ABSTRACT

In modern mathematics, fixed point theory is one of the most powerful tools to the problems arising in differential and integral equations. Fixed point theorems are related to the existence of fixed point and its various properties. The theory is deeply concerned with the result that states that under certain conditions a self-map $f$ on a set $X$ has one or more fixed point. The theory has wide range of applications in the field of mathematics, engineering, physics, economics, game theory, biology, chemistry etc.

The theory started with the fundamental principle of contraction mapping known as Banach Contraction Principle [1], on a complete metric space. Banach Contraction Principle provided solution to maximum problems raised in mathematics. Banach result was a platform for many researchers to work on fixed point theory. Many generalizations came into being just by weakening the contractive nature of the map as in [5-9]. Some of the generalizations developed by weakening the topology see [10, 11]. Nadler [12] also extended the Banach fixed point theorem.

In 1992, Matthew [15] followed the steps of metric space to introduce the notion of Partial metric space. He [15, 16] generalized metric space in such a way that the distance of point from itself may not be zero. The Partial metric space was further generalised by O’Neil [92] to bridge the structures and topological aspects of the domain theory. The fixed point theory in Partial metric space find its maximum use in mathematics, computer science, matrix equations and ordinary differential equations. For detail one can refer to various papers ([93-111]).

In 2000, Hitzler and Seda [14] introduced a new abstract space called Dislocated metric space in which self-distance of a point need not be equal to zero. They also obtained characterizations of Banach
Contraction Principle in this space. Dislocated metric space has been the centre of vigorous research activity as it plays an important role in topology, logical programming and in electronics engineering. Mathematicians as Aage and Salunke [87], Zeyada et al. [88], Jha and Panthi [89] and Rao and Rangaswamy [90] proved some productive results on the fixed point theorems in Dislocated metric space.

Although fixed point theory enjoyed remarkable success in different fields of research but was facing the problem of uncertainty and vagueness. To answer the question of vagueness, Zadeh [114] introduced the fuzzy set theory in 1965. Researchers as Deng [116], Erceg [117], Kramosil and Michalek [118] made many efforts for the growth of the fuzzy mathematics. In 1994, George and Veeramani [17] introduced the concept of Fuzzy metric space on the basis of t-norm. Many authors studied fixed point theorems in Fuzzy metric space. Some of the interesting references are of Altun and Turkoglu [47] Butnariu [119], Chang [120], Mishra et al. [121], Cho [122], Chang et al. [123].

In the beginning of twenty first century that is in the year 2002, Sushil Sharma [18] propounded the notion of Fuzzy 2-metric space. For this, he considered Gahler’s [19] properties on 2-metric space as a base to introduce the notion of Fuzzy 2-metric space. In this context a series of research papers [124-129] appeared.

In 1984, Dhage [22] generalized 2-metric space to D-metric space. But somehow, the fixed point theory was not successful on D-metric space, as some topological properties were not valid see [23, and 24]. In 2006, Sedghi et al. [21] introduced $D^\#$-metric space as a probable definition of D-metric space. They studied those topological properties on $D^\#$-metric space which were not valid in D-metric space. In the same year 2006, Sedghi and Shobe [20] introduced the concept of M-fuzzy metric space as a generalisation of George and
Veeramani’s [17] Fuzzy metric space. Researchers established the existence and uniqueness of fixed point results with different mappings and conditions which can be viewed in ([45, 48, 154-157]). Naschie ([140-143]) applied fuzzy topology to Quantum Physics particularly in connection with both string and e-infinity theory.

In 2007, Saadati et al. [25] introduced the notion of $L$-fuzzy metric space with the help of continuous t-norm. $L$-fuzzy metric space is a generalization of George and Veeramani’s [17] Fuzzy metric space and Park and Saadati’s [178] Intuitionistic fuzzy metric space. To introduce the notion of $L$-fuzzy metric space, Saadati et al. [25] used the idea of Goguen’s [164] $L$-fuzzy sets. The space being the active field of research made different authors to prove their fixed point results which appeared in ([171-179]).

Fixed point theory is valuable from a numerical point of view as it is used in various numerical methods to estimate an error for the iterative scheme. Fixed point iterative schemes have wide range of applications in Industrial and Applied Mathematics. It is used in Computational Techniques to compare the rate of convergence of different iterative schemes as Rhoades and Solutz [191], S. L. Singh [195], Berinde [188], Phuengrattana and Suantai [185] did it for their results.

The objective of the present work is to do comparative study of fixed point theorems in different spaces with different conditions and mappings described in the various chapters.

The thesis consists of seven chapters showing work with our objective to do a comparative study of fixed point theorems on various spaces.

In Chapter 1, some basic definitions and results required to prove fixed point theorems in different spaces as Dislocated metric
space, Partial metric space, Fuzzy 2-metric space, M-fuzzy metric space, $\mathcal{L}$-fuzzy metric space are stated. Notions of compatible mappings, weakly compatible mappings, occasionally weakly compatible mappings and compatible mappings of type (P) are described. A number of examples are cited to justify the remarks related to the defined notions. Concepts of weakly increasing maps, biased maps of type (RM) and cyclic contraction mappings are also discussed. A brief description of each chapter is also given. It also includes the notions of different iterative procedures as Mann [180], Ishikawa [181], Aggarwal et al. [182], Thainwan [183], SP [185], CR [186] and Noor [184] to support the present work on newly introduced K-iterative scheme.

Chapter 2 presents two results on fixed point theorems. The first result is a theorem for six self-mappings in Dislocated metric space for occasionally weakly compatible mappings. The second result is a fixed point theorem on Partial metric space for two weakly increasing maps.

In Chapter 3, the author proves fixed point theorems in Fuzzy 2-metric space. In this chapter, a new definition of sub-compatibility of type (A) is introduced and is used with reciprocal continuity to prove fixed point result in Fuzzy 2-metric space. The concept of absorbing maps [121] has been used successfully to prove the second main result on fixed point theorem in Fuzzy 2-metric space.

In Chapter 4, the author works on M-fuzzy metric space. Here a new class of implicit relations is defined to present the author’s work for six weakly compatible mappings. For this, property E is used. The other result on fixed point theorem is an extension of Koireng and Rohen’s [161] work proved on Fuzzy metric space. It is now proved on M-fuzzy metric space using compatible mappings of type (P).
Chapter 5 deals with work on \(\mathcal{L}\)-fuzzy metric space. The notion of \(\mathcal{L}\)-fuzzy metric space was given by Saadati et al. [25]. The chapter presents work on biased maps of type \((R_M)\) introduced in Fuzzy metric space. The result is obtained by using the Fang’s [78] property of \(C\). The chapter also comprises of a common fixed point theorem for cyclic weak \(\emptyset\)-contraction mapping. To prove this fixed point theorem, the triangular condition on \(\mathcal{L}\)-fuzzy metric space is defined.

Chapter 6 deals with work on iterations. The new iterative scheme named as K-iterative scheme is introduced. It is proved that K-iterative scheme converges to the same fixed point with lesser number of iterations as compared to other iterative procedures like Mann, Picard, Aggarwal et al., SP, CR and Noor. Various illustrations are given to justify the fact. The comparison is done on the basis of C-language programming. Figures are traced to show the rate of convergence with the help of MAT LAB.

Chapter 7 is the chapter of Conclusions and Discussions. It includes conclusion of above mentioned chapters. The future perspectives of the present work are also suggested.