Chapter 7

Spatiotemporal Dynamic of a Toxin Producing Phytoplankton (TPP)-Zooplankton Interaction

7.1 Introduction

Mathematical models for the understanding of the occurrence of red tides have been proposed based on the idea that they are caused by toxin-producing phytoplankton (TPP) (Chattopadhay et al., 2002a; Chattopadhyay et al., 2002; Roy et al., 2006, 2007). Observational evidences for the patchiness of phytoplankton is abundant and is relatively easy to obtain (Siegel et al., 1999). Information on zooplankton distribution, vital for theoretical modeling of the dynamics, is however scarcer and difficult to obtain. Many authors have included the spatial variation of phytoplankton population in bloom models (Mathews and Brindley, 1996; Pitchford and Brindley, 2001). Recently, spatial movements of planktonic systems in the presence of toxin-producing phytoplankton have been found to generate and maintain inhomogeneous biomass distribution of competing phytoplankton as well as grazer zooplankton (Roy, 2008). To explain the periodicity of bloom, Mukhopadhyay and Bhattacharyya (2006) have considered a plankton interaction model which exploits spatial variation of plankton with self and cross diffusion induced by toxin producing phytoplankton population. Tiana et al. (2011) have also shown in their study that cross diffusion of plankton population lead to the formation of spatial patterns and hence induces diffusive instability. Instabilities and patterns in phytoplankton-zooplankton with self diffusion have been
discussed by Thakur et al. (2012). In the present chapter, we have considered the interaction of toxin producing phytoplankton (TPP)-zooplankton with diffusion and the main aim is to obtain the effect of this spatial heterogeneity on the stability of TPP and zooplankton interaction. The organization of this chapter is as follows: In the first section, we develop mathematical model followed by its stability analysis in section 7.3, where bifurcation analysis with and without diffusion stating its dynamic flow is discussed. The stability and bifurcation properties are provided in subsection 7.3.3. The justification of our analytical findings are provided numerically followed by conclusion in the final sections.

7.2 The Mathematical Model

The mathematical model representing the TPP-zooplankton interaction is governed by the following system of ordinary differential equation,

\[
\begin{align*}
\frac{dp}{dt} &= rp(1 - p/k) - c\frac{p}{\gamma + p}z + d_1 \nabla^2 p \\
\frac{dz}{dt} &= c_1\frac{p}{\gamma + p}z - \delta_2 z - \eta\frac{p}{\gamma + p}z + d_2 \nabla^2 z \\
\partial_n p &= \partial_n z = 0 \\
p(0, x) &= p_0 \geq 0, z(0, x) = z_0 \geq 0
\end{align*}
\] (7.1)

where \( p(t,x) \) and \( z(t,x) \) denote the population densities of prey (toxin producing phytoplankton) and predator (zooplankton) species at time ‘t’ and space ‘x’, respectively; the positive constants ‘\( d_1 \)’ and ‘\( d_2 \)’ represent the diffusion rates of prey and predator species, respectively; \( r > 0 \) denotes the intrinsic growth rate of prey species, \( k > 0 \) denotes the carrying capacity of prey species, \( c > 0, c_1 > 0 \) be the capturing rate and conversion rate of the predator population; \( \delta > 0 \) denotes the death rate of predator species, \( \gamma > 0 \) is the half saturation constant, \( \eta > 0 \) be the rate of toxication by the TPP population and \( \nabla^2 \) denotes the Laplacian operator

7.3 Stability Properties of Equilibrium

All solutions of (7.1) are nonnegative and bounded for all \( t > 0 \) when \( d_1 = 0, d_2 = 0 \). If \( r > \delta_1 \) and \( c_1 - \eta > \delta_2 \), system (7.1) has a unique interior equilibrium \( E^*(p, z) \), where \( p = \frac{\gamma \delta_2}{c_1 - \eta - \delta_2} \) and \( z = \frac{r(1 - \frac{c}{\gamma})}{c}(\gamma + p) \). In fact, under the Neumann boundary conditions, we know that \( E^* \) is still the steady-state solutions of system (7.1). From the point of view of ecology, the properties of positive constant steady-state solutions are important and
interesting. Therefore, in the following, we focus on the stability of \( E^*(p, z) \) and the existence of Hopf bifurcation. Using transformation \( u = p - p^* \) and \( v = z - z^* \), system (7.1) can be rewritten as,

\[
\begin{align*}
\frac{du}{dt} &= a_{10}u + a_{01}v + d_1 \frac{\partial^2 u}{\partial z^2} + f(u, v, d_1) \\
\frac{dv}{dt} &= b_{10}u + b_{01}v + d_2 \frac{\partial^2 v}{\partial z^2} + g(u, v, d_2)
\end{align*}
\]

(7.2)

Where

\[
\begin{align*}
f(u, v, d_1) &= a_{20}u^2 + a_{11}uv + a_{02}v^2 + a_{30}u^3 + a_{21}u^2v + \ldots \\
g(u, v, d_2) &= b_{20}u^2 + a_{11}uv + +b_{30}u^3 + a_{21}u^2v + \ldots \text{h.o.d.}
\end{align*}
\]

(7.3)

\[
\begin{align*}
a_{10} &= r - \frac{2rp^*}{K} - \frac{c\gamma z^*}{(\gamma + p)^2}, \quad a_{01} = -\frac{cp^*}{\gamma + p}, \quad a_{20} = -\frac{2r}{K} + \frac{2c\gamma z^*}{(\gamma + p)^2}, \quad a_{11} = -\frac{c\gamma}{(\gamma + p)^2}, \quad a_{02} = 0, \\
a_{30} &= -\frac{6c\gamma z^*}{(\gamma + p)^2}, \quad a_{21} = \frac{2c\gamma}{(\gamma + p)^2}, \quad a_{30} = 0, \quad b_{10} = \frac{(c_1 - \eta)\gamma z^*}{(\gamma + p)^2}, \quad b_{01} = \frac{(c_1 - \eta)\gamma}{\gamma + p} - \delta_2, \quad b_{20} = -\frac{2(c_1 - \eta)\gamma z^*}{(\gamma + p)^2}, \quad b_{11} = \frac{(c_1 - \eta)\gamma}{(\gamma + p)^2}, \quad b_{02} = 0, \quad b_{30} = \frac{6(c_1 - \eta)\gamma z^*}{(\gamma + p)^2}, \quad b_{21} = -\frac{2(c_1 - \eta)\gamma}{(\gamma + p)^2}, \quad b_{03} = 0.
\end{align*}
\]

### 7.3.1 Stability without Diffusion

**Theorem 7.1.** In the absence of diffusion, choosing \( \eta \), the rate of toxin liberation as the bifurcation parameter;

(i) System (7.1) remains asymptotically stable if \( T_0 < 0 \) and \( D_0 > 0 \) which is equivalent to \( \eta > \eta_0 \), \( r > Max\left(\frac{d_2}{2}, 1\right) \) where \( \eta_a = c_1 - \frac{\delta_2(1-r) + 2r(d_2 + \gamma)}{2r - \delta_2} \).

(ii) A Hopf-bifurcation occurs as \( \eta \) passes through its critical value \( \eta_0 \).

**Proof** The Jacobian matrix of the system (7.2) at \( (u^*, v^*) \) is

\[
J = \begin{pmatrix}
a_{10} & a_{01} \\
b_{10} & b_{01}
\end{pmatrix}
\]

The characteristic equation is given by,

\[
\lambda^2 - T\lambda + D = 0
\]

(7.4)

where \( T = a_{10} + b_{01}, \; D = a_{10}b_{01} - a_{01}b_{10} \)

It is well known that the stability of trivial solution of (7.2) depends on the locations of roots of (7.4) and when all roots of (7.4) have negative real parts, the trivial solution of (7.2) is stable; otherwise, it is unstable.

Thus if \( T < 0 \) and \( D > 0 \), then \( E^* \) is locally asymptotically stable.

Next, we analyze the Hopf bifurcation occurring at \( E^* \) by choosing \( \eta \) as the bifurcation parameter.
For the occurrence of hopf bifurcation at $\eta = \eta_0$, jacobian $J$ has a pair of imaginary eigenvalues, say $\lambda = \pm \sqrt{D}$ and let $\lambda(\eta) = \sigma(\eta) + \omega(\eta)$ be the root of (7.4), then

$$\sigma(\eta) = \frac{T}{2}, \quad \omega(\eta) = \frac{\sqrt{4D - T^2}}{2}$$

and, $|\frac{\partial \sigma(\eta)}{\partial \eta}|_{\eta = \eta_0} = \frac{-2r\delta_2^2}{K(c_1 - \eta - \delta_2)^2} + \delta_2(1 - r\gamma K) (c_1 - \eta)^2 \neq 0$

By the Hopf-bifurcation theorem, we know that system (7.1) undergoes a Hopf bifurcation at $E^*$ when $\eta = \eta_0$.

### 7.3.2 Diffusion-driven Instability of the Equilibrium Solution

In this part, we derive conditions for the diffusion-driven instability with respect to the equilibrium solution $E^*$, the spatially homogenous solution of (7.1).

It is well known that the operator $-\Delta \phi = \lambda \phi, x \in \Omega$, with the above no-flux boundary condition has eigenvalues and eigenfunctions as follows:

$$\mu_0 = 0, \quad \phi_0(x) = \sqrt{\frac{1}{\pi}}, \quad \mu_k = k^2, \quad \phi_k(x) = \sqrt{\frac{2}{\pi}} \cos(kx), \quad k = 1, 2, 3, \ldots$$

From the standard linear operator theory, it is known that if all the eigen-values of the operator $L$ have negative real parts, then $E^* = (u_1, u_2)$ is asymptotically stable, and if some eigen-values have positive real parts, the $E^* = (u_1, u_2)$ is unstable.

We consider the following characteristic equation of the operator $L$,

$$L \begin{pmatrix} \phi \\ \psi \end{pmatrix} = \mu \begin{pmatrix} \phi \\ \psi \end{pmatrix}$$

Let $(\phi(x), \psi(x))^T$ be the eigen-function of $L$ corresponding to eigen-value $\mu$ and let

$$\begin{pmatrix} \phi \\ \psi \end{pmatrix} = \sum_{k=0}^{\infty} \begin{pmatrix} a_k \\ b_k \end{pmatrix} \cos(kx)$$

where $a_k$ and $b_k$ are coefficients, we obtain that

$$-k^2 D \sum_{k=0}^{\infty} \begin{pmatrix} a_k \\ b_k \end{pmatrix} \cos(kx) + J \sum_{k=0}^{\infty} \begin{pmatrix} a_k \\ b_k \end{pmatrix} \cos(kx) = \mu \sum_{k=0}^{\infty} \begin{pmatrix} a_k \\ b_k \end{pmatrix} \cos(kx)$$

$$(J - k^2 D) \sum_{k=0}^{\infty} \begin{pmatrix} a_k \\ b_k \end{pmatrix} = \mu \sum_{k=0}^{\infty} \begin{pmatrix} a_k \\ b_k \end{pmatrix} \quad (k = 0, 1, 2, \ldots)$$

Denote

$$J_k = J - k^2 D = \begin{pmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{pmatrix} - \begin{pmatrix} d_1 \mu_k & 0 \\ 0 & d_2 \mu_k \end{pmatrix} = 0$$

It follows from this, that the eigen-values of $L$ are given by the eigen-values of $J_k$ for $k=0, 1, 2, \ldots$. 

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The characteristic equation of $J_k$ is

$$\mu^2 - \mu T_k + D_k = 0, \quad k = 0, 1, 2, \ldots$$  \hfill (7.5)$$

where

$$T_k = (d_1 + d_2)k^2 - (a_{10} + b_{01})$$
$$D_k = d_1d_2k^4 - (d_1b_{01} + d_2a_{10})k^2 + (a_{10}b_{01} - a_{01}b_{10})$$

**Theorem 7.2.** (i) The positive equilibrium $E^*$ is locally asymptotically stable in the presence of diffusion if and only if $T_k < 0$ and $D_k > 0$ i.e. (7.5) do not possess a positive root for any $k \geq 0$

(ii) Diffusion instability (Turing instability) (Turing, 1952) occurs if the following inequality holds, i.e. $4d_1d_2(a_{10}b_{01} - a_{01}b_{10}) < (d_1b_{01} + d_2a_{10})^2$.

**Proof.** From the definition of $T_k$, we have $T_k < 0 (T_0 < 0)$ for all $k > 0$ satisfying $k < \frac{\eta - \eta_0}{d_1 + d_2}$ where $\eta > \eta_0$.

Thus, the diffusion driven instability only occurs if $D_k(k^2) = d_1d_2k^4 - (d_1b_{01} + d_2a_{10})k^2 + (a_{10}b_{01} - a_{01}b_{10}) < 0$,

i.e. (7.5) has at least one positive root.

Since, $D_k$ is quadratic in $k^2$ and the graph of $D_k = 0$ is a parabola. The minimum of $D_k(k^2)$ is occur at $k^2 = k^2_{\min}$, where, $k^2_{\min} = \frac{d_1b_{01} + d_2a_{10}}{2d_1d_2} > 0$

Consequently, the condition for diffusive instability is $D_k(k^2) < 0$, which on simplification gives,

$$4d_1d_2(a_{10}b_{01} - a_{01}b_{10}) < (d_1b_{01} + d_2a_{10})^2.$$  

### 7.3.3 Properties of Bifurcating Solutions

The PDE (7.1) possesses any periodic solution of corresponding ODE as a spatially homogeneous periodic solution, including the ones from Hopf bifurcation. We can also perform a Hopf bifurcation analysis of PDE (7.1) at the same bifurcation point of ODE and bifurcating spatially homogeneous periodic solutions exist near $\eta = \eta_0$.

So, we shall applying the normal form theory and center manifold theorem introduced by Hassard et al. (1981) to study the direction of Hopf bifurcation. To determine the stability of bifurcated periodic solutions, we need to know the restriction of the system to its center manifold at $\mu = \mu_0$. Denote by $L$ the operator

$$
\begin{pmatrix}
u
\end{pmatrix} = L \begin{pmatrix}
u
\end{pmatrix}
$$

with domain, $D= \{(u, v) \in H^2(\Omega) \times H^2(\Omega) | \partial u_w, \partial v_w = 0, x \in \Omega\}$, where the $H^2(\Omega)$ is the standard Sobolev space and
\[ L^* = \begin{pmatrix} a_{10} + d_1 \Delta & b_{10} \\ a_{01} & b_{01} + d_2 \Delta \end{pmatrix}. \]

In fact, we can choose
\[ q = \begin{pmatrix} 1 \\ \frac{\omega_0 - a_{10}}{a_{01}} \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}, \]
\[ q^* = D^* \begin{pmatrix} \frac{-\omega_0 - a_{10}}{a_{01}} \\ \iota \end{pmatrix} = \begin{pmatrix} a_* \\ b_* \end{pmatrix}, \quad D^* = \frac{a_{01}}{2\pi \omega_0} \]

For all \( \alpha \in D_{L^*} \), \( \beta \in D_L \), it is not difficult to verify that
\[ <L^* \alpha, \beta> = <\alpha, L \beta>, \quad Lq = \iota \omega_0 q \]
\[ L^* q^* = -\iota \omega_0 q^*, \quad <q^*, q> = 1 \text{ and } <q^*, \bar{q}> = 0. \]

Where, \( <\alpha, \beta> = \int_{\Omega} \bar{\alpha}^T \beta dx \) denotes the inner product in \( L^2(\Omega) \times L^2(\Omega) \).

Noticing that,
\[ (u, v) = zq + \bar{z} \bar{q} + W, \quad z = <q^*, (u_1, u_2)^T > \]
Then
\[
\begin{align*}
    u &= z + \bar{z} + W_1 \\
    v &= z \left( \frac{\omega_0 - a_{10}}{a_{01}} \right) + \bar{z} \left( \frac{-\omega_0 - a_{10}}{a_{01}} \right) + W_2
\end{align*}
\]
System in \((z, W)\) coordinates becomes,
\[
\begin{align*}
    \frac{dz}{dt} &= \iota \omega_0 z + <q^*, \tilde{f}> \\
    \frac{dW}{dt} &= LW + [\tilde{f} - <q^*, \tilde{f} > q - <q^*, \tilde{f} > \bar{q}]
\end{align*}
\]
where \( \tilde{f} = (f, g) \) defined in (7.3).

Then, straightforward but tedious calculations show that,
\[
\begin{align*}
    <q^*, \tilde{f}> &= \frac{a_{01}}{2 \omega_0} \left[ \left( \frac{\omega_0 - a_{10}}{a_{01}} \right) f + ig \right] \\
    <q^*, \tilde{f}^*> &= \frac{a_{01}}{2 \omega_0} \left[ \left( \frac{\omega_0 + a_{10}}{a_{01}} \right) f - ig \right] \\
    <q^*, \tilde{f} > q &= \frac{a_{01}}{2 \omega_0} \left[ \left( \frac{\omega_0 - a_{10}}{a_{01}} \right) f + ig, \left( \frac{\omega_0 + a_{10}}{a_{01}} \right) f + ig \right] \\
    <q^*, \tilde{f}^* > q &= \frac{a_{01}}{2 \omega_0} \left[ \left( \frac{\omega_0 + a_{10}}{a_{01}} \right) f - ig, \left( \frac{\omega_0 - a_{10}}{a_{01}} \right) f - ig \right]
\end{align*}
\]
Noticing that,
\[
H = \frac{H_{20}}{2} z^2 + H_{11} z \bar{z} + \frac{H_{02}}{2} \bar{z}^2 + \ldots
\]
\[
W = \frac{W_{20}}{2}z^2 + W_{11}z\bar{z} + \frac{W_{02}}{2}\bar{z}^2 + \ldots
\]  

(7.9)

On the center manifold, we have
\[
\begin{cases}
(2i\omega - L)W_{20} = H_{20} \\
(-L)W_{11} = H_{11} \\
W_{02} = W_{20}
\end{cases}
\]

and
\[
< q^*, \tilde{f} > q^* + < \bar{q}^*, \tilde{\bar{f}} > \bar{q} = (f, g)
\]

\[
H(z, \bar{z}, W) = < q^*, \tilde{f} > q^* + < \bar{q}^*, \tilde{\bar{f}} > \bar{q} - (f, g) = 0
\]

This implies that,
\[
W_{20} = W_{02} = W_{11} = 0
\]

Therefore,
\[
\frac{d\bar{z}}{dt} = i\omega z + \frac{g_{20}}{2}z^2 + \frac{g_{11}}{2}z\bar{z} + \frac{g_{02}}{2}\bar{z}^2 + \frac{g_{21}}{2}z^2\bar{z} + O(|z|^4)
\]  

(7.10)

where
\[
\begin{align*}
g_{20} &= 2D^* [\tilde{a}_0^*(a_{20} + b_0a_{11} + b_0^2a_{02}) + \tilde{b}_0^*(b_{20} + b_0b_{11})], \\
g_{11} &= D^*[\tilde{a}_0^*(2a_{20} + 2Re(b_0)a_{11} + 2a_{02}b_0\bar{b}_0) + \tilde{b}_0^*(2b_{20} + 2Re(b_0)b_{11})], \\
g_{02} &= 2D^*[\tilde{a}_0^*(a_{11} + a_{12}b_0 + a_{02}\bar{b}_0^2 + 2a_{21}\bar{b}_0) + \tilde{b}_0^*(2b_{20} + \bar{b}_0b_{11} + 2\bar{b}_0b_{21})], \\
g_{21} &= 2D^*[\tilde{a}_0^*(3a_{30} + a_{21}\bar{b}_0 + 2a_{21}b_0) + \tilde{b}_0^*(3b_{30} + 2(2b_0 + \bar{b}_0)b_{21})]
\end{align*}
\]

According to Hassard et al. (1981), we can obtain
\[
c_1(0) = \frac{i}{2\omega_0} \left\{ g_{20}g_{11} - 2|g_{11}|^2 - \frac{|g_{02}|^2}{3} \right\} + \frac{g_{21}}{2}
\]

\[
\mu_2 = -\frac{\text{Re}\{c_1(0)\}}{\text{Re}\{\lambda'(\eta)\}}
\]

\[
\beta_2 = 2\text{Re}\{c_1(0)\},
\]

### 7.4 Numerical Simulation

In this section, we numerical verify the results derived analytically in above sections to observe the effects of diffusion on the given plankton system. We have considered the following model system,

\[
\begin{align*}
\frac{dp}{dt} &= 3p(1 - p/105) - 0.7\frac{p}{5+p}z + d_1\nabla^2 p \\
\frac{dz}{dt} &= 0.6\frac{p}{5+p}z - 0.364z - 0.2\frac{p}{5+p}z + d_2\nabla^2 z
\end{align*}
\]

(7.11)

which has an interior equilibrium point \(E^* (52.80, 84.40)\) in the absence of diffusion. From the sign of \(\text{T}(\text{trace})=-0.0976\) and \(\text{D}(\text{det})=0.0314\), it is clear that (7.4) has eigen
Figure 7.1: Left: the positive equilibrium is asymptotically stable and right: positive equilibrium is unstable, and there exists a stable limit cycle.

Figure 7.2: Numerical simulations of the stable equilibrium solution of system (7.1). The solution appears to converge to a homogeneous steady state.
Figure 7.3: Numerical simulations of an unstable homogeneous equilibrium solution driven by diffusion. Left: component p (unstable); right: component z (unstable).

Figure 7.4: Numerical simulations of inhomogeneous stable periodic solution of given system with diffusion. Left: orbitally stable periodic solution (component p); right: orbitally stable periodic solution (component z).
values with negative real parts which ensures the local asymptotic stability of $E^*$ when $\eta < \eta_0 = 0.1871$. When $\eta = \eta_0 = 0.1871$ and $K=108$, it can be calculated that the jacobian has a pair of imaginary eigen values and system enters into a Hopf-bifurcation with the existence of limit cycles (see fig 7.1). For PDE, taking $d_1 = 1 \times 10^{-3}$, $d_2 = 1 \times 10^{-3}$, $K=105$ and $\eta = 0.2071$, it is found that the homogenous equilibrium solution which are also spatially homogeneous is stable and is shown in fig 7.2. Again, taking $d_1 = 1 \times 10^0$, $d_2 = 1 \times 10^0$, $K=113$ and $\eta = 0.2071 > \eta_0$, it can be found that the homogenous equilibrium solution which are also spatially homogeneous become unstable (or Turing unstable) as shown numerically in fig 7.3. Choosing another set of parameters, $d_1 = 1 \times 10^{-3}$, $d_2 = 1 \times 10^{-5}$, $K=108$ and $\eta = 0.1871$, we have obtained that a Hopf-bifurcation occurs at $\eta = \eta_0$, the direction of the bifurcation is supercritical and the bifurcating periodic solutions are unstable. This is shown in fig 7.4., where the initial condition is taken at $(52 + \sin x, 84 + \cos x)$.

### 7.5 Conclusion and Discussion

In this chapter, a TPP-zooplankton system is considered in spatiotemporal domain. We have first analyzed the given system by taking diffusion coefficients $d_1 = d_2 = 0$ and has observed that the interior equilibrium $E^*$ remains LAS by satisfying certain conditions. Further, a large amplitude oscillations appears when $\eta$, the rate of toxication passes from its bifurcation value $\eta = \eta_0$. In the presence of diffusion, it is found that the spatially homogeneous solution remains stable under certain conditions and Turing instability arises when some of these conditions are violated. Using normal form, formulae to determine the direction and stability of spatially homogeneous periodic solutions are derived.
Scope of Future Work

In this thesis, we have proposed some mathematical models using Ordinary differential equations, Delay-differential equations, and Partial differential equations to understand the dynamic of plankton system. We have discussed the following aspects in this thesis:

1. The effect of different delays; gestation, predation and toxin liberation on the plankton dynamic
2. The role of toxicating phytoplankton as a controlling parameter for the occurrence and termination of blooms
3. Introduction of various types of response functions in toxin liberation process and studied their role in exploring possible mechanisms behind the planktonic blooms
4. Bifurcation analysis for the model systems with respect to different control parameters
5. An optimal harvesting policy of phytoplankton-zooplankton system using Pontragin’s maximal principle

Keeping in view the work discussed in this thesis, the future work may be extended in the following direction:

1. The functional responses considered in the models of this thesis can be modified by some density dependent type functional responses and then the dynamics of the system can be investigated
2. The effect of delays in the presence of diffusion (non linear diffusion and cross diffusion) can be studied for pattern generation in plankton dynamics
3. Some field study would be carried out in the light of existing models for their validation
4. Different algorithms can be evolved to develop an optimal harvesting policy for maintaining the requisite fish stock and sustainable development of the ecosystem
5. Mathematical models for studying the interaction between the viral infected plankton and higher aquatic animals in homogeneous and heterogeneous domain with and without delay can be developed for finer and deeper insight into the underlying mechanisms responsible for the formation of blooms.