CHAPTER V

OPERATIONS RESEARCH TECHNIQUES

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CHAPTER V

OPERATIONS-RESEARCH TECHNIQUES

5.0 HISTORICAL DEVELOPMENT:

The existence of optimization methods can be traced to the days Newton, Langrange and Cauchy. The development of differential calculus methods of optimization was possible because of the contributions of Newton and Leibnitz to calculus. The foundations of calculus of variations were laid by Bernoulli, Euler, Lagrange and Weistrass. The method of optimization for constrained problems, which involves the addition of unknown multipliers became known by the name of its inventor, Lagrange. Cauchy made the first application of the steepest descent method to solve unconstrained minimization problems. In spite of these early contributions, very little progress was made until the middle of the twentieth century, when high-speed digital computers made the implementation of the optimization procedures possible and stimulated further research or new methods. Spectacular advances followed, producing a massive literature on optimization techniques. This advancement also resulted in the emergence of several well defined new areas in optimization theory.
It is interesting to note that the major developments in the area of numerical methods of unconstrained optimization have been made in the United Kingdom only in the 1960's. Programming problems first arose in economics, where the optimal allocation of resources has long been of interest to economists. More specifically, however, programming problems seem to be a direct outgrowth of the work done by a number of individuals in the 1930's. One outstanding theoretical model developed then was Von Neumann's Linear model of an expanding economy, which was part of the efforts of a number of Austrian and German economists and mathematicians who were studying generalizations of Walrasian equilibrium models of an economy. Latteron a more practical approach was made by Leontief, who developed input-output models of the economy. His work was concerned with determining how much various industries would have to produce to meet a specified bill of consumer demands. Input-output models did not actually involve any optimization; instead they required the solution of a system of simultaneous linear equations.

During World War-II, a group of scientists, under the direction of Marshall K. Wood worked on allocation problems for the United States Air force. Generalizations of Leontief-type models were developed to allocate resources in such a
Although no single technique has been found to be universally applicable for non-linear programming problems, the works by Carroll and Fiacco and McCormick made many a different problem to be solved by using the well-known techniques of unconstrained optimization with ease. Geometric programming was developed in the 1960's by Duffin, Zener and Peterson. Gomory did pioneering work in integer-programming, which is one of the most exciting and rapidly developing areas of optimization. The reason for this is that most of the real-world applications fall under this category of problems. Dantzig and Charnes and Cooper developed stochastics programming techniques and solved problems by assuming design parameters to be independent and normally distributed. A. Charnes, who did some important theoretical work, and W.W. Cooper took the lead in encouraging industrial applications of linear programming. The desire to optimize more than one objective or goal while satisfying the physical limitations led to the development of multi-objective programming methods. Goal programming is well known technique for solving specific types of multiobjective optimization problems. The Goal programming was originally proposed for linear problems by Charnes and Cooper in 1961.
Problems of the linear-programming type had been formulated and solved before the pioneering work of Dantzig. In 1941 Hitchcock formulated the solved the transportation problem, which was independently solved by Koopmans in 1947. In 1942, Kantorovitch (Russian) also formulated the transportation problem, but did not solve it. The economist Stigler worked out and minimum cost diet in 1945. Although this problem can be formulated as a linear programming problem, Stigler did not use this technique. It was not until Zantzig's work, however, that the general linear programming problem was formulated as such and a method devised for solving it.

Network analysis methods are essentially management control techniques and were developed during the years 1957 and 1958. The foundations of game theory were laid by Von-Neumann in 1928 and since it has been applied to solve several mathematical economics and military problems. Only during the last few years, game theory has been applied to solve some of the engineering design problems.
5.1 METHODS.

The optimum seeking methods are also known as Mathematical programming techniques and are generally studied as a part of operations research. Operations Research is a branch of Mathematics and Statistics which is concerned with the application of scientific methods and techniques to decision making problems and with establishing the best or optimal solutions. Following are the various Mathematical programming techniques along with the other well defined areas of Operations Research.

Methods of Operations-Research

(A) Mathematical Programming Techniques.

(1) Calculus Method
(2) Calculus of Variations
(3) Non-Linear Programming
(4) Geometric-Programming
(5) Quadratic Programming
(6) Linear Programming

(7) Dynamics Programming
(8) Integer Programming
(9) Stochastic Programming
(10) Separable Programming
(11) Multiobjective Programming
(12) Network-Methods: CPM and PERT
(13) Game Theory.

(B) Stochastic Process Techniques:
(1) Statistical decision Theory
(2) Markov-process
(3) Queueing Theory
(4) Renewal Theory
(5) Simulation Method
(6) Reliability Theory

(C) Statistical Methods:
(1) Regression Analysis
(2) Cluster Analysis pattern recognition
(3) Design of Experiments
(4) Discriminant Analysis (factor analysis).
5.2. OPERATIONS RESEARCH:

The term 'Operations Research', was first coined in 1940 by McClosky and Trefthen in a small town, Bowdsey, of the United Kingdom. This new science came into existence in military context. During World War-II, military management called on scientists from various disciplines and organised them into groups to assist in solving strategic and tactical problems i.e. to discuss, evolve and suggest ways and means to improve the execution of various military projects. By their joint efforts, experience and deliberations, they suggested certain approaches that showed remarkable progress. This new approach to systematic and scientific study of the operations of the system was called the "Operations Research" or "Operational Research".

After the end of the World-War, the success of military groups attracted the attention of industrial managers who were seeking solutions to their complex executive type problems. During the year 1950, Operations-Research got recognition as a subject of academic interest in the Universities. Since then, the subject has been gaining more and more importance for students of Economics, Management, Public Administration, Behavioural sciences, Social Work,

5.3. Operations Research in India:

In India, Operations Research came into existence in 1949 with the opening of an operations Research unit at the Regional Research Laboratory at Hyderabad. At the same time, another group was setup in the Defence Science Laboratory which devoted itself to the problems of stores,
purchase and planning. In 1953, an Operations Research unit was established in the Indian Statistical Institute, Calcutta, for the application of O.R. methods in national planning and survey. Operations Research society of India was formed in 1955. It became a member of the International Federation of Operations Research Societies in 1959. The first conference of Operations Research Society of India was held in Delhi in 1959. It was felt that the primary necessity of the country is to produce well-trained Operations-Research practitioners who can tackle practical problems. So some of the institutions started producing Operations Research workers to meet the present and future needs of India. It was also decided to start a journal of Operations Research, which took practical shape in 1963 in the form of "OpSearch". The Indian Institute of Industrial Engineers has also promoted the development of "Operations Research", and its journals "Industrial Engineering" and "Management" publish several papers related to the area. Other journals which deal with Operations Research are: Journal of the National Productivity Council, Materials Management Journal of India and the Defence Science Journal.
Towards the application of Operations Research Techniques in India, Prof. Mahalanobis made the first important application. He formulated the second Five-Year Plan with the help of Operations Research techniques to forecast the trends of demand, availability of resources and for scheduling the complex schemes necessary for developing our country's economy. It was estimated that India could become self-sufficient in Food production merely by reducing the wastage of food by 15 percent. Now a days Operations Research techniques are being used to achieve this goal. Planning Commission of India, made the proper use of operations research techniques for planning the optimum size of the caravelle fleet of Indian Airlines.

In the industrial sector, in spite of the fact that opportunities of operations Research work at present are very much limited. Organized industries in India are gradually becoming conscious of the role of Operations Research and a good number of them have well trained Operations Research teams. Most popular practical application of Operations Research in India has been mainly that of Linear Programming problems. With the exception of the Government and textile industries, applications of Operations Research
in other industries have been more or less equally
distributed. Operations Research techniques are efficiently
used in Agricultural sector to study the optimum use of
water and land management to obtain the maximum net returns.

5.4 "Operation Research and Its meaning:
(Nature of O.R.)

Operations Research can be considered as a source
to other new sciences. The word 'Operations' may be
defined as some action that we apply to some problems or
hypotheses and the word 'Research' is an organised process
of seeking out facts about the same. In fact, it is very
difficult to define O.R., mainly because of the fact that
its boundaries are not clearly marked, O.R. has been vari-
ously described as the "Science of use" , "quantita-
tive common sense", "Scientific approach to decision
making problems", etc. But only a few are commonly used
and widely accepted, namely,

(i) "O.R. is the art of giving bad answers to problems
    which otherwise have worse answers" , . . . . . T.L.Saaty

(ii) "O.R. is a scientific method of providing executive
departments with a quantitative basis for decisions under
their control" --- P.M. Morse and G.E.Kimball.
(iii) "O.R. is a application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solutions to the problem." ——— Churchman, Ackoff and Arnoff.

(iv) "O.R. is applied decision theory. It uses any scientific, mathematical, or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problems ",


(v) "O.R. is the attack of modern science on problems of likelihood that arise in the management and control of men and machines, materials and money in their natural environment, its special technique is to invent a strategy of control by measuring, comparing and predicting probable behaviour through a scientific model of a situation."

——— Kantiswarup, P.K. Gupta and Manmohan.
5.5 **SCOPE OF O.R.**

Operations Research is mainly concerned with the techniques of applying scientific knowledge, besides the development of science. It provides an understanding which gives the expert/manager new insights and capabilities to determine better solutions in his decision making problems, with great speed, competence and confidence. Following are the major areas of research that have proved amenable to the particular technique of Operations Research.

1. O.R. is useful to the Directing Authority in deciding optimum allocation of various limited resources such as men, machines, material, time, money, etc. for achieving the optimum goal.

2. O.R. is useful to production specialist in

   (i) designing, selecting and locating sites,

   (ii) determining the number and size,

   (iii) scheduling and sequencing the production runs by proper allocation of machines, and

   (iv) calculating the optimum production.
(3) O.R. is useful to the Marketing Manager/Executive in determining:

(i) how to buy, how often to buy and what to buy at the minimum possible cost;

(ii) distribution points to sell the products and the choice of the customers.

(iii) Minimum per unit sale price.

(iv) the customers preference relating to the size, colour, packaging, etc. for various products and the size of the stock to meet the future demand and

(v) Choice of different media of advertising.

(4) O.R. is useful to the Personnel Administrator in finding out:

(i) Skilled person at minimum cost

(ii) the number of persons to be maintained on full time basis in a variable workload, like freight handling, etc. and

(iii) the optimum manner of sequencing personnel to a variety of jobs.
(5) O.R. is useful to the Financial Controller to:
   (i) find out a profit plan for the company
   (ii) determine the optimum replacement policies, and (iii) find out the long-range capital requirements as well as the ways and means to generate these requirements.

   Apart from the above few cited applications, it may not be surprising to say that O.R. techniques are used in almost every walk of life, wherever a decision is sought.

5.6 FEATURES OF O.R.

   Following are the salient features of O.R.:

   (1) Operation Research is the inter-disciplinary team approach to find out the optimum returns.

   (2) O.R. uses techniques of scientific research to arrive at the optimum solution.

   (3) O.R. emphasises on the overall approach to the system i.e., all the aspects of the problem under consideration.

   (4) O.R. tries to optimize the total output by maximising the profit and minimising the loss or cost.

   (5) O.R. gives only bad answers to the problems where otherwise worse could be given, i.e. it cannot give perfect answers to the problems. Thus O.R. improves only the quality of the solution.
5.7 Operations Research in Decision-Making:  

"Operations Research is a tool employed to increase the effectiveness of managerial decisions as an objective supplement to the subjective feeling of the decision-maker. O.R. may be regarded as a tool that enables the decision-maker to be objective in creating alternatives and choosing an alternative which is best from among these. O.R. is an aid for decision maker in his creative and judicious role.

(i) Better Decisions:  
O.R. models, frequently yield actions that do improve on intuitive decision-making. A situation may be complex so that the human mind can never hope to assimilate all the significant factors without the aid of O.R. guided computer analysis.

(ii) Better Co-ordination: Same times operations research has been instrumental in bringing order out of chaos. For instance, an O.R. oriented planning model becomes a vehicle for co-ordinating marketing decisions within the limitations imposed on manufacturing capabilities.
(iii) **Better Control:** The managements of large organizations recognize that it is extremely costly to require continuous executive supervision over routine decisions. An O.R. approach, thereby gained new freedom to the executives to devote their attention to more pressing matters. The most frequently adopted application in this category deals with production scheduling and inventory replenishment.

(iv) **Better System:**

Often, an O.R. study is initiated to analyze a particular decision problem, such as whether to open a new warehouse. Afterwards the approach is further developed into a system to be employed repeatedly. Thus, the cost of undertaking the first application may produce benefits.

**Operations Economy:**

O.R. is a problem solving and a decision making science. Whenever, we have conflicts, uncertainty and complexity in any situation, O.R. can help in the end to reduce costs and improve profits and effect substantial "Operation Economy" O.R. provides scientific decision rules for setting the parameters and reducing acquisition costs, inventory carrying costs and shortage costs considerably. The
The technique of linear programming helps in maximising profit or minimising costs in product mix, advertising mix, transportation, assignment problems etc. Once the old approach of management by intuition is buried, a scientific approach to decision making is bound to help.

5.8 Operations Research Models:

"Model building is the essence of the Operations Research approach".

Classification of O.R. models is a subjective problem. Some of the basic types of O.R. models are given as follows:

(1) Iconic (Physical) Models:

These are pictorial representation of real systems and have the appearance of the real thing. These kinds of models are called "Iconic" because they are "look-alike" items to understand and interpret. An iconic model is said to be 'scaled-down' or 'scaled-up' according as the dimensions of the model are smaller or greater than those of the real item. For instance in biology the structure of a cell may be illustrated by an enlarged (scaled-up) iconic model for teaching purposes. Iconic models are easy to observe, build and describe, but are difficult to manipulate and not very useful for the purpose of prediction. Commonly these models represent a static event.
(2) **Analogue Models:**

These models are more abstract than the iconic ones for there is no 'look-alike', correspondance between those models and real items. They are built by utilizing one set of properties to represent another set of properties. Graphs and maps in various colours are analogue models in which different colours correspond to different characteristics, Demand curves, flow charts in production control and frequency curves in Statistics are analogue models of the behaviour of events. Graphs of time series, stock-market changes are other examples of analogue models. These models are easier to manipulate and can present dynamic situations.

(3) **Mathematical (symbolic) Models** are most abstract in nature, they employ a set of mathematical symbols to represent the component of the real system. These models are most general and precise. However, it is not always possible to depict a real systems in Mathematical formulation, some times it is easier to use mathematical symbols for describing the relationship of the components and some times an analogue model may express the pattern of this relationship in a better way.
Formulating the Problem:

The initial phase of the O.R. study involves the formulation of the problem in an appropriate form. This clearly yields a statement of the problem's elements that include the controllable variables, the uncontrollable parameters, the restrictions or constraints on the variables, and the objectives for defining a good or improved solution.

Construction of Model:

The second phase of the investigation is concerned with the choice of a proper data inputs and the design of the appropriate information output. It requires identification of both static and dynamic structural elements and the representation of inter-relationship among the elements in terms of Mathematical formulae.

The mathematical model should include mainly the following three basic sets of elements:

(i) Decision Variables and Parameters:

Decision variables are those unknowns that are to be determined from the solution of the problem, where as parameters are the given uncontrollable variables of the model.
(ii) **Constraints or Restrictions:**

The model must include constraints in order to account for the physical limitations of the system. Let $X_1, X_2$ denote the number of produce of type $T_1$ and $T_2$ respectively per day and let $a_1$ and $a_2$ be their respective unit cost of production (parameters). If the total budget of the producer allows a maximum of Rs.$r$ to be spent per day, then the corresponding constraint is $a_1X_1 + a_2X_2 \leq r$.

(iii) **Objective Function:**

The model must also include an objective function which defines the measure of effectiveness of the system as a mathematical function of its decision variables. Objective function acts as an indicator for the achievement of the optimum solution to the model and thus a poor formulation of the objective function can only lead to a poor solution to the problem.

Let the decision variables $X_i$, $i=1,2,...,n$ which will optimize.
the objective function

\[ Z = f(x_1, x_2, \ldots, x_n) \]

subject to the constraints:

\[ g_j(x_1, x_2, \ldots, x_m) \leq b_j \quad j = 1, 2, \ldots, m \]

where the function \( f \) is a objective function, while \( g_j \leq b_j \) represents the \( j \)th constraint, where \( b_j \) is a constant.

**Deriving the solution:**

The third phase of the study deals with the mathematical calculations for obtaining the solution to the model. Frequently, a solution of the model means those values of the decision variables that optimize one of the objectives and give permissible levels of performance on any other of the objectives.

**Updating the Model:**

The fourth phase of the study involves checking the validity of the model used. A model may be said to be valid if it can give a reliable prediction of the system's performance.
Controlling the Solution:

The fifth phase of the study establishes control over the solution by proper feedback to the information variables which deviated significantly. As soon as one or more of the controlled variables change significantly, the solution goes out of control. In such situations the model may accordingly be modified.

Implementing the Findings:

The sixth and final phase of the study deals with the implementation of the tested results of the model. This would basically involve a careful explanation of the solution to be adopted and its relationship with the operating realities. This stage of O.R. investigation is executed primarily through the co-operation of both the O.R. experts and those who are responsible for managing and operating the system. Implementation is the corner stone of any decision making. Implementation is culture bound.