
</tr>
<tr>
<td>Karnataka</td>
<td>4,48,17,398</td>
<td>66</td>
</tr>
<tr>
<td>Kerala</td>
<td>2,90,11,337</td>
<td>80</td>
</tr>
<tr>
<td>Andhra Pradesh</td>
<td>7,15,00,000</td>
<td>54</td>
</tr>
</tbody>
</table>

Assignment:

1. Which state has maximum percentage of literacy?
2. Which state has the lowest percentage of literacy?
3. What is the number of literate people in Maharashtra?
4. Which of the states given in above table has the second lowest population and yet the highest literacy?

Observe and Understand

Example 1: In a water molecule, there is 88 8/9 % oxygen and the rest is hydrogen. How many grams of oxygen and hydrogen are in 450 gms of water?

Solution: 88 $\frac{8}{9}$ % = $\frac{800}{9}$

Oxygen in 450 gms. of water = $\frac{800}{9}$ % of 450

= $\frac{800}{9}$ / 100 x 450

= $\frac{800}{9}$ x $\frac{1}{100}$

450 = 450 gms.

Hydrogen = 450 - 400 = 50 gms.
Example 2

If the taxable income of Manohar is Rs. 40,000 and that of Pramod is Rs. 70,000 find out how much income tax is to be paid by each one of them?

Solution: Income tax is not charged upto the taxable income
Rs. 28,000.
Rs. 40,000 – 28,000 = Rs. 12,000
On Rs. 20,000 tax is charged at the rate of 20%
Total income tax = 20% of 12,000

\[
\frac{20}{100} \times 12000 = 2400
\]

Manohar has to pay tax of Rs. 2400/-

Pramod’s Taxable income is Rs. 70,000
50,000 – 28,000 = Rs. 22,000
Income tax on Rs. 22,000 = 20% of Rs 22,000

\[
\frac{20}{100} \times 22000 = 4400
\]

70,000 – 50,000 = Rs. 20,000
Income tax on Rs. 20,000 = 30% of Rs. 20,000

\[
\frac{30}{100} \times 20,000 = 6000
\]

Total income tax = 444 + 6000 = Rs. 10,400
Assignment:

Solve:

1. 50 Students have passed in grade 1 out of total 80 students. What is the percentage of students passing in grade 1?

2. A customer paid sales tax 4% on some goods. He paid in all Rs. 60 as sales tax. Find the cost of goods.

3. In brass 70% is copper and the rest is Zinc. Find how many grams of copper does the metal brass weighing 2 kgs contain?

4. An alloy of tin and silver weights 25 kg. Tin is 90% in the alloy and the rest is silver. Find the weight of each in alloy?

5. Arundhati's taxable income during the economic year 1994-95 was Rs. 69,990. Find out how much income tax did Arundhati pay?
Exploration:

Calculate total money spent (C.P.) and total money earned by selling the items (S.P.). Is it profit or loss?

Find out percent profit or percent loss whichever may be the case.

Tabulate the Results:

<table>
<thead>
<tr>
<th>Group No.</th>
<th>C. P. (Rs.)</th>
<th>S. P. (Rs.)</th>
<th>Profit</th>
<th>Loss</th>
<th>% Profit</th>
<th>% Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assignment:

Solve the following problems:

1. Ganu bought 32 hens for Rs. 1600. He sold 16 hens with 10 p.c. profit and remaining hens with 5 p.c. loss. Find out whether he had profit or loss in this deal and how much? Find the percentage.

2. Sonu bought a washing machine and sold it to Chandu with 10 p.c. profit. Chandu spend Rs. 1,000 on its repairs and sold it to Rupa for Rs. 16,000. Chandu had no profit or loss in this deal. What is the C.P. of the machine for Sonu?
13. Profit and Loss

Activity: Market Day

Make groups of 10 students in each in your class. Each group has to buy some item such as vegetables, fruits, stationary etc and sell it in your school premises. At the end of the day make a record of sale as follows:

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Item</th>
<th>Money Spent (Rs.)</th>
<th>Money earned (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vegetable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Fruits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Stationary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Crafts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Key – Chains</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If your groups has earned more money than money spent, you are in profit. If otherwise, you are at loss!

Calculate profit or loss in Rs. for your groups.

Mathematical Note:

Cost of the item is known as its cost price (C.P.) and the amount at which it is sold, is called selling price (S.P.)

If C.P. < S.P. it is profit and if C.P. > S.P. it is loss.

Percent profit = \( \frac{\text{C.P.}}{100} \times \text{Profit (Rs.)} \)

Percent Loss = \( \frac{\text{C.P.}}{100} \times \text{Loss (Rs.)} \)
Exploration:

Calculate total money spent (C.P.) and total money earned by selling the items (S.P.). Is it profit or loss?

Find out percent profit or percent loss whichever may be the case.

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<table>
<thead>
<tr>
<th>Group No.</th>
<th>C. P. (Rs.)</th>
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<th>Profit</th>
<th>Loss</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
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2. Sonu bought a washing machine and sold it to Chandu with 10 p.c. profit. Chandu spend Rs. 1,000 on its repairs and sold it to Rupa for Rs. 16,000. Chandu had no profit or loss in this deal. What is the C.P. of the machine for Sonu?
Activity: Play a game:

Find your partner!

Divide the class into two groups A and B such that there are equal number of students in each group, say 1 to 24 in group A and 25 to 48 each group B. Each student will be given a flash card. The students from group A have to write any number from 1 to 24 on their own card. The students from group B have to write any number falling between 1-600. Each student from group A will multiply number on his/her card by itself and will read out the product. If any student from group B is having that number written on his/her card, group A student finds a partner. e.g. Group A No. on the card is 15. 15X15=225. Check if anyone from group B is having a card with 225 written on it.

Exploration:

Thus 225 is obtained by multiplying 15 by 15. Hence the square of 15 is 225. 15 is called square – root of 225. Also look for the digits that are not found in the units place of any square. Discuss the strategy to guess whether the given numbers is likely to be a perfect square or not.
**Mathematical Note:**

Perfect squares usually have the digits 0, 1, 4, 5, 6, 9 in the units place. If the digits such as 2, 3, 7 and 8, appear in the units place of the number, then that number is definitely not a perfect square.

**Factorization:** You can find a square root of given number by factorization. For example, find a square root of 1764

\[
1764 = 2 \times 882 = 2 \times 2 \times 441 = 2 \times 2 \times 21 \times 21
\]

\[\sqrt{1764} = 2 \times 21 = 42\]

**Assignment:**

Find the square roots of the following numbers by factorization 225, 784, 1024, 2025.
Activity

Make a paper cut out of a right angled triangle having length of hypotenuse 10 cm and lengths of remaining two sides 8 cm and 6 cm respectively. Also make paper cut-outs of squares having length of each side 10 cm, 8 cm and 6 cm respectively. Arrange each square on the appropriate side of the right-angled triangle. Calculate area of each square.

What do you observe?

Diagram:

Mathematical Note:

Area of the square built on the hypotenuse of right angled triangle is equal to the sum of the areas of squares built on remaining two sides of the triangle.

This statement is called theorem of Pythagoras. Thus we can say that,

\[(\text{hypotenuse})^2 = (\text{one side})^2 + (\text{other side})^2\]

Pythagoras triplets: Pythagoras triplet is a group of three numbers in which the square of the largest number is equal to the sum of the squares of remaining two numbers.
Exploration:

Your note book, classroom may be rectangular or square in shape. A diagonal divides them into right-angled triangles. Confirm whether the right angled triangles thus obtained follow the theorem of Pythagoras.

List some other objects that are right-angled triangles in shape. Check whether they satisfy the theorem of Pythagoras.

Discuss the practical applications of theorem of Pythagoras especially in case of stability of constructions.

Assignment:

Solve:

1. A ladder is placed in such a way that its top reaches the windows of the house which is at a distance 6 ft. from the ground. The base of the ladder is 8 ft. away from the ground. Find the length of the ladder.

2. 25 ft. long guy rope is tied to the top end of the pole. The other end of the rope is fixed into the ground 20 ft. away from the base of the pole. Find the height of the pole.

3. Given the lengths of two sides of right-angled triangle, find the length of hypotenuse.
   (a) 3,4    (b) 18,24    (c) 12, 16

4. Given the length of the hypotenuse and one side, find the length of the other side.
   (a) \sqrt{7} , \sqrt{5}    (b) 8, 4    (c) 22, 18
Activity:

Draw a circle of radius 4 cm on a thick card paper. Cut it out from the paper. Take two straws of equal lengths and paste them on the circle so that the straws represent the congruent chords of the circle. (The length of the chord is less than that of the diameter or in other words, the diameter is the largest chord of the circle.)

Exploration:

Check the following

1. Perpendicular distance between the center of the circle and each cord.
2. Length of each part of the chord formed due to a perpendicular drawn from the centre of the circle to the chord.

Compare your results with those of your group-mates. Derive the conclusions.

Mathematical Note:

1. A perpendicular drawn from the center of the circle on the chord bisects the chord.
2. Congruent chords are equidistant from the centre of the circle.

Activity: Note down the properties of the circle that you learnt from the previous activity. Now you may use the same cut-out of the circle and the straws for verifying some other properties of circle. Join two ends of both the chords to the center of circle. Measure the angles formed at the centre of the circle. Measure the angles formed at the points on the circle.
The angle formed at the centre of the circle by joining two ends of the chord to the centre is called central angle. Central angles subtended by congruent chords are congruent.

The angle formed at the point on the circle by joining two ends of the chord to any point on the circle is said to be subtended by the chord. Angles subtended by congruent chords are congruent.

Figures:

**Activity:**

Draw a chord AB so that the circle is divided into two unequal parts. The smaller part is known as minor arc and the bigger part is known as major arc. Take two points C and D on a minor arc and join two ends of the chord to each of these two points. The angles thus formed are $\angle ACB$ and $\angle ADB$. Measure each of them. Also take two points X and Y on the major arc and obtain the angles AXB and AYB. Measure each of them. What do you learn from the measures of these angles?

**Exploration:**

Diagram:
In above figure arc ABC or arc AOB is a minor arc and arc AXB or arc AYB is a major arc.

Angles subtended by arc ACB are \( \angle AXB \) and \( \angle AYB \). \( \angle AXB \equiv \angle AYB \)

Angles subtended by arc AXB arc \( \angle ACB \) and \( \angle ADB \). \( \angle ACB \equiv \angle ADB \)

Angles subtended by arc AXB arc \( \angle ACB \) and \( \angle ADB \). \( \angle ACB \equiv \angle ADB \)

Angles subtended by arc AXB arc \( \angle ACB \) and \( \angle ADB \). \( \angle ACB \equiv \angle ADB \)

What is the measure of angle inscribed in a semicircle?

Assignment:

1. \( \angle ACB \) and \( \angle ADB \) are opposite to angles AXB and AYB. Try to correlate their measures. What property do you observe? Name the quadrilaterals formed by joining any for points on the circle. What can you say about the measures of opposite angles of such quadrilateral?

2. List all properties of circle that you learnt from the above activities. Demonstrate these properties in your group by using paper models or on the black board.
17. Product of Algebraic Expressions

Discussion:

What are the algebraic expressions?

These are the mathematical expressions that contain letters of alphabet alongwith the numbers.

Any letter of alphabet such as a, b, x, y can be used to denote the unknown quantity and is called variable. For example, 3x contains the variable ‘x’. 3 is called its coefficient.

What are the types of algebraic expressions?

Types of algebraic expressions are viz. monomial, binomial, trinomials and polynomial.

Monomials contain only one term. E.g. $y^2$, $2x$, 7 etc

Binomials contain two terms e.g. $3x + 7$, $x^2 - 2$, etc.

Trinomials contain three terms e.g. $(a^2 + 2ab + b^2)$

Polynomials contain more than three terms e.g. $a^2 + bx + cy + b^2$

Mathematical Note:

An expression is monomial if –

1. Multiplication is the only operation involved in the expression.
2. The index of the variable is either a positive integer or 0, when it is in the numerator.

E.g. $3x^2$, $m$, 0, 25, $4x^2y$ are monomials
$3/x$, $1/m$, $8x^2/yz$, $5xy^3$, $1/x$ are not monomials
1. Which of the following expressions are monomials?
   \(2y^2, 3x^2, \frac{1}{m^3}, -5, \frac{1}{2}xy^{-2}, m^{-1}n^{-1}, 0\)

2. Multiply
   1. \(8m^2 \times 2m\)
   2. \(mn \times m^3n^2\)
   3. \(15 \times 1/3 xy\)
   4. \(9xy^2 \times 1/3\)
   5. \(12ab \times \frac{1}{4} bc\)

3. Multiply
   1. \(5a \times (7ab + 2)\)
   2. \(4 \times (3 + 2x)\)
   3. \(6y^2 \times (y^2 + 2xy)\)
   4. \((3x^2 - 2y^3) \times x\)

4. Multiply and verify for \(x=1, y=2, p=2, q=3, b=4\)
   1. \(49(p^2 + y^2)\)
   2. \((3x^2y - 2ay) \times 5 bq\)

5. Multiply using vertical arrangement
   1. \(2x + 3y \times a\)
   2. \(3x - 2y \times 2a\)
   3. \(4x + 5y \times x\)
   4. \(mn - 2 \times 3mn\)
6. Expand
   1. $3a(4ab - 10)$
   2. $6X(2 - 5b^2c)$
   3. $15X(1/5 ab + 2/15 bc)$

7. Multiply a binomial by a binomial
   1. $(x + 1) (x +2)$
   2. $(2x - 3) (x +5)$
   3. $(x+y) (x +2y)$
   4. $(2x^2+x) (y^2+3y)$
18. Identities

**Mathematical Note:**

The equation which is satisfied by any value of the variable used in it, is called identity

**Example:**

In the following equations replace the variable ‘X’ by 0, 1, 2 and identify which are identities

1. \(3x + 5 = 2x + 7\)
2. \(x(2x + 1) = 2x^2 + x\)

**Discussion:**

1. \(3x + 5 = 2x + 7\)
   
   Replacing \(x\) by 0 we get
   
   \[
   \text{LHS} = 3(0) + 5 \quad \text{RHS} = 2(0) + 7 \\
   \text{LHS} = 5 \quad \text{RHS} = 7
   \]
   
   \(\text{LHS} \neq \text{RHS}\) when \(x = 0\)
   
   The eqn \(3x + 5 = 2x + 7\) is not an identity

2. \(x(2x + 1) = 2x^2 + x\)
   
   Replacing \(x\) by 0, we get
   
   \[
   \text{LHS} = 0(2(0)+1) \quad \text{RHS} = 2(0)+0 \\
   \text{LHS} = 0 \quad \text{RHS} = 0
   \]
   
   Replacing \(x\) by 1 we get
   
   \[
   \text{LHS} = 1(2+1) \quad \text{RHS} = 2(1)^2 + 1 \\
   \text{LHS} = 3 \quad \text{RHS} = 3
   \]
   
   Replacing \(x\) by 1 we get
   
   \[
   \text{LHS} = 1(2+1) \quad \text{RHS} = 2 \times 2^2 + 2 \\
   \text{LHS} = 10 \quad \text{RHS} = 10
   \]
It is found that the eqn \(x (2x + 1) = 2x^2 + x\) is satisfied for \(X = 0, x = 1\) and \(x = 2\).

Hence \(x (2x+1) = 2x^2+x\) is an identity

**Assignment:**

In the following equations replace the variable by 0, 1, 2 and identify which of them are identities

1. \(4x + 7 = (4x+5) + 3\)
2. \(8x + 15 = 4 (2x+5) - 3\)
3. \(6x + 9 = 3 (3x+2)\)
4. \(2x (x+1) = 2x^2 - x\)

**Observe and Understand**

Example 1

\[(x+y)^2 = (x+y) (x+y)\]
\[= x (x+y) + y (x+y)\]
\[= x^2 + xy + xy + y^2\]
\[= x^2 + 2xy + y^2\]

\((x+y)^2 = x^2 + 2xy + y^2\)

LHS = RHS

Thus the square of sum of two terms =

Square of first term + Twice the product of first term and second term + Square of second term

Example 2

\[(2x+3)^2 = (2x)^2 + 2 \times 2x + 3^2\]
\[= 4x^2 + 12x + 9\]

Expand \(= (3x + 5)^2\)
Example 3
Find the square of 105
\[(105)^2 = (100 + 5)^2\]
\[= 100^2 + 2 \times 100 \times 5 + 5^2\]
\[= 10000 + 1000 + 25\]
\[= 11025\]

Observe and Understand

Example 4
Expand \((x-y)^2\)
\[(x-y)^2 = (x-y) \times (x-y)\]
\[= x (x-y) - y (x-y)\]
\[= x^2 - xy - xy + y^2\]
\[= x^2 - 2xy + y^2\]
\[= x^2 - 2xy + y^2\]

Square of the difference between two terms =
(First term)\(^2\) - 2 x (First term x Second term) + (Second term)\(^2\)

LHS = RHS

Observe and Understand

Example 5
Expand \((3a-2b)^2\)
\[(3a-2b)^2 = (3a)^2 - 2 \times 3a \times 2b + (2b)^2\]
\[= 9a^2 - 12ab + 4b^2\]

Expand \((x/y - 2)^2\)
Example 6
Find the Value of $98^2$

$$98^2 = (100 - 2)^2$$

$$= (100)^2 - 2 \times 100 \times 2 + 2^2$$

$$= 10000 - 400 + 4$$

$$= 9604$$

Example 7
Expand $(x + y) (x - y)$

$$(x + y) (x - y) = x (x - y) + y (x - y)$$

$$= x^2 - xy + xy - y^2$$

$$= x^2 - y^2$$

$(x + y) (x - y) = x^2 - y^2$

LHS = RHS
Thus the sum of two terms X the difference between the same terms

$= (\text{First term})^2 - (\text{Second term})^2$

Example 8
Expand $(2x + y) (2x - y)$
First term is $2x$ and second term is $y$

$$(2x + y) (2x - y) = (2x)^2 - (y)^2$$

$$= 4x^2 - y^2$$

$(2x + y) (2x - y) = 4x^2 - y^2$

Expand : $(3x + 4y) (3x - 4y)$
Observe and Understand

Example 9:
Multiply 101 X 99

101 = 100 + 1 and 99 = 100 – 1

101 X 99 = (100-1) (100-1)
Firs term is 100 and 2nd term is 1

101 X 99 = 100^2 – 1^2
= 10000 – 1
= 9999

Multiply 102 X 98

Assignment:

1. Expand
   1. (x + 3y)^2
   2. (4 + 5y)^2
   3. (x + 1/x)^2
   4. (2y + 1/y)^2

2. Find the value using expansion formula
   1. (65)^2
   2. (39)^2
   3. (x +1/x)^2
   4. (2y +1/y)^2

3. Write the squares of the following
   1. 4x – 3y
   2. mx – ny
   3. a – 3b
   4. 2m – 5

4. Find the value using expansion formula
   1. (98)^2
   2. (96)^2
   3. (89)^2
   4. (79)^2
5. Expand
1. \((a + b)(a - b)\)  
2. \((x - 1/x)(x + 1/x)\)  
3. \((2a + 3b)(2a - 3b)\)  
4. \((mn + pq)(mn - pq)\)

6. \((4 + 6x)(4 - 6x)\)

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**Mathematical Note:**

Following are some mathematical identities

- \((x+y)^2 = x^2 + 2xy + y^2\)
- \((x-y)^2 = x^2 - 2xy + y^2\)
- \((x+y)(x-y) = x^2 - y^2\)
Mathematical Note:
While factorizing an algebraic monomial, obtain the prime factors of the coefficient and factorizes the variable part such that the index of each variable is 1.

e.g. $6x^2y^2$
- Coefficient is 6
- Factors of 6 = 2 X 3 (prime number)
- Variable is $x^2y^2$
  \[ x^2y^2 = x \times x \times y \times y \]
- Factors of $6x^2y^2 = 2 \times 3 \times x \times x \times y \times y$

Assignment
Factorise:
1. $7x^2$
2. $10m^2n$
3. $36q^4$

Mathematical Note:
To find common factor of the given monomial is to find the greatest common factor of them.

e.g. $4xy, 6x^2y$
- $4xy = 2 \times 2 \times x \times y$
- $6x^2y = 2 \times 3 \times x \times x \times y$
- Common factor = $2 \times x \times y = 2xy$
Observe and Understand

Example 1
Find the common factors of $6x^3y^3$ and $15x^3y^2$

$6x^3y^3 = 2 \times 3 \times x \times x \times x \times y \times y \times y$

$15x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$

Common factor = $3 \times x \times x \times x \times y \times y = 3x^3y^2$

Find common factor of $16pq^2$ and $8p^2q$

Observe and Understand

Example 2
Find common factors of $13x^3y^3z$

$39xyz$ and $52x^2y^4z$

C. G. D of 13, 39 and 52 = 13

The lowest power of common variable $x$ is $x$

The lowest power of common variable $y$ is $y^2$

The lowest power of common variable $z$ is $z$

Common factors = $13 \times x \times x \times y^2 \times x \times z = 13xy^2z$

Find common factor of $7m^3n$, $14m^2n^3$, $28mn$

Mathematical Note:
When a monomial is divided by one of its factors the quotient thus obtained is the other factor.
Example 3:
First factor of monomial \(36x^2 y^2 z\) is \(6xy^2\)

Find the Second factor

\[
\text{Second factor} = \frac{\text{Monomial}}{\text{First Factor}} = \frac{36x^2 y^2 z}{6xyz} = \frac{2 \times 2 \times 3 \times 3 \times x \times x \times y \times y \times z}{2 \times 3 \times x \times y \times z}
\]

Second factor \(= 6xy\)

First factor of monomial \(25pq^2 r^2\) is \(5pqr\). Find the second factor

Example 4
Factorize: \(6a^2b - 4ab^2\)

Common factor of coefficients 6 and 4 is 2
The lowest power of common variable \(a\) is \(a\)
The lowest power of common variable \(b\) is \(b\)

Common factor of two factor of two monomials = \(2ab\)

To find 2nd factor of \(6a^2b\), we write

\[
\frac{6a^2b}{2ab} = \frac{2 \times 2 \times 3 \times a \times a \times b}{2 \times a \times b} = 3a
\]

To find 2nd factor of \(4ab^2\), we write

\[
\frac{4a^2b}{2ab} = \frac{2 \times 2 \times a \times a \times b \times b}{2 \times a \times b} = 2b
\]

\(6a^2b - 4ab^2 = 2ab(3a - 2b)\)

Factorize: \(16m^2n^2 - 4m^2n^2\)

Example 5:
Factorize \(ax + bx + 3a + 3b\)

\((ax + bx) + (3a + 3b)\)
\[= x(a + b) + 3(a+b)\]
\[= (a + b)(x + 3)\]
Factorize: \(am - bm + an - bn\)

**Mathematical Note:**
Factors based on identities

We know that, 
\[(a+b)^2 = a^2 + 2ab + b^2\]
\[(a+b)^2 = a^2 - 2ab + b^2\]

Hence, a perfect square trinomial can be expressed in the form of factors. To factorize a trinomial of the above types, we must determine whether the given trinomial is a perfect square or not. If the first and third terms are positive perfect squares and the middle term is twice the product of the square roots of the first and third terms then the expression is a perfect square.

**Observe and Understand**

Example 6

Identify which of the following expressions is a perfect square
a. \(x^2 + 6xy + 9y^2\)  
   b. \(3x^2 - 6xy + y^2\)

The first term \(x^2\) and the third term \(9y^2\) are positive perfect squares

The middle term 
\[= 2 \times \sqrt{x^2} \times \sqrt{9y^2}\]
\[= 2 \times x \times 3y = 6xy\]

Hence, \(x^2 + 6xy + 9y^2\) is a perfect square trinomial

b. \(-3x^2 + 6xy + y^2\)

The first term \(3x^2\) is not a perfect square

The expression \(3x^2 - 6xy + y^2\) is not a perfect square.
Identify the perfect squares from the following
1. \( q^2 + 4ab - 4b^2 \)  
2. \( 9a^2 - 12ab + 4b^2 \)  
3. \( 3x^2 - 6xy + 4y^2 \)  
4. \( 25a^2 - 40a + 16 \)

**Observe and Understand**

Example 7: Find the binomial of which the following trinomial is a square
\( 25x^2 + 30xy + 9y^2 \)

Solution:
First term = \( 25x^2 \)
\( \sqrt{25x^2} = 5x \)
Second term = \( 9y^2 \)
\( \sqrt{9y^2} = 3y \)
Also, \( 2 \times 5x \times 3y = 30xy \) i.e. the middle term
\( 25x^2 + 30xy + 9y^2 \) is a square of \( (5x + 3y) \)

Find the binomials of which the following trinomials are squares
1. \( 16a^2 - 40ab + 25b^2 \)  
2. \( 4a^2 + 12ab + 9b^2 \)  
3. \( a^2 - 8ab + 16b^2 \)  
4. \( 4y^2 - 28y + 49 \)

**Observe and Understand**

Example 8: Factorize \( 64p^2 - 81q^2 \) using the formula
\( 64p^2 \) and \( 81q^2 \) are perfect squares
Formula: \( a^2 - b^2 = (a+b)(a-b) \)
\( 64p^2 - 81q^2 = (8p+9q)(8p-9q) \)
Factorise \( x^2/4 - 9 \) using formula
Observe and Understand

Example 9:

Find the product of $1\times3 \times 97$ using formula

$$103 \times 97$$

$$= (100 + 3)(100 - 3)$$

$$= 100^2 - 3^2$$

$$= 10000 - 9$$

$$= 9991$$

Find the product $48 \times 52$ using formula

Assignment:

1. Factorise
   1. $18a^2c$
   2. $72a^2bc^3$

2. Find the common factor by observation
   1. $p^2q^3r^4$, $p^2q^3r^7$
   2. $45m^3n^4$, $36m^2n^5$, $27m^4n^7$

3. The first factor of the given monomial is below. Find the second factor.

<table>
<thead>
<tr>
<th>Monomial</th>
<th>First factor</th>
<th>Second factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15a^3$</td>
<td>3a</td>
<td></td>
</tr>
<tr>
<td>$36m^3n^6$</td>
<td>$3m^3n^4$</td>
<td></td>
</tr>
<tr>
<td>$28a^3b^5c^3$</td>
<td>$4a^3b^3c^3$</td>
<td></td>
</tr>
<tr>
<td>$72m^3n^8$</td>
<td>$3m^3n^3$</td>
<td></td>
</tr>
<tr>
<td>$4a^3x^6$</td>
<td>$2/3a^3x$</td>
<td></td>
</tr>
</tbody>
</table>

4. Identify the perfect square trinomials
   1. $36a^2 - 72ab + 36b^2$
   2. $16m^2 - 16m - 4$

5. Factorise
   1. $x^2 + 8xy + 16y^2$
   2. $a^2 - 50ab + 625b^2$

3. $196m^2 - 225n^2$
20. Simple Interest

Mathematical Note:

1. One needs some amount of money to run any business. If he has less amount, he borrows it from a bank or a person. The borrowed amount is 'Principal' = P.
2. The period for which the money is borrowed is 'Time' = N.
3. As the person used this amount for a certain period of time, he pays on extra amount while returning the same. The total extra amount charged is 'Interest' = I.
4. This extra amount is calculated by fixing certain amount to be paid per every 100 of the borrowed money per year which is known as the 'Rate of Interest' = R.
5. The total amount to be returned is the sum of the principal and the interest. It is called 'Amount' = A.
6. Simple interest = \( \frac{P \times N \times R}{100} \)
7. \( A = P + SI \)

Observe and Understand

Example 1: What will be the interest at the rate 7 p.c. per year on Rs. 1000 for 4 years?

Given: \( P = \text{Rs. 1000}, \text{N} = 4 \text{ yrs}, \text{R} = 7 \)

Solution: \( S.I. = \frac{P \times N \times R}{100} \)

\[ \frac{1000 \times 4 \times 7}{100} = 280 \]

\( S.I. = \text{Rs. 280} \)
Example 2: The interest on a certain amount at 8 pcpa for 2 yrs is Rs. 256. Find the principal
To find the : Principal
Given : S. I. = Rs. 256, R = 8 pcpa, N = 2yrs

Solution:

\[
\frac{PNR}{100} = \frac{256}{P \times 2 \times 8}
\]

Therefore, \( P \) = Rs. 1600

Example 3: Sadoba borrowed Rs. 500 for 6 months. He returned Rs. 512 including interest and principal. Find the rate of interest.
To find R = ?
Given P = Rs. 500, N = 6 months = \( \frac{1}{2} \) year, A = 512

Solution:

Amount returned = Rs. 512
Amount borrowed = Rs. 500
Interest = Rs. 12

Therefore, \( \frac{PNR}{100} = \frac{12}{500 \times \frac{1}{2} \times R} \)

Therefore, \( R \) = 4.8 p.c.p.a.
Assignment:

1. Fill in the missing entries in the following table

<table>
<thead>
<tr>
<th>P (Rs.)</th>
<th>N (yrs.)</th>
<th>R (p.c.p.a)</th>
<th>S.I. (Rs.)</th>
<th>A (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>4 ½</td>
<td>7.5</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1500</td>
<td>3</td>
<td>–</td>
<td>720</td>
<td>–</td>
</tr>
<tr>
<td>750</td>
<td>–</td>
<td>5</td>
<td>–</td>
<td>900</td>
</tr>
<tr>
<td>–</td>
<td>4</td>
<td>4 ½</td>
<td>288</td>
<td>–</td>
</tr>
</tbody>
</table>

2. Solve:

1. Aamir and Salman had borrowed Rs. 5600 each at the same rate of interest. 2 years, Aamir cleared the loan by returning the amount of Rs. 6500. What amount did Salman pay after 3 years?

2. After 1 ¼ years an amount of Rs. 575 is obtained on a certain principal at the rate of 10 p.c.p.a. What amount will be obtained after 2 years from the same principal at the rate of 12 p.c.p.a.?

3. Some amount has been deposited at the rate of 12 ½ p.c.p.a. After how many years will it be doubled?

3. Project:

1. Visit a nearby bank and find out information about any of their fixed deposit or short term deposit scheme.

2. Also find if the bank offers loan for any purpose. If yes, find out the eligibility conditions for availing loan, rate of interest maximum period of loan repayment etc.
Activity:

Discover the hidden figures.

Materials: Card paper, Scissors, Adhesive

1. Draw a parallelogram on white card paper and cut it out with a cutter or scissors. Follow the steps as given below.

Diagram:

2. Remove piece A from the cut out. Place it as shown below.

Diagram:

3. The figure thus obtained is a rectangle
   The length of this rectangle is base of the parallelogram
   The breadth of this rectangle is height of the parallelogram
   Therefore, Area of parallelogram = area of rectangle
   But area of rectangle = length X breadth
   Area of parallelogram = base X height

4. Draw a diagonal of this parallelogram cum rectangle

Diagram:
Diagonal thus divides a parallelogram cum rectangle into two triangles. Therefore, area of triangle = \(\frac{1}{2}\) (the area of parallelogram) 
Area of triangle = \(\frac{1}{2} \times \text{base} \times \text{height}\)

5. Cut the portion from a rectangle as shown below.

Diagram:

The figure thus obtained is a rhombus

Area of rhombus = area of a rectangle = Length \(\times\) breadth

But length of rectangle = length of one diagonal (QS)
And breadth of rectangle = \(\frac{1}{2}\) the length of 2\(^{nd}\) diagonal (PR)
Area of rhombus = length of one diagonal \(\times\) \(\frac{1}{2}\) length of 2\(^{nd}\) diagonal.

Assignment:

Use the formula learnt above and solve the following:

1. Area of parallelogram is 120 sq.cm and its height is 8 cm. Find its base.
2. Area of parallelogram is 180 sq.cm. and its base is 15cm. Find its height.
3. In \(\triangle\ ABC\), \(\text{seg AD} \parallel \text{seg BC}\) if \((BC) = 10\) cm and \((AD) = 8\) cm. Find the area of \(\triangle\ ABC\).
4. The sides of right angled triangle forming a right angle are 8 cm and 15 cm respectively. Find the area of the right angled triangle.
5. The diagonals of rhombus are 10 cm and 15 cm respectively. Find its area.
Trapezium: It is a quadrilateral in which only one pair of opposite sides is parallel and the other pair is non-parallel.

Diagram: 

\[ \text{The area of trapezium} = \frac{1}{2} \times \text{sum of lengths of parallel sides} \times \text{height} \]

Examples:

1. The parallel sides of a trapezium are 8 cm and 11 cm. The perpendicular distance between two parallel sides is 6 cm. Find its area.

2. If the parallel sides of a trapezium are 40 cm and 60 cm and if its area is 1800 sq.cm. what is its height?
22. Discount, Commission and Rebate

Discussion : Discount
Following is an advertisement published in the newspaper

<table>
<thead>
<tr>
<th>MONSOON SAREE SALE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banglore Silk</td>
</tr>
<tr>
<td>Italian Crepe</td>
</tr>
<tr>
<td>Printed Saree</td>
</tr>
<tr>
<td>Shaloo</td>
</tr>
</tbody>
</table>

What do you understand from this advertisement?

When we buy an item on sale, we receive the same item in less price than the original price i.e. we get discount means reduction from the original cost price on printed price.

Discount is given on 100 or percentage basis.
Thus, Printed price – Discount = New Selling price

Discount (Rs.) = \( \frac{\text{Percent Discount}}{100} \times \text{Printed price} \)

Observe and Understand

Example 1 : Mr. Ashok Jain offered 20 p.c. discount on the labelled price of Maths text book. The labelled price is Rs. 60. What is the new selling price of the book?

To find : New S. P. = ?

Given : Printed Price = Rs. 60
Discount = 20%

Solution : Discount (Rs) = \( \frac{\text{Percent Discount}}{100} \times \text{Labelled price} \)

= \( \frac{20}{100} \times 60 \)

= 12
New S.P. = Labelled price – discount
= 60 – 12 = Rs. 48

**Observe and Understand**

Example 2: A bookseller gets a commission of 25% on the printed price of books from the shopkeeper. After deducting the commission the bookseller paid Rs. 1500. What was the printed price of the books?

To find: Printed price of the books = ?

Given Commission = 25%, S. P. = Rs. 1500

Printed price S.P.

\[
\begin{array}{c|c}
100 & 75 \\
\hline
x & 1500 \\
\end{array}
\]

\[x = \frac{1500 \times 100}{75} = Rs. 2000\]

**Mathematical Note:**

When an institution or an individual undertakes the task of selling goods or articles, some amount is given to them in return, which is termed as commission.
The rate of commission is expressed as a percentage.

**Assignment:**

1. Pandoba sold his piece of land through an agent. The price of that land was fixed for Rs. 46,000. He gave the agent 2% commission. What did Pandoba get in selling the land?

2. An agent sold goods worth Rs. 9200 to a tradesman for a commission of the rate of 2% each from both the parties. Find the total commission.
Example 3 : Ushadevi purchased the following articles from the Vidarbha Handloom Board.

1. 2 sarees for Rs. 275 each
2. 3 bedsheets for 120 each

Find the total rebate on the above purchase if the rebate amounted was 10%. Find the amount for which Ushadevi bought all these articles.

Solution:

To find the amount paid = ?

List price of 2 sarees = 275 X 2 = Rs. 550
List price of 3 bedsheets = 120 X 3 = Rs. 360
Total L. P. = Rs. 550 + Rs. 360 = Rs. 910

\[
\text{10} \quad \text{Rebate obtained} = \quad \frac{10}{100} \quad \text{\times} \quad 910 = \text{Rs. 91}
\]

Amount paid by Ushadevi = 910 - 91 = Rs. 819

Mathematical Note:

To promote the sale of some articles such as khadi or handicrafts, the government asks the traders to sell these articles at the price less that the printed price. This loss is covered by the government and such compensation is known as rebate. Rebate is given on percentage basis.
1. At Agakhan Palace Khadi Bhandar, Uma purchased some articles such as 4 napkins each for Rs. 8, 3 handkerchiefs each for Rs. 4 and 12 metre khadi cloth at the rate Rs. 36 per metre. There was 12% rebate on the sale of these articles. Find the amount paid by Uma.

2. Sangram purchased following articles from Sahakari Bazar.
   1) 10 honey bottles each for Rs. 75/-. 
   2) 5 pairs of Kolhapuri Chappal each pair for Rs. 110/-
   The rebate on the sale was 15%. Find out how much amount was paid by Sangram.
23. Average

**Observe and Understand**

**Example 1**: Following table shows the runs made by Sachin Tendulkar in five one day matches in a series.

Find the average.

<table>
<thead>
<tr>
<th>Match No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runs</td>
<td>27</td>
<td>105</td>
<td>10</td>
<td>55</td>
<td>45</td>
</tr>
</tbody>
</table>

**Solution**: Total runs of all five matches

\[= 27 + 105 + 10 + 55\]

\[= 297\]

No. of matches = 5

Avg. runs per match = \[
\frac{\text{total runs}}{\text{no. of matches}}
\]

\[= \frac{297}{5} = 59.4\]

**Mathematical Note**: To find an average, we add all the quantities and then divide the total by the number of quantities.

\[\text{Average marks} = \frac{\text{Total marks}}{\text{No. of Subjects}}\]

\[\text{Average speed} = \frac{\text{Total distance covered over total time taken}}{\text{time taken}}\]
Assignment

1. Vijay Co. exported 4,820 boxes of mangoes in the first year and 6,150 boxes of mangoes in the second year. What was the yearly average of the export of boxes?

2. The daily earnings of a family for five days are: Rs. 110, Rs. 140, Rs. 96.50, Rs. 128 and Rs. 120.50. What is the average daily earning of the family?

Observe and Understand

Example 2: The average weight of 8 boys is 40 Kg. The average weight of 4 of them is 45 Kg. What is average weight of the remaining 4 boys?

Solution:
Total wt. of 8 boys = 8 x 40 = 320 Kg.
Total wt. of 4 boys = 4 x 45 = 180 Kg.
Total wt. of remaining 4 boys = 320 - 180 = 140 Kg.
Average wt. of 4 boys = 140 / 4 = 35 Kg.

Assignment:

1. Shailaja secured 63 marks on an average in 6 subjects in Std. 6 examination. How many marks in all did she get in the six subjects?

2. The average attendance of a school for the first four days of a week was 135. The total attendance for the first three days of a week was 522. What was the attendance on the 5th day?

3. A hawker earns Rs. 50 on an average for the five days of the week. For the remaining two days the average earning is Rs. 185. What is the daily average earning of the hawker?

4. The average income of 3 shows of drama is Rs. 9640. Find the total income of all the 3 shows. If the income of the first show is Rs. 8420 and that of the other two shows equal, find the income of the third show.
Example 1: Study the following data and its graphical representation (Bar graph)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of villages (thousands)</td>
<td>350</td>
<td>370</td>
<td>390</td>
<td>410</td>
<td>440</td>
</tr>
</tbody>
</table>

Graph

Discussion:

Answer the following questions based on above data and graph.

1. Which two quantities are shown in the graph?
2. What is the horizontal straight line named as?
3. What is the vertical line named as?
4. Which quantity is taken on X-axis?
5. Which quantity is taken on Y-axis?
6. Which bar has the maximum height?
7. Which bar has the least height?
Mathematical Note:

A graph is a diagram. It shows relation between two quantities.
When two straight lines intersect each other in a point, the plane is divided into four equal parts. Each part is called a quadrant.
The horizontal line is called X-axis. The vertical line is called Y-axis.
The point of intersection of X-axis and Y-axis is called origin.
A graph is a graphical representation of the given quantities using bars of the same width.

Assignment:

Draw the following bar graphs of the following:

1. On Y-axis, 1 cm = 200 crores

<table>
<thead>
<tr>
<th>Export of things</th>
<th>Tea</th>
<th>Coffee</th>
<th>Minerals</th>
<th>Garments</th>
<th>Rice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Export (Rs)</td>
<td>600</td>
<td>340</td>
<td>640</td>
<td>2400</td>
<td>440</td>
</tr>
</tbody>
</table>

Population of Indian states in 1994 is as follows:

2. On Y-axis, 1 cm = 50 lakh population

<table>
<thead>
<tr>
<th>State</th>
<th>Maharashtra</th>
<th>Gujarat</th>
<th>M.P.</th>
<th>Andhra</th>
<th>Karnataka</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (lakhs)</td>
<td>820</td>
<td>430</td>
<td>700</td>
<td>715</td>
<td>460</td>
</tr>
</tbody>
</table>

Literacy of Indian states is given below:

3. On Y-axis, 1 cm = literate people 5%

<table>
<thead>
<tr>
<th>State</th>
<th>Maharashtra</th>
<th>Gujarat</th>
<th>M.P.</th>
<th>Andhra</th>
<th>Karnataka</th>
</tr>
</thead>
<tbody>
<tr>
<td>literacy (%)</td>
<td>75</td>
<td>71</td>
<td>54</td>
<td>54</td>
<td>66</td>
</tr>
</tbody>
</table>
REFERENCES