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• S. Asha and R. Kala, Continuous Monotonic decomposition of some special graphs, Communicated to Bulletin of the Allahabad Mathematical Society.

• S. Asha and R. Kala, Continuous Monotonic double decomposition of certain graphs, Communicated to International Journal of Pure and Applied Sciences and Technology.
Chapter 1

Preliminaries

In this Chapter we collect the basic definitions and theorems which are needed for the subsequent chapters. For graph theoretic terminology we refer Harary [6] and for definitions of special classes of graphs we refer [9] and [10].

Definition 1.0.1. [6] A graph $G = (V, E)$ consists of a finite non-empty set $V$ of vertices together with a set $E$, whose elements are unordered pairs of distinct vertices of $V$. The number of vertices in the graph is called the order of the graph and the number of edges of the graph is called the size of the graph. The order and size of $G$ are denoted by $n$ and $m$ respectively.

Definition 1.0.2. [6] If $e = \{u, v\}$ is an edge, we write $e = uv$; we say that $u$ and $v$ are adjacent vertices; $u$ and $v$ are incident with $e$.

Definition 1.0.3. [6] A graph $G_1$ is isomorphic to a graph $G_2$ if there exists a bijection $\phi$ from $V(G_1)$ to $V(G_2)$ such that $uv \in E(G_1)$ if and only if $\phi(u)\phi(v) \in E(G_2)$. If $G_1$ is isomorphic to $G_2$, we write $G_1 \cong G_2$.

Definition 1.0.4. [6] A graph $H$ is called a subgraph of a graph $G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. A spanning subgraph of $G$ is a subgraph $H$ with $V(H) = V(G)$. For any set $S$ of vertices of $G$, the induced subgraph $\langle S \rangle$ is the maximal
subgraph of $G$ with vertex set $S$. Thus two vertices of $S$ are adjacent in $<S>$ if and only if they are adjacent in $G$.

**Definition 1.0.5.** [6] The *degree* of a vertex $v$ in a graph $G$ is the number of edges of $G$ incident with $v$ and is denoted by $\text{deg } v$ or $d(v)$. The minimum and maximum degrees of $G$ are denoted by $\delta(G)$ and $\Delta(G)$ respectively. A vertex of degree 0 in $G$ is called an *isolated vertex*.

**Definition 1.0.6.** [6] A graph $G$ is *complete* if every pair of its vertices are adjacent. A complete graph on $n$ vertices is denoted by $K_n$.

**Definition 1.0.7.** [6] A *bipartite* graph is a graph whose vertex set $V(G)$ can be partitioned into two subsets $V_1$ and $V_2$ such that every edge of $G$ has one end in $V_1$ and the other end in $V_2$; $(V_1, V_2)$ is called a *bipartition* of $G$. If further every vertex of $V_1$ is joined to every vertex of $V_2$, then $G$ is called a *complete bipartite* graph. The complete bipartite graph with bipartition $(V_1, V_2)$ such that $|V_1| = m$ and $|V_2| = n$ is denoted by $K_{m,n}$.

**Definition 1.0.8.** [6] The complete bipartite graph $K_{1,n-1}$ is called a *star*. The vertex of degree $n - 1$ in $K_{1,n-1}$ is called its *center*. The graph obtained from $K_{1,r}$ and $K_{1,s}$ by joining their centers with an edge is called a *bistar* and is denoted by $B(r, s)$.

**Definition 1.0.9.** [6] Let $u$ and $v$ be (not necessarily distinct) vertices of a graph $G$. A $u-v$ *walk* of $G$ is a finite, alternating sequence $u = u_0, e_1, u_1, e_2, \ldots, e_n, u_n = v$ of vertices and edges beginning with vertex $u$ and ending with vertex $v$ such that $e_i = u_{i-1}u_i$, $i = 1, 2, \ldots, n$. The number $n$ is called the *length* of the walk. A walk in which all the vertices are distinct is called a *path*. A closed walk $(u_0, u_1, u_2, \ldots, u_n)$ in which $u_0, u_1, u_2, \ldots, u_{n-1}$ are distinct is called a *cycle*. A path on $n$ vertices is denoted by $P_n$ and a cycle on $n$ vertices is denoted by $C_n$.

**Definition 1.0.10.** [6] A graph $G$ is said to be *connected* if any two distinct
vertices of $G$ are joined by a path. A maximal connected subgraph of $G$ is called a component of $G$. Thus a disconnected graph has at least two components.

**Definition 1.0.11.** [6] A connected graph without cycles is called a tree.

**Definition 1.0.12.** [7] Let $G$ and $H$ be two graphs. The Corona of $G$ and $H$, denoted by $G \circ H$ is a graph constructed as follows: Take one copy of $G$ and take $|V(G)|$ copies of $H$ corresponding to each vertex of $G$. Each vertex $v$ of $G$ is made adjacent with every vertex of $H$ in the copy corresponding to $v$.

**Definition 1.0.13.** [3] Let $G = (V, E)$ be a simple graph of order $n$ and size $m$. If $G_1, G_2, \ldots, G_k$ are edge-disjoint subgraphs of $G$ such that $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_k)$, then $\{G_1, G_2, \ldots, G_k\}$ is said to be a decomposition of $G$.

**Definition 1.0.14.** [4] A decomposition, $\{H_1, H_2, \ldots, H_k\}$ $\forall k \in \mathbb{N}$, is said to be Continous Monotonic Decomposition (CMD) if each $H_i$ is connected and $|E(H_i)| = i \forall i \in \mathbb{N}$.

**Definition 1.0.15.** [14] Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The tensor product $G = G_1 \wedge G_2$ is defined as a graph with vertex set $V_1 \times V_2$. Edge set is defined as follows: If $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are two vertices of $G$ with $u_i \in V_1$ and $v_i \in V_2 (i = 1, 2)$ then $w_1 w_2 \in E(G)$ if and only if $u_1 u_2 \in E_1$ and $v_1 v_2 \in E_2$.

**Definition 1.0.16.** [14] Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The cartesian product $G = G_1 \times G_2$ is defined as a graph with vertex set $V_1 \times V_2$. Edge set is defined as follows: If $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are two vertices of $G$ with $u_i \in V_1$ and $v_i \in V_2 (i = 1, 2)$ then $w_1 w_2 \in E(G)$ if and only if either $u_1 = u_2$ and $v_1 v_2 \in E_2$ or $v_1 = v_2$ and $u_1 u_2 \in E_1$.

**Notation 1.0.17.** For any real number $x$, $\lfloor x \rfloor$ denotes the largest integer less than or equal to $x$ and $\lceil x \rceil$ denotes the smallest integer greater than or equal to $x$. 

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Definition 1.0.18. [6] A unicyclic graph is a connected graph having just one cycle.

Definition 1.0.19. [6] A vertex of degree one in a graph $G$ is called a pendent vertex. The vertex that is adjacent to a pendent vertex is called a support.