CHAPTER I
INTRODUCTION AND SUMMARY

1.1. Introduction

Statistical inference is a branch of statistics which is concerned with using probability concept to deal with uncertainty in decision making. The entire body of classical statistical inference techniques is based on fairly specific assumptions regarding the nature of the underlying population distribution; usually its form and some parameter values must be stated. Given the right set of assumptions, certain estimators for the parameters can be derived and test statistic can be developed, which are frequently elegant and beautiful. In real world problems, everything does not come packaged with labels of population of origin. A decision must be made so as to what population properties may judiciously be assume for the model. If the reasonable assumptions are such that the traditional techniques are applicable, the classical methods might be used and inferences can be drawn only with the appropriate qualifiers.

A major branch of statistical inference which is of the interest in the thesis is inference on reliability. When we buy a product we expect it to function properly for a reasonable period of time. Failure of the components may be due to substandard equipment and raw material used or random causes, wear-out caused by the ageing of the components etc. The problem of increasing reliability of any system become more significant in many fields of industry, communications technology etc., with the complex mechanization and automation of industrial processes. Underestimation and overestimation of factors associated with
reliability may cause great losses. The knowledge of reliability of a system can lead to savings in its cost of production and maintenance.

Since any component is likely to fail at any time, one can assume that the life of the component is a random variable. The life time of a component is defined as time for which the component carries its predefined function satisfactorily and passes in to a ‘failed’ state. As a random variable, a lifetime is completely characterized by its distribution function.

Let $X$ be a continuous positive random variable representing the life time of a component with distribution function $F(t)$. Then the reliability of the component at a time $t$ is given by $R(t) = P(X \geq t) = 1 - F(t)$. In other words, Reliability is defined as the probability that a component, equipment, or a system will satisfactorily perform its intended function under given circumstances, such as environmental conditions, limitations as operating time, and frequency and thoroughness of maintenance for a specified period of time.

As the complexity of system increases, its reliability decreases unless the compensatory measures are taken. System reliability can be increased by increasing the component reliability. One of the important models that is studied in the literature is stress-strength model. The term “Stress-Strength” can be described as an assessment of reliability of a component in terms of random variables $X$ representing “Stress” experienced by the component and $Y$ representing “Strength” of the component available to overcome the stress. Reliability is thus defined as the probability of not failing. For the first time the formal term “Stress-Strength” appeared in the title of Church and Harris (1970).
In a stress-strength system, a component fails if at any moment the applied stress exceeds the component’s strength. The stress is a function of the environment in which the component is located and can be estimated from the available technological knowledge about the relevant conditions of the system and the manner they interact. Stress can be treated as a random variable based on a priori considerations whereas strength cannot be computed from a priori considerations and can only be estimated by means of statistical methods from the results of the tests specifically geared for this purpose.

An alternative to the stress-strength model to increase the system reliability is to have standby components in the system and hence the study of stress-strength model with standby is necessitated. A standby system is having a number of components which works with only one active component at a time with others being idle. When an impact of stress exceeds the strength of the active component, it fails and a component from standbys, if there is any, is activated. The system fails when all the components fail.

A certain question in reliability theory pertains to the modeling of the probability distributions of random variables representing the life time of units such as human beings, animals, radioactive substances, components and system of components. The life distributions are classified according to the ageing properties of the component or system. It can be no ageing, positive ageing or negative ageing distribution. Translated into reliability terms no ageing simply means that the probability distribution of the life time of the unit does not change with the knowledge that the unit has already survived for a given time. Positive
ageing explains the situation where the performance of a component or system deteriorates with the age where as negative phenomenon possesses a beneficial effect of age.

Reliability theory make extensive use of the exponential distribution. Because of the memoryless property of this distribution, it is well-suited to model the constant hazard rate portion of the bathtub curve used in reliability theory. A popular model is the exponential distribution which is useful whenever the ‘no ageing’ phenomenon is evident. As against this, many units exhibit positive ageing phenomena. The exponential hypothesis is important because of its implications concerning the random mechanism operated in the experiment being considered. Tests of exponentiality are subject to the usual dilemma concerning goodness of fit tests. Only when the hypothesis is rejected, we have a significant result. On the other hand when a test rejects an exponential model it justifies the use of other complicated models and probabilistic and statistical methods go along with these models. Classes of life distributions based on the notion of ageing have been introduced in the literature. Some of the classes of life distributions based on ageing are Increasing Failure Rate (IFR), Increasing Failure Rate Average (IFRA), New Better than Used (NBU), New Better than Used of specified age (NBU-\(t_0\)) and New Better than Used in the tail \(NBU - [t_0, \infty)\).

The chain of implication of these notions is given by

\[ IFR \Rightarrow IFRA \Rightarrow NBU \Rightarrow NBU - [t_0, \infty) \Rightarrow NBU - t_0. \]

In the thesis, the problems considered are

(i) Statistical inference on system reliability of stress-strength models.
(ii) Estimation of system reliability of stress-strength models with standby.

(iii) Nonparametric tests for comparing two life distributions possessing IFR and NBU property.

1.2. Chapter wise Summary

In this thesis chapter I gives a thorough introduction to research work undertaken along with a brief summary.

In chapter II, a stress-strength model is formulated for a multi-component system consisting of $k$ identical components. The $k$ components of the system with random strengths $(X_1, X_2, \ldots, X_k)$ are subjected to one of the $r$ random stresses $(X_{k+1}, X_{k+2}, \ldots, X_{k+r})$. The estimation of system reliability based on maximum likelihood estimators (MLEs) and Bayes estimators in $k$ components system are obtained when the system is either parallel or series with the assumption that strengths and stresses follow exponential distribution. A simulation study is conducted to compare MLE and Bayes estimator through the mean squared errors (MSEs).

In chapter III, the estimation of reliability of Exponential-Lindley Stress-Strength model with multi-component system with more than two stresses is studied. Estimation of $R_p = P[\text{Max}(Y_1, Y_2, \ldots, Y_r) < \text{Max}(X_1, X_2, \ldots, X_k)]$ and $R_s = P[\text{Max}(Y_1, Y_2, \ldots, Y_r) < \text{Min}(X_1, X_2, \ldots, X_k)]$ when $X_1, X_2, \ldots, X_k$ are strengths subjected to one of the stresses $Y_1, Y_2, \ldots, Y_r$, assuming that $X_1, X_2, \ldots, X_k$ follow independent Lindley distribution and $Y_1, Y_2, \ldots, Y_r$ follow independent exponential distribution is discussed. The expression for system reliability of series and parallel systems for an exponential-Lindley stress-strength model is
derived. MLEs for the parameters and reliability functions are derived and their asymptotic distributions are established. The Bayes estimators are derived under squared error loss function. Also the performance of MLEs and Bayes estimators of reliability functions are studied by computing the MSEs through simulations.

In chapter IV, the estimation of $R_s = P[Max(X_{k+1}, X_{k+2}) < Min(X_1, X_2, ..., X_k)]$ when $X_1, X_2, ..., X_k$ are strengths subjected to one of the stresses $X_{k+1}, X_{k+2}$ is studied, assuming that $X_1, X_2, ..., X_k, X_{k+1}, X_{k+2}$ follow independent Pareto distributions. The expressions for MLEs for the parameters and reliability function are derived and their asymptotic distributions are established. The problem of testing the equality of the parameters when samples are drawn from two independent Pareto distributions is discussed. Likelihood Ratio Tests (LRTs) are derived for testing equality of shape parameters. Tests for equality of scale parameters are also studied. Some simulation study is conducted to estimate the power of the test.

Recently, the problem of estimation of reliability assuming distributions of life times of the components of the system as a mixture of two distributions has been studied in the literature. This motivated us to study the estimation of reliability of multi-component Stress-Strength model with mixture of two exponential as distribution of stress in detail. In chapter V, estimation of $R_p = P[Y < Max(X_1, X_2, ..., X_k)]$ and $R_s = P[Y < Min(X_1, X_2, ..., X_k)]$ is studied when $X_1, X_2, ..., X_k$ are independent strengths subjected to an independent stress $Y$. Assuming that $Y$ follow mixture of two exponential distributions, the system reliabilities are derived where $X_1, X_2, ..., X_k$ follow independent and
identical exponential or two parameter exponential or Power function or Lindley distribution. MLEs for the parameters and reliability functions are derived. Also the performance of MLEs is studied by estimating the MSEs through simulations.

One of the technique to improve reliability of a system is to use standby redundant components. In chapter VI, a \( n \) standby system is studied with initially \( n \) components, out of which only one is active under the impact of stresses and the remaining \( (n – 1) \) components are standbys is considered. Expressions for System Reliability and MLEs of the parameters are derived in the following cases.

(i) Stress follows three parameters generalized Lindley distribution and strength follows a mixture of two exponential distributions.

(ii) Stress follows two parameter gamma distribution and strength follows a mixture of two exponential distributions.

(iii) Stress follows Gompertz distribution and strength follows a mixture of two exponential distributions.

(iv) Stress follows Log-Logistic distribution and strength follows a mixture of two exponential distributions.

In chapter VII, a simple test procedure is proposed for testing the null hypothesis that two life distributions \( F \) and \( G \) are identical against the alternative that \( F \) is more IFR than \( G \). The asymptotic efficacies due to Pitman (1948) are computed to indicate its performance. Also, a test procedure is proposed for testing the null hypothesis that two life distributions are identical against the alternative that one is more new better than used than the other. Also the
asymptotic relative efficiency of the proposed $U$-statistic $T_{k,n}$, relative to the $V_{k,n}$ test statistic given by of Hollander, Park and Proschan (1986) for the two pairs of distributions $(F_{1,\theta}, G)$ is studied by assuming $G$ is an exponential distribution with mean one and $F_{1,\theta}$ as Weibull distribution and $F_{2,\theta}$ as Linear failure rate distribution. The Asymptotic relative efficiencies of the proposed test $T_{k,n}$ relative to the test due to Hollander, Park and Proschan (1986) $V_{k,n}$ for the various alternatives are computed.