CHAPTER - I

INTRODUCTION
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1.1 Introduction:

Secretary problem deals with the sequential decision procedure. Let there be N candidates appearing for an interview for a particular post. The interviewer takes the interviews of the candidates one by one. The ordering of the candidates is random. All N! possible orders are equally likely. Further, the interviewer has to select only one candidate from those N available candidates. The interviewer must decide immediately after the interview, whether to accept the present candidate or not. If that candidate is not accepted the next is called for an interview. When, one candidate is accepted, the process of interview stops, resulting in the selection of that candidate. If all (N - 1) candidates are rejected then, Nth candidate i.e. the last one must be selected. Like this the observer has to take decision sequentially. Here the interviewer knows the relative ranking of only the present and the previously observed candidate at each stage. The main aim is to give a stopping rule or a selection procedure so that, the procedure results in the selection of a 'suitable' candidate.

In this problem observer is never able to go back and choose a previously presented candidate, which in retrospect, turns out to be the
best. He clearly has to balance the danger of stopping too soon and accepting an apparently desirable candidate when an even better one might be still to come against that of going on too long and finding that, the best candidate was rejected earlier on. The golden mean is to obtain the optimal stopping with the best candidate.

According to Dimitris, A Sardelis,(1999) the importance of this optimal stopping is that, it provides an artificial idealized simulation of sequential decision process. Indeed every day life reveals that, almost all successful decisions are preceded by learning period during which one observes, classifies and ranks experiences.

Secretary problem was studied by the host of researchers from different angles and using different approaches, which is discussed in section 2.2.

The consensus among the many who have worked on this problem is that, the observer's best strategy is to let a certain fraction of total number of candidates passed and take the next one who has a score better than that any of the ones seen thus far. Hence, out of N candidates before the observer, he inspects first r candidates without selecting any. Select the first candidate there after which is better than the best of first r candidates. If none of the first (N-1) candidates is selected then select the last one.
1.2 Origin of Secretary Problem:

The idea of secretary problem originated with the game called 'Googol' which is as follows,

On a table place some slips of papers on which different positive numbers are written. The numbers may range from 1 to a number, which is the size of googol. These slips are turned face down and shuffled over a top of a table. Turn the slips to face up at a time. The aim of the player is to stop turning when the number appeared is guessed to be the largest amongst all. The player cannot go back and pick up a previously turned slip. Also, there is one more condition that, if the player turns over all the slips on the table, then he must pick up the last one turned. N, the total number of slips is known. The secretary problem is viewed as a modification of googol with the modification that, the numbers written on the slip are not simply positive numbers but in fact are the relative ranks of the persons to be interviewed.

The originator of the secretary problem is unknown. Mosteller was told the problem in 1955 by Gleason who claimed to have heard it from another. The secretary problem, also known as the "Sultan's Dowry Problem" is believed to have been first stated in Martin Gardner's Mathematical recreations column in the February 1960 issue of the 'The Scientific American' and because of its practical implications has created
a subject of study of its own in the field of Management Science. Gardner (1960) attributed the problem to Fox and Marnie-Bissinger and Siegel (1963) posed the special case with $N = 1000$ and the solution was given by Bosch (1964) and by twelve other people. Lindley (1961) for the first time gave solution to the standard problem.

1.3 Organization of the thesis:

The second chapter of the thesis contains review of other developments of secretary problem, along with the historical background of the problem. Introduction of some important versions of the secretary problem namely original secretary problem, secretary problem with measurable character and two tailed secretary problem, along with the joint and marginal distribution of two well defined random variables $X$ and $Y$ are also discussed in the same chapter.

In the third chapter we introduced two tailed secretary problem with measurable character. A well defined parameter $t$ for this version governs the joint probability distribution of $X$ and $Y$ and hence, consequently five joint probability distributions of $X$ and $Y$ for five different cases are obtained with different ranges of $t$.

The fourth chapter contains the rank analysis of TTSP with measurable character, which includes the derivation of marginal distribution of random variable $X$. Obviously there are five different
probability distributions of $X$ and $Y$ with respect to different values of $t$. This marginal distribution of $X$ is important in the sense that, it throws light on the probability of the best selection i.e. $P(N)$. Many important results connected with $P(N)$ are stated. Here role of $t$ in the analysis of $P(N)$ is studied.

The fifth chapter consists of stopping time analysis of TTSP with measurable character, which constitutes marginal distribution of $Y$ in all five cases with respect to $t$. $E(Y)$ is discussed in detail with some interesting results.

In the sixth chapter we carried out comparisons of various versions of the secretary problem. TTSP with measurable character is compared with TTSP without measurable character. Also, TTSP with measurable character is compared with the original secretary problem with measurable character.

Conclusion and prominent applications are stated here. The last section of the chapter includes some open problems. All open problems connected with the results related with secretary problem in the thesis are presented here.

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