Chapter 1

Introduction

1.1 Optical control technique for diode lasers

This thesis is concerned about the study and coherent control of the complex dynamical behavior of diode lasers subjected to external optical perturbation that is able to considerably affect the behavior and characteristics of the lasers. The laser is a convenient and versatile tool for reproducing and understanding the nonlinear dynamics observed in other fields. The fundamental nonlinear character of interaction between the electric field and the active medium plays an important role in the onset of these complex dynamics. The increase in broadband deterministic noise in the domain of the complex dynamics has led to seeking ways to suppress or control these to improve laser performance in applications. However, control perturbations can also alter the dynamics of the system in ways which can serve as a test of those dynamics and lead to new insights. We thus consider ‘control’ as a general phenomenon that alters the laser dynamics in some predictable manner, rather than simply converting unstable behavior to steady-state emission. Several control methods have been developed and studied, some of which suppress the unstable dynamics entirely, and others which instead seek to modify or regulate the dynamics.

Diode lasers have the same three ingredients like other lasers, namely, a) an active medium hosting stimulated emission, b) a pumping source responsible of creating the necessary population inversion, and c) a cavity to provide a feedback and frequency selection mechanism. But the working principle is different from other lasers because usually a p-type semiconductor material (active layer) is placed between two passive sections (one n-doped, the other p-doped). The gain is provided by the application of forward-biased electric field, and the feedback is provided from the semiconductor facets. Contrary to many other laser types, in diode lasers the lasing transition takes place between two energy bands, which leads to a shift of the refractive index dispersion curve relative to the gain curve, because gain changes with carrier density and correspondingly, refractive index (which depends on carrier density)
changes around the lasing frequency. The coupling between carrier concentration induced variation of real and imaginary parts of the refractive index commonly describes the phase-amplitude coupling factor $\alpha$. Because of this coupling factor, any fluctuation in the field amplitude is coupled to phase fluctuations, leading to additional broadening of linewidth. That is why this phase-amplitude coupling factor is called the linewidth enhancement factor. The relation between the different (material, field, and carrier density) decay rates ($\gamma_P \gg \gamma_E, \gamma_N$ directly related to the possible dynamics present in the laser) and the non-zero phase amplitude coupling factor make the diode laser different from other lasers. Since in a semiconductor the relaxation time for the polarization is much shorter than that for the electric field and carrier density decay times, the adiabatic elimination of the material polarization allows the system to be described by two first-order ordinary differential equations, which are known as the semiconductor rate equations. These equations describe the temporal evolution of the electric field and carrier density inside the laser cavity and are found to adequately predict many aspects of diode laser behavior including the output dynamics.

Now-a-days diode lasers are widely used in photonics technologies because of their compactness, low cost, small size and easy to modulate. The GHz-scale frequencies associated with diode laser instabilities require special and difficult high-speed techniques that have not been applied with significant success in experimental systems. The pure optical control technique offers a means of manipulating the laser system, either for control and regulation, or as a basis for creating or investigating fundamental dynamic phenomena. Diode lasers have some drawbacks - these are very sensitive to external perturbation (optical feedback or optical injection), and have large linewidths because of multimode operation. The problem of coherent control of complex dynamics is of long standing for diode lasers. Different techniques for control the diode laser output are being used including optoelectronic feedback control, second optical feedback. The drawback of existing optoelectronic feedback technique is that the optical phase does not play any role due to insensitivity of the photo-detectors to the phase of electric field, low bandwidth, low response time limited by electron velocity and coupling strength is limited by lasers own output. A second optical feedback technique has been used to suppress the low-frequency fluctuations produced in an external cavity by the first optical feedback [1, 2, 3, 4]. The use of second external cavity to suppress the un-
stable dynamics created by first cavity and in this technique the controlling signal depends on the system's dynamical state. It is typically used to generate the control perturbation after detecting the state of the system. The drawback of this technique is that it gives very low output power and does not give any particular route to reach the stable state and also very difficult to implement in real experiment because of mode matching problem. Dynamic targeting [5] is other control technique involve making clear adjustments to the parameters of the system to create a desired change of dynamics, rather than sensing the system and responding accordingly. Dynamic targeting seek to place the system in an existing or newly-created stable state, and keep it there by simultaneous adjustment of system parameters. The control parameter is manipulated in such a way that the condition of maximum gain mode is always fulfilled, otherwise the laser operates alternately between stable, high-power operation and strongly fluctuating states with lower average intensity [6]. Another technique of modifying system parameters for control purposes has been investigated by Heil [7, 8], who stabilized laser system by reducing the linewidth enhancement factor $\alpha$. The lowering $\alpha$ result in increasing the average time duration over which stable emission was observed via enlarging the basin of attraction of maximum gain mode and by making them more distant from unstable mode. However, complete stabilization was not attained in this system, even at its minimum attainable $\alpha$. In order to improve the diode laser characteristic in terms of narrowing of linewidth and wavelength tunability, optical feedback in an external-cavity is used. But along with the improvement of laser characteristic by external cavity, the diode laser exhibits a large variety of complex dynamical behavior, such as periodic and quasi-periodic oscillations, chaos, coherence collapse and low frequency fluctuations or pulsations.

In many dynamical systems, chaotic behaviour is found only in a subset of their phase space. The cases of most interest arise when the chaotic behaviour takes place on an attractor, since then a large set of initial conditions will lead to orbits that converge to this chaotic region. On the one hand, such dynamical instability in a laser may disturb the application where one needs constant output power from the laser, and on the other hand, these dynamics can be used optical chaotic communication using chaos synchronization. One needs to understand and control these dynamics for diode laser stabilization technologies, high-speed long-haul communication and for various applications in photonics technology.
In order to overcome the problems mentioned above in different techniques, we use pure optical technique to control the laser output via optical injection because the optical phase is the crucial parameter for phase locked lasers to provide high as well as ultra-stable power in well collimated (diffraction limited) coherent beam in a mutually-coupled diode laser system. Our technique is attractive because it does not require any modification of laser diode parameter like $\alpha$ or any externally fast electronic servo system. Mutually coupled diode laser systems have attracted much attention not only because of potential application in modern technologies but also as the fundamental tool for study and control of complex dynamics of nonlinear coupled oscillators. The existence of time delay in the coupling makes the system infinite dimensional, which can provide many advantages including laser linewidth reduction [9, 10], suppression of mode hopping, reduction of mode-partition noise [11] and a variety of functionalities that might be useful but not achievable through the free-running lasers [12, 13, 14, 15, 16, 17]. The first optical injection locking was demonstrated by Stover and Steier using two He-Ne lasers in 1966 [18]. The physical process behind the optical injection locking is that the externally injected signal from the master laser is regeneratively amplified inside the slave laser diode cavity and produces an amplified output power at the injected frequency. The amplification of this signal increases as it moves closer to the free-running frequency of the solitary diode (maximum of the gain curve). If the frequency detuning between the slave and master lasers is made small enough, or the injected signal power large enough, then the amplified injected signal begins to saturate the laser gain, and the free-running laser oscillation is turned off, leaving only the amplified injected signal. Such a state is known as injection locking; the slave laser is forced to oscillate at the injected signal frequency and is locked to its phase. The carrier density in the active region of an injection-locked laser is reduced. This leads to an increase in the index of refraction, which gives rise to a decrease in the cavity resonance frequency (increase in wavelength). This leads to locking that prefers the longer wavelengths and the spontaneous emission of the slave laser is significantly suppressed throughout the locking range, resulting in RIN reduction of the injection-locked laser. Instead of single diode laser we take two diode lasers and both lasers are driven to each other. This sort of mutual coupling between the diode lasers via optical injection has some interesting points in terms of enhancement of modulation.
bandwidth, quantum phase noise reduction, fascinating collective behaviors such as phase locking or phase synchronization and amplitude death. The time delay in coupling could be the stabilizing factor for control of complex nonlinear dynamics. New opportunity also arises due to resonant or non resonant coupling between the lasers for giving the in-phase or anti-phase amplitude death state. For a coupled diode laser system driven by mutual delayed optical injection, the result of adjustment of rhythm among collective behavior is characterized by phase locking. The another fascinating behavior where both laser oscillators drive each other to stop the oscillatory behavior is called amplitude death.

In this thesis, we study the particular route to ultimate amplitude death state and stable phase locking phenomena in a mutually delay-coupled diode laser system. We also give the global view of different dynamical regime in a coupling strength $\eta$ and delay time $\tau$ parameter space. We vary the coupling strength from very low to very high values and check how many dynamical states one encounters on the way to reach the amplitude death state. We have been able to see the phase-flip bifurcation in the coupled diode laser system. In this pure optical control technique, we also encounter robust phase locking between the diode lasers for ultimate laser power stabilization and find a strange bifurcation window between the amplitude death islands.

The cause of amplitude death response of lasers output is the existence of resonant or non-resonant coupling between the lasers, which leads to interference between the laser light and the injected light that controls the laser output. In non-resonant coupling, the injected light does not meet the round trip phase condition and depletes the carrier density and hence increases the refractive index. Only few carriers take a part in amplification because the injected light makes a single round trip before being lost. The importance of amplitude death arises from the fact that it gives rise to an ultimate stable state of no power fluctuations in a coupled laser system. One of the main reasons to work on injection locking is to investigate whether the strange bifurcation window and death island state can be utilized to control the complex dynamics and thereby lock. Such system can have numerous applications in gravitational wave detection and optical parametric oscillators. We study the phase locking via visibility and cross-correlation measures. It is worth noting that the coherent coupling is determined by optical phase locking as electric field fluctuates very fast in comparison to
intensity fluctuations. The phase locking between the intensity oscillations does not mean phase locking in the fields.

The Chapter is organized as follows. In Sec. 1.2 we give the working principle of diode lasers along with some laser characteristics like threshold condition, phase noise, linewidth enhancement factor. In Sec. 1.3, we introduce the optical feedback scheme. Section 1.4 gives the solution of Lang-Kobayashi model and the consequence of coherent optical feedback as low frequency fluctuation (LFF), coherence collapse (CC). In Sec. 1.5, we give the mechanism of optical injection locking in coupled diode laser system with some collective behavior like phase locking, amplitude death etc. In Sec. 1.6, we mention some applications of two coupled diode lasers. And finally in Sec. 1.7, we present an outline of the thesis.

1.2 Working principle of diode lasers

Semiconductors are based on p-n junctions of semiconductor materials having the separation of valence and conduction bands on the order of a few eV, which is comparable to energy of photons in the visible spectrum. A range of materials have been developed to produce wavelengths over a wide spectrum from 0.1 \( \mu \)m to 3 \( \mu \)m. In semiconductors the gain is achieved via carriers, which consist of an electron hole pair, where the electron lies in bottom of the conduction band and the hole (absence of electron) resides near the top of the valence band. In thermodynamic equilibrium, the electrons and holes distributed over a quasi-continuum range of energies described by Fermi-Dirac statistics and may be described by a density of states. The carriers serve as a source for stimulated emission, as photons may induce radiative recombination of the electron and hole producing a photon with identical characteristics. However, the carriers may spontaneously recombine to form photons that do not share characteristics with the coherent state. In addition, absorption of light in the gain medium may produce carriers and various mechanisms for nonradiative recombination due to material defects are present as well. Nonradiative depletion of carriers also occurs in Auger recombination, where an electron and a hole recombine but the energy of this transition is used to further excite an electron.

It is clear that with many competing recombination and absorption processes in the gain medium, the rate of stimulated emission is extremely small. Many carriers must be
Laser action arises when a sufficient density of carriers are injected into a gain medium from an electrical contact. For very small injection currents, the light is primarily produced by spontaneous emission, as a majority of the photons are absorbed before they may initiate stimulated emission with remaining carriers. When the injection current is tuned high enough such that enough light is generated to overcome absorption, then the material is transparent. Upon a further increase of injected charge carriers, stimulated emission may begin but the coherent light produced is attenuated as it travels through the material. Dissipation processes in the gain medium and losses of photons out of the injected region must be compensated by a further increase of the injection current. Once these are overcome, lasing threshold is reached and stimulated emission will dominate the light output processes producing a coherent light source of light as shown in Fig. 1.1. Further increases of the injection current at this point translate into a proportional increase of the output intensity of the laser (losses along the direction of propagation), as the number of carriers in the medium are clamped to the amount produced at the lasing threshold.

As discussed above, the electrons and holes must be densely concentrated into a small region to generate enough stimulated transitions for sufficient light output. This localization is commonly achieved in practice with the use of a heterostructure. This arrangement is depicted in Fig. 1.2. The region where gain is achieved, the active layer, is sandwiched
between two semiconductor materials which each possess a wider band-gap. This feature more efficiently confines charge carriers injected into the active layer when forward bias is applied so the lasing threshold may be reached for lower levels of the injection current. Furthermore, the potential barrier created by the band-gap difference allows the thickness of the active layer to be tightly controlled. Standing waves in the cavity form the longitudinal modes available for lasing emission. The reflectivity of the facets is generally only around 30%, much lower than most other lasing mediums with mirrors reflecting near all of the light in the gain medium. The reflectivity of one facet is often coated with a dielectric material to establish a single output for the light. The cladding layers confining the carriers are chosen to have a lower index of refraction then the active layer which serves to concentrate the optical field in the active medium. The operating principles of a diode laser are similar to those of any other laser system. The device consists of a gain medium with feedback providing the gain is sufficient to overcome the losses in the system then lasing can occur. Current semiconductor devices are fabricated with heterostructure that allow room-temperature low-threshold cw operation. The heterostructure widths can be chosen to produce either a bulk, or a quantum-well gain medium. A simple heterostructure device, the double heterostructure, which comprises a narrow bandgap, usually $p$-type, semiconductor material (the active layer) that is placed between two passive sections (one $n$-doped, the other $p$-doped), is illustrated schematically in Fig. 1.3. Gain is provided by the application of a forward biased electric
field and feedback is provided from the semiconductor facets. The facets have an uncoated reflectance of 32% (for GaAs/AlGaAs devices) due to Fresnel reflections, so high reflectance or anti-reflectance coatings are often added to improve performance. The forward biased current results in an injection of electrons (from the n-doped layer) and holes (from the p-doped layer) into the active layer. These excess carriers are confined to the active layer by the bandgap differential. They can then recombine via nonradiative recombination, stimulated emission (lasing) or spontaneous emission.

1.2-1 Threshold condition and rate equations

The diode laser threshold gain can be estimated from the unity round trip condition for the intensity in a Fabry-Perot cavity

$$r_1 r_2 e^{-2l(\alpha_m-\alpha_m^l)} = 1$$  \hspace{1cm} (1.1)

where $r_1$ and $r_2$ are the power reflectivity of the front and rear facets of the laser diode, respectively. The laser cavity length is given by $l$ and $\alpha_m$ account for the internal optical losses. Solving Eq. (1.1) for the threshold gain per unit length gives

$$g_{th} = \alpha_m - \frac{\ln(r_1 r_2)}{2l}.$$  \hspace{1cm} (1.2)

The gain per unit length is converted to gain per unit time $G$ by multiplying with $v_g = \frac{c}{n}$, with $c$ being the speed of light and $n$ denoting the real part of the refractive index.
Dynamic behavior, such as the modulation characteristics of diode lasers are commonly modeled using the following rate equations for the cavity photon numbers $P(t) = |E(t)|^2$ as $E(t) = \sqrt{P(t)}e^{i\phi(t)}$, the phase $\phi$, and the carrier density $N$ [19, 9].

\[
\frac{dP}{dt} = (G(N) - \frac{1}{\tau_p})P(t) + R_{sp} + F_s(t),
\]

\[
\frac{d\phi}{dt} = \frac{\alpha}{2}[G(N) - \frac{1}{\tau_p}] + F_\phi(t),
\]

\[
\frac{dN}{dt} = \frac{I(t)}{e} - \frac{N(t)}{\tau_N} - G(N)P(t) + F_N(t),
\]

where $I(t)$ denotes the laser injection current, $e$ is the electron charge, $\tau_N$ is the carrier lifetime, $\tau_p$ is the photon lifetime, $\alpha$ is the linewidth enhancement factor, and $G(N)$ is the modal gain per unit time, $R_{sp}$ is the spontaneous emission rate and $F_s(t)$, $F_\phi(t)$ and $F_N(t)$ are the Langevin noise term [20] for photon numbers, phase, carrier density, respectively. Spectral hole burning and other effects leading to gain compression are neglected and the gain is assumed to be a linear function of carrier density $N$.

\[
G(N) = G_0(N - N_{th}),
\]

\[
G_0(N - N_{th}) = A_g\Gamma(N - N_{th}),
\]

where $G_0$ is the gain coefficient and $A_g$ is the differential gain. The Confinement factor $\Gamma$ describes the overlapping between the lasing mode and the active region. The Carrier number for transparency (or zero gain) is denoted by $N_{th}$. The photon lifetime $\tau_p$ depends on the cavity losses as

\[
\tau_p = \frac{n}{c} \left[ \alpha_m - \frac{\ln(\tau_1\tau_2)}{2l} \right]^{-1}.
\]

Using the relation in Eq. (1.6) and Eq. (1.7) together with Eq. (1.8), we can write the gain at threshold as

\[
G(N_{th}) = \frac{1}{\tau_p},
\]

which allows us to express the rate equations in the following form:

\[
\frac{dP}{dt} = G_0(N(t) - N_{th})P(t),
\]

\[
\frac{d\phi}{dt} = \frac{\alpha}{2} G_0(N(t) - N_{th}),
\]

\[
\frac{dN}{dt} = \frac{I(t)}{e} - \frac{N(t)}{\tau_N} - G_0(N(t) - N_{th}).
\]
The above equations have been derived following a number of simplifying assumptions. First, the slowly varying envelope approximation (SVEA) is employed. This relies on the difference in the time scale at which the electric field changes in time and space. It is considered that the electric field amplitude $E(t)$ varies slowly compared to the optical carrier frequency $\omega$. In the second order differential equation for the electric field, derived from the electromagnetic wave equation, the second derivatives can be discarded, as they are very small compared with the first order terms $\frac{d^2}{dt^2} \ll \frac{d}{dt} \ll \omega^2$. Secondly, the polarization has been adiabatically eliminated. This is possible because the interband polarization adjust to changes in the electron and hole distributions (carrier density fluctuations) on a sub-picoseconds time scale. This is significantly faster than the photon life time. Therefore, as the temporal variation in the carrier density and field amplitude are followed adiabatically by the polarization, the polarization term in the wave equations can be ignored. This approximation is no longer valid if the photon life time becomes comparable to the intraband scattering time, or if the electric field varies substantially within the cavity round trip time.

1.2-2 Laser phase noise and linewidth

The fundamental or inherent linewidth is caused essentially by altering of phase due to the fluctuations caused by spontaneous emission. This is shown in Fig. 1.4, where the electric field of single mode laser is given by

$$E(t) = [E_0 + \Delta E(t)]e^{i(2\pi \nu t + \Delta \phi(t))},$$  \hspace{1cm} (1.13)

where $E_0$ being the nominal amplitude of the field and $\nu$ denoting the optical frequency. $\Delta \phi$ and $\Delta E$ are instantaneous changes of the phase and amplitude caused by a spontaneous emission event. Due to the incoherence of spontaneous emission its phase is random, which results in diffusion of phase angle from its original value. It can be shown that the phase has a Gaussian probability distribution and the Fourier transform of the autocorrelation of the resulting field produces a power spectrum with a Lorentzian line shape. The corresponding linewidth is given by the modified Schawlow-Townes formula [9].

$$\Delta \nu_{ST} = \frac{\pi \hbar \nu (\Delta \nu_c)^2}{P_i}.$$ \hspace{1cm} (1.14)

In Eq. (1.14), $\Delta \nu_c$ is the cold cavity linewidth of the laser, $P_i$ is the intra-cavity power of the
lasing mode. From the above relations we can see that the linewidth is inversely proportional to the \( P_i \). This is due to the fact that above threshold any increase in excitation lead to an increase in stimulated emission, while the spontaneous emission rate remains essentially unchanged. However, in diode lasers the lasing transition takes place between two energy bands, and the lower band is generally not empty. As a result, the laser photons can be absorbed and again re-emitted, possibly in spontaneous emission, thus leading to additional phase fluctuations. The corresponding increase in line broadening is by the spontaneous emission factor \( n_s \), which is the ratio of spontaneous emission rate per mode to the stimulated emission rate per laser photon. In diode lasers, the amplitude fluctuations are coupled to phase fluctuations. For this reason, any fluctuation of field amplitude leads to additional broadening of the linewidth. This effect is taken into account by multiplying the linewidth by \( 1 + \alpha^2 \), where \( \alpha \) is the linewidth enhancement factor. The perturbations of the field amplitude are restored to the steady-state value via damped relaxation oscillations that have a frequency typically in the order of few gigahertz. After including the aforementioned effect to the modified Schawlow-Townes formula, the fundamental linewidth of diode laser is written as

\[
\Delta \nu_0 = \frac{\pi h \nu (\Delta \nu)^2}{P_i} n_s (1 + \alpha^2).
\] (1.15)
The cold cavity linewidth $\Delta \nu_c$ depends on the cavity quality factor $Q$, and is for a diode laser expressed as

$$\Delta \nu_c = \frac{2\pi \nu}{Q}$$  \hspace{1cm} (1.16)

$$= \frac{c}{2\pi n_d l} (\alpha_m l - \ln \sqrt{R_1 R_2}),$$  \hspace{1cm} (1.17)

where $n_d$ is the refractive index of the cavity medium. The quality factor of normal solitary diode laser is relatively poor due to short cavity and low reflectance of the output mirror. The cavity quality factor can be improved, and the linewidth significantly reduced, by employing the laser diode in an external cavity.

### 1.2-3 Linewidth enhancement factor

In most of the lasers, the laser transition occurs between two discrete energy levels and the gain curve is symmetric around its maximum value. The refractive index dispersion curve is also symmetric and has a zero at the frequency of the maximum gain, as is shown in Fig. 1.5 a. The gain (imaginary part of the refractive index) and (the real part of) the refractive index are coupled together via Kramer’s-Kronig dispersion relation [21]. Contrary to many other laser types, in diode laser the lasing transition takes place between two energy band, which leads to a shift of refractive index dispersion curve relative to the gain curve (Fig. 1.5 b). This is because of the asymmetry of the gain curve. The asymmetry is for the reason that the gain material is transparent at long wavelengths where the photon energy is below the bandgap, while higher energy photons are strongly absorbed. As the gain changes with carrier density, also the refractive index around the lasing frequency is dependent on the carrier density. The coupling between carrier-concentration induced variations of the real and imaginary part of the relative index is commonly described using the linewidth enhancement factor $\alpha$ which is defined as [22]

$$\alpha = -2k \frac{dn}{dN} / \frac{dg}{dN},$$  \hspace{1cm} (1.18)

where $k$ is the free-space wave vector, $n$ is the real part of refractive index, and $g$ is the gain per unit length. The linewidth enhancement factor is typically between 2 and 10 for semiconductors. This parameter is important not only because it has a significant effect on spectral properties of solitary diode laser but also because it has major contribution on
how the laser behaves under optical feedback and optical injection. In next section, we will discuss about the dynamical behavior of single diode laser when subjected to self optical feedback.

1.3 Diode laser with optical feedback: Lang-Kobayashi model

The low quality factor of diode lasers makes them extremely sensitive to optical feedback. This sensitivity is further enhanced due to the flatness of the gain curve. The actual effect of the feedback depends on phase, magnitude, and polarization of the light fed back to the laser, and also on the feedback distance (i.e., the delay time). The rate equations that describe the solitary diode laser are modified to account for optical feedback by including feedback terms in both the photon number and phase equations. These modified rate equations are known as the Lang-Kobayashi rate equations: The rate equations that describe the solitary diode laser are modified to account for optical feedback by including feedback terms in both the photon number and phase equations. The single-mode Lang-Kobayashi equation for the complex electric field in a compound cavity consisting of the active gain medium (with internal round-trip time $\tau_{\text{int}}$ and the passive external resonator round-trip time $\tau$) is as
follows:

\[
\frac{d}{dt} E(t)e^{i\omega t} = [i\omega_N(N(t)) + \frac{1}{2}(G(N(t)) - \Gamma)]E(t)e^{i\omega t} + \eta E(t) - \tau) e^{i\omega(t-\tau)}. \tag{1.19}
\]

The slowly-varying envelope of the electric field is modulated rapidly by the solitary emission frequency \(\omega_0\). The first term on the right-hand side reflects the dependence of the longitudinal mode resonant frequency

\[
\omega_N = \frac{N\pi c}{nl} \tag{1.20}
\]
on the carrier number \(N\) which occurs via the refractive index. The integer \(N\) defines which longitudinal mode is exhibited in the cavity, \(c\) is the light speed, and \(l\) is the length of the laser diode cavity. The second term accounts for the balance between the gain amplification \(G(N)\) due to stimulated emission and the attenuation of the light described the cavity decay rate \(\Gamma = \frac{1}{\tau_p}\), where \(\tau_p\) is the lifetime of the photon. The photon losses stem from light exiting through the material layers and the polished facets of the resonant cavity. The rate equation (1.19) is coupled with the evolution of the carrier number

\[
\frac{d}{dt} N(t) = \frac{I}{ed} - \frac{N(t)}{\tau_e} - G(N)|E(t)|^2. \tag{1.21}
\]

The first term represents the injection rate per unit volume of the excited carriers in terms of the injection current density \(J\), the electronic charge \(e\), and the active layer thickness \(d\). The carrier lifetime \(\tau_e\) reflects the decay of the excited states as a result of the spontaneous recombination of electrons and holes as well as nonradiative recombination from defects.
and Auger recombination. The third term accounts for the depletion of the carriers due to stimulated emission. This equation additionally assumes that carrier diffusion is not relevant in describing the dynamics of the compound cavity, a reasonable assumption for the index-guided lasers realized in our experiments.

We now apply a linear approximation for the gain and refractive index to evaluate how changes in the carrier number affect the resonant frequency $\omega_N$ [9]. For small variations $\delta N(t) = (N(t) - N_{th})$ of the carrier number around the solitary lasing threshold $N_{th}$, the refractive index may be described as

$$n(t) = n_{th} + \delta N \frac{\partial n}{\partial N}. \quad (1.22)$$

Plugging this relation into Eq. (1.20) the longitudinal mode frequency assumes the form

$$\omega_N = \omega_{th} - \frac{\omega_{th}}{n_{th}} (\delta N \frac{\partial n}{\partial N}). \quad (1.23)$$

As the gain is similarly dependent on the number of carriers in the active layer, we may approximate the gain near threshold as

$$G(N) = G_{th} + \delta N \frac{\partial G}{\partial N}. \quad (1.24)$$

However, the threshold condition for the solitary laser requires the gain to match the photon losses in the cavity, hence $G_{th} = \Gamma$ and we may express as $\delta N$

$$\delta N = \frac{G(N) - \Gamma}{\frac{\partial G}{\partial N}}. \quad (1.25)$$

When this expression is utilized in Eq. (1.23), then the resonant frequency relative to threshold reads as

$$\omega_N - \omega_{th} = \alpha \frac{G(N) - \Gamma}{2}, \quad (1.26)$$

where $\alpha$ is the phase-amplitude coupling which has important consequences in describing response characteristics and the line-width broadening observed in semiconductor laser dynamics.

In later chapters we utilize a normalized version of the Lang-Kobayashi equations [23]

$$\frac{d}{dt} E(t) = (1 + i\alpha)(N(t)E(t) + \eta E(t - \tau)e^{-i\omega \tau}, \quad (1.27)$$

$$T \frac{dN}{dt} = J - N(t) - (1 + 2N(t))|E(t)|^2. \quad (1.28)$$

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in terms the deviation of the carrier density from threshold. From this representation of the Lang-Kobayashi field equation, it is clear that explicitly connects fluctuations in the amplitude to changes in the phase. As this factor enhances the line-width of the resonant frequency, we will later see that it additionally plays a substantial role in opening up compound cavity modes available to the system in the presence of optical feedback. In this version of the delay-differential equations the parameters are scaled to the photon lifetime $\tau_p$ and the field and carrier number are normalized to remove the differential gain. Here $E(t)$ is the normalized complex optical field and $N(t)$ is the difference in carrier number with respect to solitary lasing conditions. The $\tau$ is the external cavity round trip time and $\omega_0$ is the dimensionless solitary laser frequency; $J$ is the dimensionless pump strength above threshold and $T$ denotes the ratio between the decay time of photons $\tau_p$ in the laser cavity and the carrier recombination time $\tau_c$. In diode lasers the competing timescales are very much shorter. The inversion can change dynamically on nanosecond timescales: $\tau_c$ and the photon lifetime $\tau_p$ in short, relatively low-finesse semiconductor laser cavities is 13 picoseconds, typically. The relaxation oscillations in this case occur at GHz frequencies $f_{ro}$. The relaxation oscillation frequency scales as the square root of the injection current for diode lasers. The relaxation oscillations are damped with a rate $f_d$ which scales as $f_{ro} = \frac{1}{2} \sqrt{\left(\frac{1}{T_c} - \frac{J}{J_{th}}\right)}$.

1.4 Solution of Lang-Kobayashi equations

We assume that the stationary solutions of electric field and carrier density equations can be expressed as $E(t) = E_s e^{i\omega_st}$ and $N(t) = N_s$. Additionally, the optical phase, defined from the slowly-varying envelope of the electric field $E(t) = \sqrt{P(t)} e^{i\phi(t)}$ with amplitude $\sqrt{P(t)}$, is conveniently represented in the analysis by the introduction of the external cavity phase shift $z(t) = (\phi(t) - \phi(t - \tau))$ and the corresponding stationary solution $z_s = \omega_0 \tau$. In the phase space of $E(t)$, $N(t)$, and $\Delta(t)$, the fixed points of the system satisfy

\begin{align*}
z_s &= -\eta \tau \sqrt{1 + \alpha^2} \sin(z_s + \omega \tau + \tan^{-1} \alpha), \\
N_s &= -\eta \cos(z_s + \omega \tau), \\
E^2_s &= \frac{J - N_s - \omega \tau}{1 + 2N_s}.
\end{align*}

\begin{align}
(1.29) & \\
(1.30) & \\
(1.31)
\end{align}
An immediate inspection of the above fixed points clearly shows that the transcendental equation for the external cavity phase shift determines the number and position of available fixed points. The intensity and carrier number follow directly from the solutions of Eqs. (1.31) and (1.30). When \( C = \eta_r \sqrt{1 + \alpha^2} < 1 \), only one solution exists close to the solitary laser frequency. As the feedback strength is increased, new fixed points are created in pairs via a saddle-node bifurcation. The emergence of new fixed points, which may also be generated by increases in the external cavity round-trip time or the linewidth enhancement factor, can be visualized in the graphical solution of Eq. (1.29) shown in Fig. 1.7.

A linear stability analysis of the fixed points [24] reveals that for every pair created, one may be identified as an external cavity mode (ECM) of the laser and the other is an inherently unstable saddle point (antimode). The ECMs shifted from the solitary laser frequency physically represent constructive interference between the external cavity and the optical field, while the antimodes correspond to destructive interference. Upon its inception, each new ECM is stable but suffers a Hopf bifurcation to periodic oscillations (relaxation oscillations become undamped) as the feedback level is incremented. At even higher levels of feedback the attractor present at each existing ECM follows a quasiperiodic route to chaos [25, 26]. Although this is the most common path for the emergence of chaotic emission, a period-doubling route has been observed when the relaxation oscillations matched a harmonic of the external cavity round-trip time.

1.4-1 Consequence of optical feedback

If the amount of feedback is sufficiently low, multiple chaotic attractors coexist at the external cavity modes with separate basins of attraction. For moderate feedback strengths the time-dependent solutions are no longer confined to a local attractor surrounding each ECM and individual attractors begin to merge. The laser emission will transiently move along the unstable chain of attractor ruins associated with the steady-state solutions, and the dynamics is said to exhibit a chaotic itinerary. The antimodes play a substantial role in regulating the individual connections between external cavity modes. The emergence of locally coupled attractor ruins is considered more concretely in Fig. 1.7 for computations of the Lang-Kobayashi equations without the inclusion of additive Langevin noise [27]. For a
low level of feedback, the time series of the external cavity phase shift wanders around a single chaotic attractor. The duration spent near individual attractors will often fall close to a multiple of the external cavity round trip time, but at higher feedback strengths mode-hopping in the light dynamics may occur at a faster time-scale. At higher levels of feedback or in the presence of a long external cavity roundtrip time (1ns) hundreds of locally-coupled attractor ruins may participate in the dynamics. When a particular ECM is visited by the system, the trajectory of the system variables is highly localized around the ECM, but since the system never settles down the resulting dynamical state observed in the system must be typified by terms of the characteristic transitions which occur between the ECMs. The dynamics of these motions is traditionally viewed in the phase space, where the fixed points lie on a tilted ellipse [28]. In this representation the ECMs lie along the lower branch of the ellipse while the antimodes comprise the upper branch. In addition to increasing the number of ECMs available to the system, the line-width enhancement factor defines the eccentricity of the ellipse and regulates the size of constituent attractors [29]. When the laser dynamics exhibit low-frequency fluctuations (LFF) as depicted in the simulated in Fig. 1.7, the system trajectory wanders among the attractor ruins close to the maximal gain mode at the tip of the ellipse shown in Fig. 1.7 until a crisis occurs when the system gets too close to the antimodes on the upper branch. To generate these simulations, a noise amplitude of $R_{sp}$ is added to the normalized Lang-Kobayashi equations to model realistic experimental conditions. This initiates a sudden shift in to solitary lasing conditions, coincident with the dropout of intensity. A stepwise, itinerant recovery of the system variables towards maximum gain follows along the lower branch of the ellipse [31]. So far we have discussed about the dynamical behavior of single diode laser subjected to self optical feedback (external cavity diode laser). Now we will discuss about the coupled diode laser system and coupling between the lasers could be unidirectional or bidirectional.

At larger injection currents, the system trajectory wanders more freely among the modes and antimodes throughout the ellipse, resulting in the signature large amplitude fluctuations of the coherence collapse (CC) regime [32]. The CC dynamics are more tightly confined near the middle of the ellipse, and there is no overall push towards the maximum gain mode or long excursions on top of the antimodes towards solitary lasing conditions. The
Figure 1.7: The open circles denote the external-cavity modes, the crosses indicate the saddle points, and the solid circle denotes the MGM. Simulations of the LFF intensity dynamics and the signature transitions between ECMs in the phase space of the external cavity phase shift $z(t)$ and carrier number $N(t)$ which typify this dynamical state [30].

qualitatively different features measured in the light dynamics are clearly manifested in the allowed interactions observed to occur between the ECMs on the ellipse.

1.5 Diode laser with optical injection: coupled diode laser system

Similar to the case of optical feedback, a dynamic analysis of a diode laser subject to optical injection is achieved via the semiconductor rate equations. The Eqs. (1.27) and (1.28) are modified to account for the interaction of the slave laser electric field with the electric field of the master laser by using $E(t) = \sqrt{P(t)}e^{i\phi(t)}$, within the slave laser active layer [33]

$$\frac{dP(t)}{dt} = (1 + i\alpha)N(t)P(t) + 2\eta \sqrt{P(t)P(t-\tau)} \cos(\phi(t) - \phi(t-\tau)) \quad (1.32)$$

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2} N(t) - \Delta\omega - \eta \sqrt{\frac{P(t-\tau)}{P(t)}} \sin(\phi(t) - \phi(t-\tau)) \quad (1.33)$$

$$\frac{dN(t)}{dt} = J(t) - N(t) - (2N(t) + 1)P(t) \quad (1.34)$$
In these equations, \( P \) denotes the photon number, \( \eta \) the coupling coefficient of the injected light into the cavity, \( P(t - \tau) \) the injected photon number, \( \phi(t) \) the photon phase inside the original cavity, \( \phi(t - \tau) \) the phase of the injected light, \( \alpha \) the linewidth enhancement factor, \( \Delta \omega \) the frequency difference, or detuning, between the two lasers.

By changing the magnitude and the optical frequency of the injection master laser, the follower laser can operate in different regimes [34]: stably locked to the injected laser frequency; amplitude modulated at the detuning frequency (beating); self-pulsating at the relaxation frequency, or become chaotic. Because of the sensitivity of such a nonlinear dynamic system to the initial conditions and the parameters describing it, immense computing power is necessary to explore all these interesting phenomena. The injection-locking rate equations are first solved in the steady-state to determine the region of stability, and to solve for the steady-state parameters. Setting the time-varying terms in the photon and phase differential equations, we obtain:

\[
(1 + i\alpha)N(t)P(t) + 2\eta\sqrt{P(t)P(t - \tau)} \cos(\phi(t) - \phi(t - \tau)) = 0 \tag{1.35}
\]

\[
\frac{\alpha}{2} N(t) - \Delta \omega - \eta \sqrt{\frac{P(t - \tau)}{P(t)}} \sin(\phi(t) - \phi(t - \tau)) = 0 \tag{1.36}
\]

Assuming the system is stability locked, the phase relation between the master and follower lasers is constant, which we denote as \( \Delta \phi = \phi(t - \tau) - \phi(t) \). Solving Eq. (1.36) for the frequency detuning \( \Delta \omega \),

\[
\Delta \omega = \eta \sqrt{\frac{P(t - \tau)}{P(t)}} \sqrt{1 + \alpha^2} \sin(\Delta \phi - \tan^{-1} \alpha). \tag{1.37}
\]

Alternatively, we can solve for the phase difference, as a function of frequency detuning and injection power.

\[
\Delta \phi = \sin^{-1} \left\{ \frac{\Delta \omega}{\eta \sqrt{\frac{P(t - \tau)}{P(t)} \sqrt{1 + \alpha^2}}} \right\} + \tan^{-1} \alpha. \tag{1.38}
\]

We can see that due to the sine term, the system is limited to a range of:

\[
|\Delta \omega| \leq \eta \sqrt{\frac{P(t - \tau)}{P(t)}} \sqrt{(1 + \alpha^2)}. \tag{1.39}
\]

However, since injection-locking increases the stimulated emission, it decreases the carrier number and decreases the gain. Thus, if it is assumed that the gain change in the follower
Figure 1.8: Schematic of the master-slave laser system. Light from the master laser is injected into the active region of the slave laser. Polarizer (POL) is used to control bidirectional coupling strength.

Laser cavity can only be negative, we find a second restriction on the locking bandwidth. The phase is bound by

$$\Delta \phi \leq \frac{\pi}{2}. \quad (1.40)$$

This leads to a reduced locking range [35], given by:

$$-\eta \sqrt{P(t-\tau)/P(t)} \sqrt{1 + \alpha^2} \leq \Delta \omega \leq \eta \sqrt{P(t-\tau)/P(t)}. \quad (1.41)$$

1.5-1 **Physical picture of optical injection locking**

One can easily gets confused in the complicated mathematics of a nonlinear dynamical system without a clear picture of its overall process. For a given slightly different control parameters or initial conditions, the behavior and outcome of the system can be completely different and difficult to comprehend. Optical injection locking of one diode laser to another is one of such example. The slave laser can be stable locked or behave chaotically depending on the laser parameters and the injection condition involved. It is of paramount interst to have a physical picture so that one will not be lost in the maze of parameter space. In this section, we will try to provide a simple physical picture of optical injection locking phenomena using the phase space concept in nonlinear dynamics. A self-sustained oscillator can be modeled as a particle moving counter-clockwise along a circle with a constant angular velocity equal to its oscillation frequency. In Fig. 1.9, the radius of the circle represents the amplitude of the oscillation and the particle position on the circle denotes its phase. When the oscillator is perturbed, the change in amplitude and phase can be visualized as the particle being pulled away from its circle track. If the oscillator is stable, which is the...
Figure 1.9: Schematic of phase space representation of self-sustained oscillator. The oscillator can be described as a particle rotating along the circle with a radius equal to its oscillation amplitude and at an angular velocity equal to the oscillation frequency (Fig. 1.9a). If a rotating reference frame is used, the particle is fixed (Point C), moving counter-clockwise or clockwise on the circle (Points A and B), depending on the difference between the reference frequency and the oscillation frequency (Fig. 1.9b).

one of interest here, the particle eventually will recover its original pace along the circle. In this phase space picture, the stability requires that any amplitude perturbation decays to zero and the particle eventually moves on the same circle. The perturbation on the phase, however, is maintained since the particle can be either ahead of or lagged behind its undisturbed position and still moves on the same circle. We are also free to choose a reference frame rotating at any angular frequency. In such a reference frame, the particle will become fixed at some point on the circle if the reference frequency is the same as its oscillation frequency. Whenever there is a frequency difference between the two, the particle will either moving clockwise or counter-clockwise with an angular velocity equal to the angular frequency difference. With this picture, we can perform a thought experiment of perturbing the previous self-sustained oscillator at \( \omega_0 \) with another oscillator at frequency \( \omega_R \). Let us arbitrarily choose the reference frame rotating at \( \omega_R \). For the case when \( \omega_0 = \omega_R \) (Fig. 1.10a), the oscillator is described as a particle at rest (Point A). The presence of the external perturbation, denoting as the vector \( \eta \) in Fig. 1.10a, shift the particle out of the
circle. Since we are only interested in a stable system, the particle will eventually drop back to the circle at point B. Again, the amplitude perturbation will decay to zero whereas the phase perturbation will be memorized by the system. In other words, only the tangential component of \( \eta \) will influence the particle in a meaningful way. Its normal component simply causes a damped transient perturbation normal to the circle and is of no significance in terms of system behavior. Consider the case when the two oscillators have different frequencies (Fig. 1.10b). Without loss of generality, we choose \( \omega_0 \neq \omega_R \) and that the rotating reference frame rotates at \( \omega_R \). When there is no perturbation, the oscillator is just a particle moving counter-clockwise with an angular velocity \( \omega_0 - \omega_R \). When the perturbation is applied, the vector \( \eta \) will have different effects on the particle depending on its the location. When the tangential component of \( \eta \) is in the same (opposite) direction of rotation, the particle movement will be accelerated or decelerated. When the direction of the \( \eta \) is normal to the circle then it will have no influence on the system. Obviously, the effect of \( \eta \) is maximum when it aligns with the tangential line of the circle. An interesting observation is that, if \( \eta \) is large enough, there exists a point on the circle where the some sort of brake introduced by \( \eta \) exactly cancels out the original angular velocity of the particle. At point C, the external force,
\( \eta \), introduced by master oscillator balances the rotation, or the frequency detuning between the two oscillators. No matter where the particle starts its rotation (the initial phase), it will stop right at such a point. Recall that a fixed point in a rotating frame phase space simply means that the oscillator is oscillating at the reference frequency. In other words, the oscillator with free running frequency \( \omega_0 \) is now oscillating at frequency \( \omega \). The same argument also applies when \( \omega_0 < \omega_R \). The only difference is that the particle will be locked at the position on the opposite side of the circle. Apparently, the position where the particle stops depends on the frequency detuning between the two oscillators and the amplitude and direction (phase) of external perturbation \( \eta \). It is worth noting that this perfect stop of the particle does not happen for any combination of external force and detuning. As mentioned earlier, the contribution of \( \eta \) to stop the particle is maximum when particle is at the position when its angular velocity aligns with \( \eta \) but at the opposite direction. So we conclude that for a large injection force, there will be a large range of detuning within which the oscillators can be locked to the injection. The locking range is asymmetric due to the change in index of refraction when the system is locked. This leads to a cavity wavelength shift in the longer wavelength direction, and finally an asymmetric locking range. For stable locking, the wavelength of slave laser must match the master laser wavelength, and the photon and electron numbers much reach a steady-state. Solution of the above rate equations yields the dynamic state of the slave laser dependent on the injected signal power and detuning. A number of different output states are observed including injection locking, limit cycle behavior, chaos, period doubling, and quasi-periodicity. The externally injected signal from the master laser is regeneratively amplified inside the slave laser diode cavity and produces an amplified output power at the injected frequency. The amplification of this signal increases as it moves closer to the free-running frequency of the solitary diode (maximum of the gain curve). If the frequency detuning between the slave and master lasers is made small enough, or the injected signal power large enough, then the amplified injected signal begins to saturate the laser gain, and the free-running laser oscillation is turned off, leaving only the amplified injected signal. Such a state is known as injection locking; the slave laser is forced to oscillate at the injected signal frequency and is locked to its phase. The range of master laser frequencies over which such a locked state occurs (the locking bandwidth) increases
with increased injected signal power. This is the dominant state for strong signal injection, i.e., when the injected signal power is comparable with the diode laser output power. The injection locked state enhances many of the solitary diode lasers operating characteristics. It suppresses mode hopping, lowers mode partition noise, narrows the linewidth and can be used to synchronize laser signals (for communications). The strong dependence of the carrier density on the refractive index in the active region also results in a large asymmetry in the locking curve, i.e., the system more readily locks for positive detuning, as opposed to negative detuning; where positive detuning is defined for an injected signal frequency that is larger than the solitary diode laser free-running frequency.

1.5-2 Physical meaning of different nonlinear dynamics

We approach the rate equations from a dynamical systems point of view and search for qualitatively different types of dynamics in \((E, N)\)-space, the three-dimensional phase space of Eqs. (1.32) and (1.34). The type of dynamics is characterized by a phase portrait. Mathematically, the phase portrait of a dynamical system is a partitioning of the phase space into orbits. Because it is impossible to draw all the orbits in a figure, the phase portrait usually includes asymptotic states (stationary points, limit cycles, tori, chaotic attractors) to which the system tends as \(t \to \infty\) together with several key orbits (especially stable and unstable manifolds of saddles) [36]. This classification automatically distinguishes between different physical processes taking place in the laser. Our aim now is to introduce different types of dynamics which may appear in the model of an injected laser and to discuss their physical meaning. Because not all phase portraits have physical meaning we focus here on dynamics which involve attractors, i.e. invariant and attracting sets in phase space. Another important notion is the basin of attraction of the attractor. It is the collection of all points in phase space that will ultimately converge to the attractor under the dynamics (the flow of the vector field) [37]. After transients, a trajectory settles down to and recovers the shape of the attractor. Optical spectra have a particular significance because most of the information about semiconductor laser dynamics obtained from experiments is based on spectral characteristics. This is so because existing oscilloscopes are still not fast enough for recording time series of semiconductor laser dynamics. This can only be done on short time intervals with a
streak camera [38]. For the physical interpretation of different phase portraits we recall that the system is written in the reference frame of the injection frequency. The time derivative of the phase of \( E \) corresponds to the difference between the instantaneous frequency at the laser output and the injected signal frequency. A stationary point corresponds to an output with constant intensity, population inversion and phase, which means that the frequency is that of the injected field. For this case, the laser is locked. A limit cycle with bounded phase describes an exchange of energy between the electric field intensity, which now oscillates, and the population inversion, relaxation oscillations being the typical example. A limit cycle with unbounded phase (running phase) corresponds to an output with oscillating intensity and a frequency that is the free running laser frequency shifted according to the new average carrier density caused by injection. Superposition of the laser field and the injected field, which have different frequencies, results in beating, that is, the signal oscillates with the optical frequency but is modulated on a scale determined by the detuning between the component fields. In this case, oscillations in the intensity of an unbounded limit cycle correspond to the offset of the beating between the laser frequency and the frequency of the injected light.

An invariant torus in the system corresponds to a competition between two oscillations, usually one associated with the detuning (forcing) and the other with the relaxation oscillations (intrinsic frequency of a semiconductor laser). It is characterized by the frequencies of these oscillations in the spectrum. On an invariant torus trajectories can either densely fill the torus, in which case the dynamics is quasiperiodic, or converge to attracting periodic orbit. In parameter space, the boundaries between these two different kinds of dynamics are formed by curves of saddle node of limit cycle bifurcations, which form the well known resonance or Arnold Tongues [36]. The standard measure for determining the transition between different dynamical state and the dynamical stability is quantified by Lyapunov Exponent by measuring the exponential rates of average divergence or convergence of near by trajectories on the attractors as a system evolves in time. Quantitatively, if two trajectories in phase space with initial separation \( \delta Z_0 \) diverge as \( |\delta Z(t)| \approx e^{\lambda t} |\delta Z_0| \), the Lyapunov exponent is

\[
\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \frac{|\delta Z(t)|}{|\delta Z_0|}.
\]

Motion on an attractor is called chaotic when it displays sensitive dependence on initial
conditions. This means that trajectories starting from two different, but very close initial conditions on the attractor diverge exponentially when $\lambda > 0$. Chaotic attractors in most cases have a very complicated structure [37], which is nicely brought out by the fractal produced in the Poincaré section. In terms of laser output, it corresponds to an irregular and unpredictable signal. Moreover, the frequency spectrum is no longer discrete but continuous. Note, however, that a chaotic frequency spectrum may still have strong periodic components. For $\lambda < 0$, the orbit attracts to a stable fixed point or stable periodic orbit. Negative Lyapunov exponents are characteristic of dissipative or non-conservative systems. For $\lambda = 0$, the orbit is a neutral fixed point (or an eventually fixed point). A Lyapunov exponent of zero indicates that the system is in some sort of steady state mode. A physical system with this exponent is conservative.

1.5-3 Synchronization

The origin of the word synchronization means to share the common time. The original meaning of synchronization has been maintained up to now in the colloquial use of this word, as agreement or correlation in time of different processes. Historically, the analysis of synchronization phenomena in the evolution of dynamical systems has been a subject of active investigation since the earlier days of physics. It started in the 17th century with the finding of Huygens that two very weakly coupled pendulum clocks (hanging at the same beam) become synchronized in phase. The modern concept of synchronization is an adjustment of rhythms of oscillating objects due to their weak interaction. According to this definition two objects are susceptible to synchronize when isolated they are able to oscillate by themselves, i.e., they are self-sustained oscillators. It is important to make clear that the resonance phenomena of forced systems that have no rhythm on their own are not synchronization processes. Moreover, it is necessary to stress that the interaction should be weak enough so that qualitatively each oscillator does not change its uncoupled dynamics. Otherwise, a too strong coupling between subsystems can be understood as we are dealing with a new unified system where each subsystem has not its own identity. It is also important to note that the coupling between different oscillatory units can be asymmetric. That is, the coupling strength may be not identical in the two directions of the interaction. The extreme
case is the unidirectional coupling, where a system (master) influences another system (slave) but the reverse is not true. In the general case, both units perturb the state of the other and it is said that there is a mutual or bidirectional coupling. In the following, the more important phenomena associated to the synchronization concept are introduced as well as some basic definitions.

1.5-4 Different kinds of synchronization

Several types of synchronization have been reported in the literature depending on the relationship that the signals $x_1(t)$ and $x_2(t)$ of two interacting systems may exhibit. The simplest case occurs when two coupled systems generate an identical output, $x_1(t) = x_2(t)$. Far from trivial, it was noticed that this phenomena could be observed even in chaotic systems and receives the name of identical or complete synchronization. This identical solution is just a form of a more general type of synchronization called generalized, where a given function $F$ relates the two outputs $x_1(t) = F(x_2(t))$. When $F$ is the identity the identical synchronization is recovered. Localized synchronization stands for the case in which both systems oscillate at common frequency but with very different amplitudes. The term phase synchronization is usually referred to the case in which the phase difference between the two outputs is bounded but their amplitudes stay uncorrelated. Another type of synchronization that shows up in different setups is the lag synchronization. This solution, which usually appears as an intermediate stage between the phase and complete synchronization, is very common in subsystems interacting with a time delay. The lag solution or synchronization stands for the equality of the two output signals once one of them has been appropriately shifted in time, i.e., $x_1(t) = x_2(t - \tau)$. Of course, some of the previous definitions can be adapted to more general cases as arrays of oscillators with a number of units larger than two or even oscillatory media.

1.5-5 Frequency locking

The main hallmark of synchronization is the so-called frequency locking. When uncoupled two oscillators use to exhibit a certain mismatch in their natural frequencies or detuning $\Delta \nu_0 = \nu_{02} - \nu_{01}$. That is, the signal of each system oscillates with a different frequency.
However, once coupled if the detuning is small enough both systems can oscillate at exactly the same frequency for a finite range of detuning. This entrainment of frequencies to a common value $\nu_{f1} = \nu_{f2} = \nu_f$ is known as frequency locking. The difference in the natural frequencies or detuning susceptible to experience the locking phenomena is dependent of the interaction strength $\eta$ between the oscillators. The region in the $\eta$ versus $\Delta \nu_0$ plane where such a locking occurs is usually called synchronization area or Arnold tongue. If the detuning parameter is very large, higher order synchronization regions may appear where the ratio between frequencies after coupling occurs is a rational number, $\nu_{f1}/\nu_{f2} = \frac{q}{p}$, of synchronization of order $p : q$.

### 1.5-6 Phase locking

Besides the amplitude of a signal its phase is also a fundamental source of information. The phase is used to parameterize the limit cycle (attractor in the phase space of the oscillatory system) on which the dynamics is taking place, so that it grows monotonically and increase a value of $2\pi$ each full oscillation. When the difference between the phases of two oscillators is bounded we say they are phase locked. More generally, we can consider phase locking of order $p : q$ if $\left| p \phi_1 - q \phi_2 \right| \leq \text{constant}$. Usually, when $|\phi_1 - \phi_2| = 0$ it is said that the oscillators exhibit in-phase dynamics, and when $|\phi_1 - \phi_2| = \pi$ it is named antiphase dynamics, as in the original observation of Huygens with the pendulum swinging in opposition. A generalization of the concept of phase $\phi(t)$ for an arbitrary signal $x(t)$ can be introduced via the construction of the complex analytical signal $x(t) + iy(t) = A(t)e^{i\phi(t)}$, where $y(t)$ is the Hilbert transform of $x(t)$

$$y(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{x(t')}{t - t'} dt'$$

(1.43)

and $P$ stands for the Cauchy principal value. Consequently, the same definition of phase synchronization is valid for this generalized phase and they can be applied now for non-periodic or even chaotic signals.

### 1.5-7 Bifurcation

The bifurcation we mean a qualitative change in the asymptotic solutions of a dynamical system when some parameter is continuously changed. These are usually related to the
change of stability of existing objects or the birth and death of asymptotic solutions. When the characterization of such a bifurcation can be reduced to the study of a vicinity of a single point in the phase space, the bifurcation is said to be local, otherwise it is qualified as global. The identity of the type of bifurcation a fixed point undergoes can be achieved by looking at how the eigenvalues of the characteristic equation cross the imaginary axis as some parameter is varied. Moreover, the symmetry of the dynamical system will impose some restrictions on the type of bifurcation we can find. First, we begin by describing some basic bifurcations of co dimension one. These are bifurcations where only one parameter is needed to be changed in order to meet the bifurcation point.

(i) Saddle-node: In this bifurcation a pair of fixed points (one attracting and one unstable) appear simultaneously as a single control parameter $\mu$ passes a threshold $\mu_0$. Near the bifurcation, the distance between the newly created steady-states scales as $(\mu - \mu_0)^{\frac{1}{2}}$. When the bifurcation occurs at $\mu_0$, a real zero eigenvalue crosses the imaginary axis.

(ii) Transcritical: It occurs when one stable and one unstable fixed point collide at the bifurcation point $\mu_0$ and interchange their stability. After the bifurcation occurs, the separation of the two fixed points varies linearly with $\mu - \mu_0$. A real zero eigenvalue is also crossing the imaginary axis at the bifurcation point.

(iii) Pitchfork: As some control parameter is changed we pass from a single stable fixed point to a situation where three fixed points are produced. In the supercritical (subcritical) version of this bifurcation, the two new steady-states are born stable (unstable) meanwhile the old fixed point gets unstable. It is important to mention that this type of bifurcation only appears in dynamical systems with an appropriate symmetry. A real zero is crossing the imaginary axis at the bifurcation point $\mu_0$. When instead of fixed points the bifurcating structures are limit cycles, the phenomenon in named as spontaneous symmetry-breaking bifurcation. This is due to the fact that in this bifurcation, the limit cycle that losses the stability is symmetric under the symmetry of the dynamical system, while the two new limit cycles created are asymmetric ones.

(iv) Hopf: In the supercritical Hopf bifurcation, a previously stable fixed point becomes unstable and a stable limit cycle is born. The limit cycle is a closed trajectory in phase space of a non-linear system having the property that at least one other trajectory spirals into it either
as time approaches infinity (stable or attractive limit-cycle) or as time approaches minus-infinity (unstable or non-attractive limit-cycle). In the subcritical case, the stable fixed point collides with an existing unstable limit cycle at the bifurcation point and as a consequence the steady-state loses its stability. In this type of bifurcations, the amplitude of the limit cycles that is born at the bifurcation point grows as the square root of the distance of the control parameter to the bifurcation point, i.e., \((\mu - \mu_0)^{\frac{1}{2}}\). The fixed point that is involved in the bifurcation gets unstable when a complex conjugate pair of eigenvalues crosses the imaginary axis in the complex plane. The imaginary part of these eigenvalues \(\lambda = \mp i \omega\) gives the angular frequency of oscillation of the periodic solution or limit cycle newly created. As we have seen, the three first types of bifurcations are occurred through the stabilization of a real zero that crosses the imaginary axis. The symmetries and other constrains are the key elements when distinguishing which of the three bifurcations occurs. A Hopf bifurcation is a local bifurcation in which a fixed point of a dynamical system loses stability as one (the largest) eigenvalue of the linearization around the fixed point becomes positive, turning the fixed point into a saddle-node, or the fixed point changes stability via a pair of purely imaginary eigenvalues, i.e., the real part of eigenvalues becomes positive (through zero).

1.6 Application of coupled diode lasers

(a) Encoding-decoding

The first demonstration of chaotic message encoding and decoding (CMA) using external-cavity semiconductor laser diodes was achieved [39, 40]. The transmitter and receiver lasers were driven into chaos by application of external-cavity feedback. Coupling between the transmitter and receiver lasers was adjusted such that the lasers synchronized. A wave message was added to the chaotic transmitter by direct amplitude modulation and message recovery was achieved by subtraction of the receiver output from the receiver input. In this case the receiver laser synchronizes to the chaotic output of the transmitter, but does not show the same level of synchronization to the message. Therefore, the input to the receiver laser contains both the chaos and the message, whilst the output from the receiver laser contains the chaos and a much reduced amplitude message. By subtracting the output of the receiver laser from the input to the receiver laser the message may be recovered. However, the
quality of message recovery depends on the synchronization quality between the transmitter and receiver. More recently, an experimental demonstration of 1 GHz bandwidth chaotic message encoding and decoding using external-cavity semiconductor laser was reported [41]. Also a 2.5 Gbits/s message encoding and decoding through synchronization of chaotic pulsing semiconductor lasers has been demonstrated [42].

(b) Coherent light beam combining

Single diode laser has some limitation (waste heat, maintaining beam quality, avoid optical damage) to deliver high power in a well collimated beam. In order to overcome these problem separate diode lasers can be combined to increase the power by matching their phases to generate a single coherent beam, or by merging beams of different wavelength. But getting it right it tricky. The goal is to make the electric field amplitude add constructively to generate a single high quality beam. But it is very difficult to adjust phases accurately enough to generate a high quality beam, particularly when combining many separate lasers beam. Complications include the need to control polarization, amplitude, power spectra of individual laser so they combine properly.

(c) Optical chaos cryptography

Chaos cryptography has been revealed as the most efficient and the most promising emerging technology attached to chaos synchronization. In particular, since the emitters and receivers in modern optical telecommunications networks are semiconductor lasers, the study of their synchronization in view of cryptographic applications has turned to be a leading topic in the nonlinear dynamics literature. In optical chaos communication schemes, information proceeding from multimedia sources (telephone, internet, etc.) are processed and multiplexed in order to generate a unique binary string. This binary information is encrypted through a mixing with an (hyper)chaotic laser radiation. After propagation in the optical fiber channel, synchronization is achieved with a receiver laser enslaved to the emitter one, and the encrypted information is thereby extracted. At last, further processing and multiplexing operations enable to restore the initial information which has therefore been securely transferred.

(e) Millimeter-wave generation

Transmission of analog or digital signals over high-frequency carriers (a few tens of GHz)
through an optical fiber has attracted great interest because optical fibers have very low propagation loss. In radio-over-fiber (RoF) systems, millimeter-wave signals are generated by optical sources in a central office (CO), and transmitted through fibers from CO to a base station (BS). Optical generation of millimeter-wave has attracted much attention because of its flexibility and applicability in broadband mobile communication systems and optical beam forming [44]. Specifically, sideband injection locking technique using a master and a slave laser is promising for generating millimeter-wave, since it is simple to implement and has high tunability. It often requires two or more light sources - a master laser and a slave laser, or a master laser and two slave lasers. Goldberg et al. first demonstrated the millimeter-wave generation by sideband injection locking [43]. In the demonstration, the coherence between two independent optical sources (two slave lasers) are achieved by locking them to two modulation sidebands of the master laser.

(f) Resonance frequency enhancement

Typically, the bandwidth of a laser is proportional to the resonance frequency, or relaxation oscillation of a laser. It has been shown that the resonance frequency of the laser can be enhanced several factors by optical injection locking [17]. The increase of resonance frequency can be explained as follows: for strong injection-locked lasers, the carrier density of the slave laser changes as a function of injection locking parameters like frequency detuning and injection ratio. This modified carrier density causes a wavelength shift of the original cavity mode of the free-running slave laser [46, 47]. As a result of an interaction between the injection-locked wavelength and the shifted cavity mode, the injection-locked laser exhibits a resonance enhancement. The properties of the resonance frequency increase depend on injection locking parameters. Simpson et al. [17] and Murakami et al. [46] demonstrated this resonance frequency increase in strong injection-locked semiconductor lasers experimentally and theoretically

$$\omega^2 = \frac{\eta g \frac{dg}{dn} P}{\tau_p} + \eta \frac{P_{m2}}{P} \tag{1.44}$$

It has been analytically predicted that the resonance frequency is expected to vary as the square root of injection power [45].
1.7 Outline of the Thesis

Chapter 1 gives the introduction dealing with coherent control of diode lasers by optical injection technique.

Chapter 2 is devoted to the design and characterization of external-cavity diode laser (ECDL) we have used for experiments on amplitude death [48] when mutually coupled with a free diode laser. Observation of amplitude death state opens up possible ways to diode laser power stabilization technologies.

Chapter 3 presents a modeling of the dynamics of mutually-coupled diode laser system and investigates different dynamical regimes [48]. The system stability is investigated as a function of coupled cavity time delay and the optical injection strength. Different dynamical regimes, spanning from 'in-phase locking' to 'out-of-phase locking' with ultimate amplitude death of low-frequency fluctuations/pulsations, are described numerically for weak to moderate injection. Qualitative agreements between numerically and experimentally observed results for amplitude quenching are shown. Numerical studies describe the shifting of phase-flip bifurcation as the optical injection strength is varied for a particular time delay.

Chapter 4 presents a theoretical analysis of strange bifurcation between the amplitude-death islands. The dynamical behaviour of the system, within the bifurcation windows between consecutive death-islands, is investigated as a function of the coupling strength $\eta$ for a fixed delay time $\tau$. The visibility and cross-correlation measures are used to unveil the signatures of global phase-locking stability. This robust phase-stability of the system may open up new ways for diode laser stabilization technology. A stability analysis is presented for the understanding of the dynamics [49].

Chapter 5 summarizes this work and provides an outlook for possible future investigations on the subject.
Finally, in the Appendix, we outline the numerical methods for the solution of coupled diode laser equations and the stability analysis of the solutions.

References


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