APPENDIX

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Effects of Porous Medium on MHD Fluid Flow along a Stretching Cylinder

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Received 12 April 2014; accepted 21 May 2014

Abstract. Numerical solution of MHD fluid flow and heat transfer characteristics of a viscous incompressible fluid along a continuously stretching horizontal cylinder embedded in a porous medium was carried out in the presence of internal heat generation or absorption. The boundary layer equations with the convective boundary conditions were transferred into a system of non-linear ordinary differential equations and solved numerically by using fourth order Runge-Kutta integration scheme with shooting method. Numerical results obtained for velocity, temperature distributions, skin friction coefficient and Nusselt number. Characteristics of the flow and heat transfer for various values of the Prandtl number, stretching parameter and magnetic parameter analyzed and presented through graphs and tables.

Keywords: Porous medium, magnetohydrodynamic, stretching cylinder, heat transfer, boundary layer

AMS Mathematics Subject Classification (2010): 76A25

1. Introduction
The heat transfer due to free and mixed convection in fluid saturated porous media has been considered in many engineering problem such as MHD power generators, petroleum industries, plasma studies, geothermal system, heat insulation, drying technology, catalytic reactors, solar power collectors, food industries and many others. Many authors have been attracted to hydrodynamic flow and heat transfer over a stretching cylinders and flat plates due to its enormous applications in industries. Carne [1] reported flow past a stretching plate. Gupta [2] obtained heat and mass transfer on a stretching sheet with suction or blowing. Grubka and Bobba [3] obtained heat transfer characteristics of a continuous stretching surface with variable temperature. Sharma [4] obtained free convection effects on the flow of an ordinary viscous fluid past and infinite vertical porous plate with constant suction and constant heat flux. Aldos [5] discussed MHD mixed convection from a vertical cylinder embedded in a porous medium. Aldos and Ali [6] studied MHD free forced convection from a horizontal cylinder with suction and
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The aim of the paper is to investigate boundary layer flow and heat transfer of a viscous and incompressible Newtonian fluid along a stretching cylinder in the presence of a constant transverse magnetic field and Variable surface temperature boundary condition.

2. Mathematical formulation

Consider the boundary layer flow due to free convection heat transfer from a horizontal cylinder of radius \( a \) and embedded in a porous medium saturated with a Newtonian fluid. It is assumed that the cylinder is stretched in the axial direction with linear velocity \( u_w(x) = Ux/l \) and the surface of the cylinder is subjected to a variable temperature \( T_w(x) = T_{\infty} + T_0(x/l)^n \). The \( x \)-axis is measured along the axis of the cylinder and \( y \)-axis is measured in the radial direction. A uniform magnetic field of strength \( B \) is acting in the radial direction. The magnetic Reynolds number is taken to be enough small such that the induced magnetic field is negligible. Under these assumptions and the boundary layer approximations, the governing equations are given by

\[
\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{u}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{\sigma B^2}{\rho} u - \frac{v}{K_v}, \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{Q}{\rho C_p} (T - T_{\infty}), \tag{3}
\]

where \( u \) and \( v \) are the velocity components along the \( x \) and \( r \) directions, respectively. \( v \) is the kinematic viscosity, \( \sigma \) is the electrical conductivity, \( \rho \) is the density of the fluid, \( K_v \) is the permeability of the porous medium, \( T \) is fluid temperature inside the thermal boundary layer, \( T_{\infty} \) is the fluid temperature in the free stream, \( \alpha \) is the thermal diffusivity and \( Q \) is the volumetric rate of heat source or sink and \( C_p \) is the specific heat at constant pressure.

The boundary conditions are

\[
r = a : u = u_w(x) = U \frac{x}{l}, \quad v = 0, \quad T = T_w(x) = T_{\infty} + T_0 \left( \frac{x}{l} \right)^n
\]

\[
r \to \infty : u \to 0, \quad T \to T_{\infty} \tag{4}
\]

where \( U \) is the reference velocity, \( l \) is the characteristic length and \( T_0 \) is the reference temperature of the fluid. Introducing the following parameters and similarity variables


\[ \eta = \frac{r^2 - a^2}{2a} \left( \frac{u_n}{v_x} \right)^{1/2}, \quad \psi = (u_n u_x)^{1/2} \phi(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_u - T_\infty}, \]  

(5)

\[ u = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \]

Into the equations (2) and (3), we get

\begin{align*}
(1+2\delta\eta) f''' + f'' + 2\delta f'' - (f')^2 - (M + \beta) f' = 0, \quad (6) \\
(1+2\delta\eta) \theta'' + 2\partial\theta' + Pr(f \theta' - nf' \theta) + Pr S\theta = 0, \quad (7)
\end{align*}

where \( \psi \) is the stream function, prime denotes differentiation with respect to \( \eta \),

\[ \delta = \left[ \omega \left( \frac{1}{\alpha U} \right) \right] \]

is the curvature parameter,

\[ M = \frac{\sigma B^2}{\rho \gamma} \]

is the magnetic parameter,

\[ \beta = \frac{\mu U}{\rho} \]

is the permeability parameter,

\[ S = \frac{\omega}{\alpha U} \]

is the heat generation parameter,

\[ Pr = \frac{\rho c_p}{\mu} \]

is the Prandtl number, \( R \) is the surface temperature exponent.

It is noticed that equation (1) is identically satisfied. The boundary conditions are reduced to

\[ f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad f'(\infty) \to 0, \quad \theta(\infty) \to 0. \quad \]  

(8)

3. Skin friction Coefficient

The shearing stress at the surface is given by

\[ \tau_s = \mu \left( \frac{\partial u}{\partial r} \right)_{r=a}, \]  

(9)

where \( \mu \) is the coefficient of viscosity.

The skin friction coefficient at the surface, is defined as

\[ C_f = \frac{2\tau_s}{\rho u_a^2}, \]  

(10)

\[ \Rightarrow \frac{1}{2} C_f \text{Re}^{1/2} = f''(0), \]  

(11)

where \( \text{Re} = \frac{U}{\nu} \) is the Reynolds number.

4. Heat transfer Coefficient

The rate of heat transfer at the surface is given by

\[ q_v = -K \left( \frac{\partial T}{\partial r} \right)_{r=a}, \]  

(12)

where \( K \) is thermal conductivity of the fluid.

The Nusselt number is defined as

\[ Nu = \frac{\kappa}{K} \left( \frac{q_v}{T_u - T_\infty} \right), \]  

(13)

\[ \Rightarrow Nu \text{Re}^{1/2} = -\theta'(0). \]  

(14)
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The governing non-linear boundary layer equations (6) and (7) are solved numerically using Runge-Kutta fourth order integration scheme with shooting integration technique. The effects of physical parameters on the velocity and temperature profiles are shown through graphs. The numerical values of skin friction and heat transfer coefficients are derived for different values of physical parameters and presented through Tables.

5. Results and discussion

It is seen from figure 1 that as $M$ increases, the velocity profiles decrease. As $M$ increases, the Lorentz force which opposes the flow also increases leads to enhance deceleration of the flow. Figure 2 depicts that fluid velocity profiles decrease due to increase in permeability parameter which agrees with natural phenomena. Figure 3 indicates the influence of the curvature parameter on velocity profiles. It is observed that velocity profiles increases as curvature parameter increases. It is seen from figure 4 that thermal boundary layer thickness increases with increasing values of the magnetic parameter. It is noted from Figure 5 that as permeability parameter increases, the temperature decreases. The temperature profiles increase as the curvature parameter increases as seen from figure 6. It is observed from figure 7 that thermal boundary layer thickness decreases with the increase of Prandtl number. From a physical point of view, if Prandtl number increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing of energy ability that reduces the thermal boundary layer. Figure 8 depicts, that as heat generation parameter increases, the temperature profiles increase. It is seen from figure 9 that as surface temperature exponent increases, the temperature profiles decrease. In order to validate the method used in this study and to judge the accuracy of the present analysis, the numerical values of skin friction coefficient and heat transfer coefficient for the stretching cylinder are compared with those of Gurbka [1985], Ali et al [2011], Abel et al [2012]. A good agreement is observed between these results shown in Table 1 and Table 2, which lends confidence in the numerical results to be reported subsequently.

![Figure 1: Velocity distribution versus $\eta$ when $Pr = 7, S = 0.5, n = 0.5, \beta = 0.5, \delta = 0.5$.](attachment:image.png)
Figure 2: Velocity distribution versus $\eta$ when $Pr = 7, S = 0.5, M = 0.5, n = 0.5, \delta = 0.5$.

Figure 3: Velocity distribution versus $\eta$ when $Pr = 7, S = 0.5, M = 0.5, n = 0.5, \beta = 0.5$. 
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Figure 4: Temperature distribution versus $\eta$ when $Pr = 7, S = 0.5, n = 0.5, \beta = 0.5, \delta = 0.5$.

Figure 5: Temperature distribution versus $\eta$ when $Pr = 7, S = 0.5, M = 0.5, n = 0.5, \delta = 0.5$.

Figure 6: Temperature distribution versus $\eta$ when $Pr = 7, S = 0.5, M = 0.5, n = 0.5, \beta = 0.5$. 

Figure 7: Temperature distribution versus $\eta$ when $S = 0.5, M = 0.5, n = 0.5, \beta = 0.5, \delta = 0.5$.

Figure 8: Temperature distribution versus $\eta$ when Pr = 7, $M = 0.5, n = 0.5, \beta = 0.5, \delta = 0.5$.

Figure 9: Temperature distribution versus $\eta$ when Pr = 7, $S = 0.5, M = 0.5, \beta = 0.5, \delta = 0.5$. 

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**Table 1:** Comparison of numerical values of \( f''(0) \) for various values of \( M \) for stretching cylinder with \( \beta = 0 \) and \( \delta = 0 \).

<table>
<thead>
<tr>
<th>( M )</th>
<th>Abel et al [2012]</th>
<th>Present study</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-0.999962</td>
<td>0.999962</td>
</tr>
<tr>
<td>0.2</td>
<td>-1.095445</td>
<td>-1.095445</td>
</tr>
</tbody>
</table>

**Table 2:** Comparison of numerical values of \(-\theta'(0)\) for various values of \( \text{Pr} \) and \( n \) with \( S = 0, M = 0, \beta = 0, \delta = 0 \).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
<td>1</td>
<td>0.8086</td>
<td>0.8086</td>
<td>0.808689</td>
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<td>1</td>
<td>1</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.000174</td>
<td>1.000000</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.1652</td>
<td>-</td>
<td>-</td>
<td>-0.038721</td>
</tr>
<tr>
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<td>1</td>
<td>1.9237</td>
<td>1.9237</td>
<td>1.923609</td>
<td>1.923700</td>
</tr>
<tr>
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<td>2.5097</td>
<td>-</td>
<td>-</td>
<td>2.515901</td>
</tr>
<tr>
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<td>-</td>
<td>-</td>
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</tr>
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<td>-</td>
<td>-</td>
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<td>3.7208</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>4.808102</td>
</tr>
</tbody>
</table>

**Table 3:** Numerical values of skin friction coefficient and Nusselt number at the surface for various values of physical parameters.

<table>
<thead>
<tr>
<th>( \text{Pr} )</th>
<th>( S )</th>
<th>( n )</th>
<th>( \beta )</th>
<th>( M )</th>
<th>( \delta )</th>
<th>(-f''(0))</th>
<th>(-\theta'(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.568286</td>
<td>2.146312</td>
</tr>
<tr>
<td>50</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.568286</td>
<td>4.485325</td>
</tr>
<tr>
<td>100</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.568286</td>
<td>6.985691</td>
</tr>
<tr>
<td>7</td>
<td>-0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.568286</td>
<td>3.227012</td>
</tr>
</tbody>
</table>
6. Conclusion
Steady two dimensional MHD boundary layer flow of an incompressible viscous fluid over a stretching cylinder is investigated with heat source or sink is placed within the flow to allow for possible heat generation or absorption effects. Numerical calculations are carried out for various values of the physical parameters and the following conclusions are made:

(i) Fluid velocity profiles decrease due to increase in the magnetic parameter and permeability parameter.
(ii) Fluid velocity profiles increase due to increase in curvature parameter.
(iii) Fluid temperature decreases due to increase in surface temperature exponent or Prandtl number.
(iv) Fluid temperature increases due to increase in permeability parameter, magnetic parameter, heat generation parameter or curvature parameter.
(v) Skin friction coefficient and Nusselt number both increase with the increase of magnetic parameter.
(vi) Nusselt number increases with the increase of Prandtl number, surface temperature exponent or curvature parameter, but it decreases due to increase of heat generation parameter or permeability parameter.
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REFERENCES

Effects Of Radiation And Viscous Dissipation On Mhd Boundary Layer Flow Due To An Exponentially Moving Stretching Sheet In Porous Medium

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Abstract: Aim of the paper is to investigate radiation effects on the MHD flow over an exponentially moving stretching sheet placed in a porous medium. A variable magnetic field is applied normal to the sheet. Similarity transformation is used to convert the governing nonlinear partial differential equations into a system of ordinary differential equations which are solved numerically using forth order Runge- Kutta integration scheme with shooting iteration technique. The effects of physical parameters on the dimensionless velocity and temperature profiles are depicted graphically and analyzed in details. Finally numerical values of physical quantities, such as the local skin friction coefficient and the local Nusselt number are presented in the tabular form.

Keywords: Stretching sheet, Porous medium, Boundary layer flow, Gebhart number.

1. Introduction


Aim of the paper is to investigate effects of radiation and viscous dissipation on steady flow of a viscous incompressible electrically conducting fluid over an exponentially moving stretching sheet in porous medium.

2. Mathematical model

Consider the two dimensional steady boundary layer flow and heat transfer through an incompressible viscous electrically conducting fluid past a semi infinite exponentially stretching sheet embedded in porous medium. The origin of the system is located at the slit from which the sheet is drawn. The x-axis is taken along the continuous stretching surface and points in the direction of motion. The y-axis is perpendicular of the plate. The sheet velocity is assumed to vary as an exponential function of distance x from the slit. The temperature away from the fluid is assumed to be $T_\infty$. The sheet ambient temperature is also assumed to exponential function of distance from the slit. A variable magnetic field of strength $B(x)$ is applied normal to the sheet. The governing continuity, momentum and heat transfer equations are given by

\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= u \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} \frac{1}{K_r(x)} u - \frac{1}{\alpha K_r(x)} u \\
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_r} \frac{\partial T}{\partial y} + \frac{\sigma B^2(x)}{\rho C_r} u^2 + \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^2
\end{align}
where \( u \) and \( v \) are velocity components in the \( x, y \)-directions, respectively \( \mu(= \mu / \rho) \) is the kinematic viscosity, \( \mu \) is the coefficient of viscosity, \( \rho \) is the density, \( \sigma \) is the electrical conductivity of the fluid, \( T \) is the fluid temperature, \( \alpha(=\kappa / \rho \c C_f) \) is the thermal diffusivity, \( \kappa \) is the thermal conductivity \( C_f \) is the specific heat at constant pressure and \( q_e \) is the radiation heat flux.

The corresponding boundary conditions are

\[
\begin{align*}
\text{u}(x,0) &= u_e(x) = U e^{xL}, \quad \nu(x,0) = 0, \\
T(x,0) &= T_e = T_u + T_c e^{xL}, \quad \text{u}(x, \infty) = 0, T(x, \infty) = T_u
\end{align*}
\] (4)

Here subscripts \( w, \infty \) refer to the surface and ambient conditions respectively. \( U \) is the characteristic velocity, \( T_u \) is the static temperature and \( L \) is characteristic length.

3. Method of solution

To facilitate a similarity solution, the magnetic field \( B(x) \) and permeability of the porous medium \( K_f(x) \) are assumed to be of the form

\[
B(x) = B_s e^{xL}, \quad K_f(x) = \rho e^{xL}
\] (5)

where \( B_s \) is a constant. It is also assumed that fluid is weakly electrically conducting so that the induced magnetic field is negligible. Following Rosseland’s approximation, the radioactive heat flux \( q_e \) is modeled as

\[
q_e = -\frac{4\sigma^*}{3k^*} \frac{\partial \eta}{\partial y}.
\] (6)

Where \( \sigma^* \) is the Stefan-Boltzman constant, \( k^* \) is the mean absorption coefficient. Assuming that the temperature difference within the flow are sufficiently small such that \( T^* \) may be expressed as a linear function of temperature \( T^* \equiv 4T_u / T - 3T_u^2 \), we have

\[
\frac{\partial \eta}{\partial y} = -\frac{16\sigma^* T_u^2}{3k^*} \frac{\partial^2 \eta}{\partial y^2}.
\] (7)

Introducing the following dimensionless variables, parameters and similarity variable

\[
\begin{align*}
\eta &= \frac{U}{2UL} y e^{-xL} \Rightarrow \eta^* = \frac{U}{2UL} y e^{-xL} \Rightarrow T_u = T_u^* + T_c e^{xL} \Rightarrow \beta = \frac{2L}{\rho U^*}, \\
\eta &= \left( \frac{U}{2UL} \right)^{1/2} y e^{-xL}, \quad M = \frac{2\sigma^* L_\nu}{\rho U^*}, \quad \kappa = \frac{4\sigma^* T_u^2}{3k^*}, \quad Pr = \frac{\mu C_f}{\kappa}, \quad Gb = \frac{U^3}{C_f T_u^2}
\end{align*}
\] (8)

Where \( \eta \) is the similarity variable, \( f(\eta) \) is dimensionless stream function, \( \theta(\eta) \) is the dimensionless temperature and the prime indicates differentiation with respect to \( \eta \).

Using equations (7) and (9) into the equations (1) to (3), we get

\[
\begin{align*}
f^* + f^* f'' - 2\left( f^* \right)^2 - (M + \beta) f' &= 0 \quad (9) \\
\left( 1 + \frac{2}{3} \kappa \right) \theta' + Pr \left( f' - 4 f'' \theta \right) + Gb \Pr \left( M f'' + (f^*)^2 \right) &= 0 \quad (10)
\end{align*}
\]

Where prime denotes differentiation with respect to \( \eta \) and \( M \) is the magnetic parameter, \( \beta \) is the local porosity parameter, \( \kappa \) is the radiation parameter, \( Pr \) is the Prandtl number and \( Gb \) is the Gebhart number.

The boundary conditions are reduced to

\[
\begin{align*}
f(0) &= 0, f'(0) = 1, \theta(0) = 0, f'(\infty) \to 0, \theta(\infty) \to 0 \quad (11)
\end{align*}
\]

4. Skin friction coefficient

The shearing stress at the surface of the wall \( \tau_w \) is given by

\[
\tau_w = -\mu \frac{\partial u}{\partial y} \bigg|_{y=0} = -\frac{\mu U}{L} \frac{Re}{2} e^{3xL} f^*(0)
\] (12)

Where \( Re = \frac{LU}{\nu} \) is the Reynolds number.

The skin friction coefficient is defined as

\[
C_f = \frac{2\tau_w}{\rho U^2}
\] (13)

\[
\Rightarrow C_f = \frac{1}{2 \mu \sqrt{Re}} e^{-3xL} = -f^*(0)
\]

5. Heat transfer coefficient

The rate of heat transfer at the surface is given by

\[
\begin{align*}
q_w &= -k \left( \frac{\partial \eta}{\partial y} \right)_{y=0} = -k \frac{T_w - T_u}{L} \frac{Re e^{3xL} \theta(0)}{2}
\end{align*}
\] (14)

The Nusselt number is defined as

\[
\begin{align*}
Nu &= \frac{x}{\kappa \left( T_u - T_w \right)}
\end{align*}
\] (15)
Eqs. (9) and (10) with boundary conditions (11) are solved numerically using Runge-Kutta fourth order integration scheme with shooting iteration technique. Numerical values of skin friction coefficient and the Nusselt number are derived and presented through tables. The effects of physical parameters on the velocity and temperature profiles are shown in figures.

6. Results and discussion

It is observed from the figure (1) that the velocity profiles decrease due to increase in the magnetic parameter as the Lorentz force opposes the flows which decelerate the flow. Figure (2) depicts that fluid velocity profiles decrease due to increase in porosity parameter which agrees with natural phenomena. It is noted from figures (3) and (5) that fluid temperature increases rapidly due to a slight increase in radiation parameter or Gebhart number respectively. Figures (4) and (6) depict that fluid temperature increases a bit due to increase in magnetic parameter or porosity parameter respectively.

Figure 1. Velocity profiles versus \( \eta \) when \( Pr = 7, K = 0.5, Gb = 0.2, \beta = 1 \).

Figure 2. Velocity profiles versus \( \eta \) when \( Pr = 7, K = 0.5, Gb = 0.2, M = 1 \).

Figure 3. Temperature profiles versus \( \eta \) when \( Pr = 7, M = 1, Gb = 0.2, \beta = 1 \).
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Figure 4. Temperature profiles versus $\eta$ when $Pr = 7, K = 0.5, Gb = 0.2, \beta = 1$.

Figure 5. Temperature profiles versus $\eta$ when $Pr = 7, K = 0.5, M = 1, \beta = 1$.

Figure 6. Temperature profiles versus $\eta$ when $Pr = 7, K = 0.5, Gb = 0.2, M = 1$.

Table 1. Comparison of the Skin friction coefficient $-f'(0)$ at the surface for various values of physical parameters.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$\beta$</th>
<th>Kameswaran et al [2012]</th>
<th>Present study</th>
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<td>--</td>
<td>2.15873</td>
</tr>
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</table>
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Table2. Numerical values of Nusselt number $- \theta'(0)$ at the surface for various values of physical parameters.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$K$</th>
<th>$Gb$</th>
<th>$\beta$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
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</tr>
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<tr>
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</tr>
<tr>
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<td>0.5</td>
<td>0.2</td>
<td>4</td>
<td>3.63242</td>
</tr>
</tbody>
</table>

The values of skin friction coefficient in the absence of some physical parameters (i.e. $Pr = 0, K = 0, Gb = 0$) are shown in Table 1. It is observed from the table 1 that skin friction coefficient increases with the increase of magnetic field or porosity parameter. Some results reported by Kameswaran et al are also included in this table. It is seen that the agreement between the previously published results with the present one is very good. We can conclude that this method works efficiently for the present problem. It is noted from the table 2 that the Nusselt number decreases with the increase of magnetic parameter, radiation parameter, Gebhart number or porosity parameter, respectively.

7. Conclusions

In this paper we have studied the two dimensional steady boundary layer flow and heat transfer through an incompressible viscous electrically conducting fluid past a semi infinite exponentially stretching sheet embedded in porous medium.

Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The results for the prescribed the skin friction coefficient and Nusselt number at the plate surface are presented and discussed. We can conclude that

1. Fluid velocity profiles decrease due to increase in the magnetic parameter.
2. Fluid velocity profiles decrease due to increase in porosity parameter.
3. Fluid temperature increases due to increase in radiation parameter, magnetic parameter, Gebhart number or porosity parameter respectively.
4. Skin friction coefficient increases with the increase of magnetic field or porosity parameter.
5. Nusselt number decreases with the increase of magnetic parameter, radiation parameter, Gebhart number or porosity parameter, respectively.

References:


Analysis of MHD Convective Flow Along A Moving Semi Vertical Plate With Internal Heat Generation

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Abstract - The objective of this work is to investigate steady convection flow of a viscous incompressible electrically conducting fluid along a semi infinite vertical plate in the presence of internal heat generation and a convective surface boundary condition. The non-linear partial differential equations, governing the problem under consideration, have been transformed using similarity transformation into a system of ordinary differential equations, which is solved numerically applying fourth order Runge-Kutta integration scheme together with shooting iteration technique. The effects of various flow parameters affecting the flow field are discussed with the help of figures and tables. Numerical data for the local skin friction coefficient and the local Nusselt number have been tabulated for various values of parametric conditions. Graphical results for the velocity and temperature profiles based on the numerical solutions are presented and discussed.

Key words- Internal heat generation, MHD, Semi vertical plate, Boundary layer flow, Local biot number.

INTRODUCTION


Aim of the paper is to investigate steady free convection flow of an incompressible viscous electrically conducting fluid along a semi vertical plate in the presence of interior heat generation and a convective surface boundary condition.

Mathematical analysis

Consider two dimensional steady laminar incompressible and natural convection boundary layer flow over the right surface of the vertical plate moving with uniform velocity $U_0$. A uniform magnetic field of strength $B_0$ is applied normal to the direction of fluid flow. It is assumed that the left surface of the plate in contact with a hot fluid while quiescence cold fluid at temperature $T_\infty$ on the right surface of the plate. The hot fluid is at temperature $T_f$ which provides a heat transfer coefficient $h_f$ and cold fluid generates heat internally at the volumetric rate $Q$. Under the above assumptions, the governing equations are

Continuity equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \text{ ,} \quad (1)
\]
Momentum equation
\[
\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{B_i^2}{\rho}u,
\] 
(2)

Energy equation
\[
\rho c_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \nu \frac{\partial^2 T}{\partial y^2} + q,
\] 
(3)

where \( u \) and \( v \) are the \( x \) and \( y \) components of the velocity, respectively; \( T \) is fluid temperature, \( \nu \) is the kinematics viscosity of the fluid, \( \rho \) is the fluid density, \( c_p \) is the specific heat at constant pressure, \( \kappa \) is the thermal conductivity of the fluid, and \( \beta \) is the thermal expansion coefficient.

The boundary conditions are
\[
u(x,0) = u(x,0) = 0, -k \frac{\partial T}{\partial y}(x,0) = h_f[T_f - T(x,0)] 
\] 
(4)

\[
u(x,\infty) = 0, T(x,\infty) = T_\infty
\] 
(5)

Introducing dimensionless variables, parameters and similarity variable
\[
\eta = \frac{y}{x}\sqrt{\frac{\nu}{Re}}. u = U_x f', v = \frac{\nu}{2x}\sqrt{\frac{\nu}{Re}}(\eta f' - f), \theta = \frac{T - T_\infty}{T_f - T_\infty},
\]
\[
B_i = \frac{h_f}{k}\left(\frac{U_x}{U_0}\right)^{1/2}, Gr_x = g\beta x \left(\frac{T_f - T_\infty}{U_0^2}\right), \lambda_x = \frac{\beta x}{\kappa Re(1 - T_f - T_\infty)}
\]
\[
Pr = \frac{\mu c_p}{\kappa}, M = B_i \sqrt{\frac{c_p x}{\rho U_0}}, Re = \frac{U_0 x}{\nu}
\] 
(6)

into the equations (1) to (3), we get
\[
f''' + \frac{1}{2} ff'' + Gr_x \theta - M^2 f' = 0,
\] 
(7)

\[
\theta'' + \frac{1}{2} Pr f \theta' + \lambda_x e^{-\eta} = 0.
\] 
(8)

where the prime denotes differentiation with respect to \( \eta \) and is the \( Bi \) Biot number, \( Gr \) is the Grashof number, \( \lambda \) is the internal heat generation parameter, \( Pr \) is the Prandtl number, \( Re \) is the Reynolds number and \( M \) is the magnetic parameter.

The boundary conditions in dimensionless form are
\[
f(0) = 0, f'(0) = 1, \theta'(0) = -Bi_x\left[1 - \theta(0)\right], f'(\infty) = 0
\] 
(9)

Moreover, equations (7) and (8) with the boundary condition (9) will definitely produce a local similarity solution for the problem. In order to have a true similarity solution, the parameters \( Gr, Bi_x \) and \( \lambda_x \) must be constant and this condition will be satisfied if we assume
\[
h_f = cx^{-1/2}, \beta = mx^{-1}, q = lx^{-1} e^{-\eta}
\] 
(10)

where \( c, m \) and \( l \) are constants but have the appropriate dimensions. Substituting (10) into the parameters \( Gr, Bi_x \) and \( \lambda_x \), we obtain
\[
Bi = \frac{c}{k}\sqrt{\frac{\nu}{U_0}}, Gr = \frac{mg(T_f - T_\infty)}{U_0^2}, \lambda = \frac{lv}{\kappa U_0(T_f - T_\infty)}
\] 
(11)

The equations (7) and (8) are a set of coupled differential equations and solved using with the shooting iteration technique under the boundary condition (9). The numerical values of surface temperature \( \theta(0) \), skin-friction coefficient \( f'(0) \) and Nusselt number \( \theta'(0) \) for various values of physical parameters are derived, discussed and presented through table.

Results and discussion

Effects of physical parameters on velocity profiles

Figure 1 shows that the velocity boundary layer thickness decreases fast with an increase in the intensity of magnetic field as the magnetic field presents a damping effect on the velocity field by creating a drag force that opposes the fluid motion. Figures 2 and 3 show that the velocity boundary layer thickness decreases with increase in the values of the Prandtl number or intensity of Biot number \( Bi_x \).

Figures 4 and 5 depict that the velocity boundary layer thickness increases with increase in the values of the Grashof number \( Gr \) or internal heat generation parameter \( \lambda_x \).

Effects of physical parameters on temperature profiles

Figure 6 depicts that fluid temperature decreases with increase in the intensity of magnetic field. Figures 7 and 8 show that the fluid temperature decreases with increase in the values of the Prandtl number or intensity of local Biot number \( Bi_x \). It is seen from figure 9 that fluid temperature decreases with the increase of Grashof number due to an increase in the intensity of buoyancy force. An increase in internal heat generation parameter causes increase in the fluid temperature as shown in figure 10.
Figure 1. Velocity profiles versus $\eta$ when $Pr = 0.72, Bi = 0.1, Gr = 0.1, \lambda = 10$

Figure 2. Velocity profiles versus $\eta$ when $M = 0.57, Bi = 0.1, Gr = 0.1, \lambda = 10$

Figure 3. Velocity profiles versus $\eta$ when $M = 0.57, Pr = 0.72, Gr = 0.1, \lambda = 10$
Figure 4. Velocity profiles versus $\eta$ when $Pr = 0.72, Bi_x = 0.1, M = 0.57, \lambda_x = 10$

Figure 5. Velocity profiles versus $\eta$ when $Pr = 0.72, Bi_x = 0.1, Gr_x = 0.1, M = 0.57$

Figure 6. Temperature profiles versus $\eta$ when $Pr = 0.72, Bi_x = 0.1, Gr_x = 0.1, \lambda_x = 10$
Figure 7. Temperature profiles versus $\eta$ when $M = 0.57, Bi_x = 0.1, Gr_x = 0.1, \lambda_x = 10$.

Figure 8. Temperature profiles versus $\eta$ when $M = 0.57, Pr = 0.72, Gr_x = 0.1, \lambda_x = 10$.

Figure 9. Temperature profiles versus $\eta$ when $Pr = 0.72, Bi_x = 0.1, M = 0.57, \lambda_x = 10$.
Figure 10. Temperature profiles versus $\eta$ when $Pr=0.72, Bi_x=0.1, Gr_x=0.1, M=0.57$

Table 1. Numerical values of $f^*(0), \theta'(0)$ and $\theta(0)$ for different values of parameters.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$Bi_x$</th>
<th>$Gr_x$</th>
<th>Pr</th>
<th>$\lambda_x$</th>
<th>$f^*(0)$</th>
<th>$\theta'(0)$</th>
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<tr>
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CONCLUSION

In this paper we have studied MHD natural convection internal heat generation on boundary layer flow over a vertical plate with a convective surface boundary condition with the surrounding in the presence of magnetic field. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The results for the prescribed local skin friction coefficient and Nusslet number at the plate surface are presented and discussed. We can conclude that

1. Skin friction coefficient at the plate surface increases rapidly with an increase in intensity of magnetic field but Nusslet number decreases slightly with an increase in intensity of magnetic field.

2. Skin friction coefficient increases with the increase of magnetic number, Biot number, Grashoff number or heat generation parameter.

3. Skin friction coefficient decreases with the increase of the Prandtl number.
4. The rate of heat transfer decreases due to increase in magnetic number, Grashoff number, Prandtl number or Biot number.
5. The rate of heat transfer increases due to increase in heat generation parameter.

REFERENCES


Effects of Heat Source/Sink on Stagnation Point Flow over A Stretching Sheet

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Abstract-Aim of the paper is to investigate the characteristics of a non linear mathematical model of MHD stagnation point flow over a stretching plate. The boundary layer equations with the convective boundary conditions are transferred by a similarity transformation into a system of non linear ordinary differential equations and solved numerically by using fourth order Runge-Kutta integration scheme with shooting method. Numerical results are obtained for velocity, temperature distributions and skin friction coefficient. Further features of the flow and heat transfer for various values of the Prandtl number, stretching parameter and magnetic parameter are analysed and presented through graphs and tables.

Keywords- Stagnation point flow, Stretching plate, MHD, Heat Transfer, Similarity transformation.

INTRODUCTION
During the past few decades there has been a growing interest to investigate the convective boundary layer flow of fluids in a continuous moving surface because of its extensive applications to many engineering and industrial problems, particularly, aerodynamic extrusion of plastic sheets, polymer, spinning of fibres, cooling of elastic sheets, plasma studies, petroleum industries, MHD power generator, cooling of nuclear reactors and so on. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. Stagnation point flow is a topic of significance in fluid mechanics, because of stagnation point appears in virtually of all flow fields of science and engineering. In some cases, the flow is stagnated by a solid wall, while in others a free stagnation point or a line exists interior of the fluid domain. Carne (1970) was the first to study the convective boundary layer flow over a stretching sheet. Gupta (1977) studied heat and mass transfer on a stretching sheet with suction or blowing. Chen and Char (1988) reported heat transfer of a stretching plate which is subjected to a convective boundary condition. Patil, Chamkha and Roy (2010) investigated steady MHD natural convection flow with variable electrical conductivity and heat generation along an isothermal vertical plate. Ishak, Nazar and Pop (2006) obtained mixed convection boundary layers in the stagnation point flow towards a shrinking sheet. Sharma and Singh (2008) presented effect of viscous dissipation and heat source/sink in steady MHD flow near a stagnation point on linearly stretching sheet. Pal (2009) considered heat and mass transfer in stagnation point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Sharma and Singh (2010) investigated steady MHD natural convection flow with variable electrical conductivity and heat generation on an isothermal vertical plate. Yacob and Ishak (2012) discussed effects of chemical reaction on mixed convection flow of a polar fluid through a porous medium in the presence of internal heat generation. Mahapatra and Nandy (2013) obtained stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation.

Aim of the paper is to investigate stagnation point flow of an incompressible viscous steady fluid over a stretching plate in the presence of magnetic field of uniform strength and heat source or sink.

Mathematical formulation
Consider the steady two dimensional MHD stagnation point flow of an incompressible viscous fluid over a stretching plate which is subjected to a convective boundary condition.

It is assumed that the external velocity \( U(x) \) and the stretching velocity \( u_w(x) \) are of the forms \( U(x) = ax \) and \( u_w(x) = bx \), respectively where \( a \) and \( b \) are constants. A magnetic field of uniform strength \( B \) is applied normal to the plate in the y-direction, which produces magnetic effect in the x-direction. A heat source or sink is placed within the flow to allow for possible heat generation or absorption effects. Under the above assumptions, the governing equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u, \tag{2}
\]
\[
\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + Q(T - T_{\infty}),
\]
(3)

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) directions, respectively, \( \nu \left( = \frac{\mu}{\rho} \right) \) is the kinematic viscosity, \( \mu \) is the coefficient of viscosity, \( \sigma \) is the electrical conductivity, \( \rho \) is the density of the fluid, \( T \) is fluid temperature inside the thermal boundary layer, \( T_{\infty} \) is the fluid temperature in the free stream, \( \alpha \) is the thermal diffusivity and \( Q \) is the volumetric rate of heat source or sink.

The boundary conditions are
\[
y = 0: \quad u = u_w(x), \quad v = 0,
\]
\[
y \to \infty: \quad u = U(x), \quad T \to T_{\infty}
\]
(4)

where \( K \) is the thermal conductivity, \( h_f \) is the heat transfer coefficient and \( T_f \) is the temperature of the hot fluid.

Introducing the following similarity variables and functions
\[
\Psi = (uxU)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_f - T_{\infty}},
\]
\[
\eta = \left( \frac{U}{ux} \right)^{1/2} y, \quad u = axf'(\eta),
\]
\[
v = -aw^{1/2} f(\eta),
\]
into the equations (2) and (3), we get
\[
f''' + ff'' - Mf' - (f')^2 + 1 = 0,
\]
(6)

\[
\frac{1}{\Pr} \theta'' + f \theta' + S \theta = 0,
\]
(7)

where, \( \Psi \) is the stream function, \( M \left( = \frac{\sigma B^2}{\rho \alpha} \right) \) is the magnetic parameter, \( S \left( = \frac{Q}{\alpha} \right) \) is the heat generation parameter, \( \Pr \left( = \frac{\nu}{\alpha} \right) \) is the Prandtl number and prime denotes differentiation with respect to \( \eta \).

The boundary conditions are reduced to
\[
f(0) = 0, \quad f'(0) = \varepsilon, \quad \theta'(0) = -\gamma [1 - \theta(0)],
\]
\[
f'(\infty) \to 1, \quad \theta(\infty) \to 0.
\]
(8)

where \( \varepsilon = \frac{b}{a} \geq 0 \) is the stretching parameter. Further,
\[
\gamma = \frac{h_f \left( \frac{\nu}{k} \right)^{1/2}}{a}
\]

is the conjugate parameter for the convective boundary condition. It is noticed that \( \gamma = 0 \) is the insulated plate and \( \gamma \to \infty \) is when the surface temperature is prescribed.

Skin friction Coefficient
The shearing stress at the surface is given by
\[
\tau_w = -\mu \left( \frac{\partial u}{\partial y} \right)_{y=0}.
\]
(9)

The skin friction coefficient at the surface, is defined as
\[
C_f = \frac{\tau_w}{\rho U^2}
\]
(10)

\[
\Rightarrow C_f \Re^{1/2} = -f''(0),
\]
(11)

where \( \Re = \frac{xU}{\nu} \) is the Reynolds number.

Heat transfer Coefficient
The rate of heat transfer at the surface is given by
\[
q_w = -K \left( \frac{\partial T}{\partial y} \right)_{y=0},
\]
(12)

where \( K \) is thermal conductivity of the fluid.

The Nusselt number is defined as
\[
Nu_x = \frac{x}{\kappa} \frac{q_w}{(T_f - T_{\infty})},
\]
(13)

\[
\Rightarrow \frac{Nu_x}{\Re^{1/2}} = -\theta'(0),
\]
(14)

The governing non-linear boundary layer equations (6) and (7) are solved numerically using Runge-Kutta fourth order integration scheme with shooting integration technique. The numerical values of skin friction and heat transfer coefficients at the surface are derived for different values of physical parameters and presented through Tables.
Effects of physical parameters on the velocity and temperature profiles are shown through graphs.

Results and Discussion

Effects of physical parameters on velocity profiles

Figure 1 indicates the influence of the stretching parameter on velocity profiles. It is observed that velocity profiles increases as stretching parameter increases. It is seen from figure 2 that as magnetic parameter ($M$) increases, the velocity profiles decrease. As magnetic parameter ($M$) increases, the Lorentz force which opposes the flow also increases leads to enhance deceleration of the flow.

Effects of physical parameter variations on temperature profiles

It is noted from Figure 3 that as stretching parameter increases, the temperature decreases, and the thermal boundary layer thickness also decreases. It is seen from figure 4 that thermal boundary layer thickness increases with increasing values of the magnetic parameter. The temperature profiles increase as the conjugate parameter ($\gamma$) increases as seen from figure 5. Figure 6 depicts, that as heat generation parameter increases, the temperature profiles increase.

It is observed from figure 7 that thermal boundary layer thickness decreases with the increase of Prandtl number. From a physical point of view, if Prandtl number increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing of energy ability that reduces the thermal boundary layer.

Figure 1. Velocity distribution versus $\eta$ when $Pr = 0.72, S = 0.5, \gamma = 1, M = 0.05$. 

Graph showing velocity distribution with varying $\eta$ values.
Figure 2. Velocity distribution versus $\eta$ when $Pr = 0.72, S = 0.5, \gamma = 1, \varepsilon = 2$.

Figure 3. Temperature distribution versus $\eta$ when $Pr = 0.72, S = 0.5, \gamma = 1, M = 0.05$. 
Figure 4. Temperature distribution versus $\eta$ when $Pr = 0.72, S = 0.5, \gamma = 1, \varepsilon = 2$. 

Figure 5. Temperature distribution versus $\eta$ when $Pr = 0.72, S = 0.5, \varepsilon = 2, M = 0.05$. 
In order to validate the method used in this study and to judge the accuracy of the present analysis, the numerical values of skin friction coefficient for the stretching sheet are compared with those of Wang [2008], Yacob and Ishak [2012], Mahapatra and Gupta [2013]. These comparisons are shown in Table 1. A good agreement is observed between these results. This lends confidence in the numerical results to be reported subsequently.

CONCLUSION
Steady two dimensional MHD stagnation point flow of an incompressible viscous fluid over a stretching plate is investigated with heat source or sink is placed within the flow to allow for possible heat generation or absorption effects. Numerical calculations are carried out for various values of the physical parameters. The following conclusions are made.
1. Fluid velocity profiles decrease due to increase in the magnetic parameter due to generation of Lorentz force.
2. Fluid velocity profiles increase due to increase in stretching parameter.
3. Fluid temperature decreases due to increase in stretching parameter or Prandtl number.
4. Fluid temperature increases due to increase in conjugate parameter, magnetic parameter or heat generation parameter.

5. Skin friction coefficient at the plate surface increases with an increase in intensity of magnetic field, but Nusselt number decreases with an increase in intensity of magnetic field.
6. Skin friction coefficient and Nusselt number both increase with the increase of stretching parameter.
7. Nusselt number increases with the increase of Prandtl number or conjugate parameter, but it decreases due to increase of heat generation parameter.

Table 1. Comparison of numerical values of \( f''(0) \) when \( M = 0 \) for stretching sheet with different values of \( \mathcal{E} \).

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Table 2. Numerical values of skin friction coefficient and Nusselt number at the surface for various values of physical parameters.

<table>
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<th>( \mathcal{E} )</th>
<th>( M )</th>
<th>( S )</th>
<th>( \text{Pr} )</th>
<th>( \gamma )</th>
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REFERENCES


