CHAPTER- 4

FIXED POINT THEOREMS IN M-FUZZY METRIC SPACE.

The purpose of this chapter is to lay emphasis on fixed point theorems in M-fuzzy metric space. The chapter has two main objectives. The first objective is to obtain some results on M-fuzzy metric space for six self-mappings. In this regard a new class of implicit relations is defined. The result is obtained by making use of weakly compatible mappings. The other objective of the chapter is to get a common fixed point theorem for six self-mappings satisfying the condition of compatible mappings of type (P) and continuity.

A part of the work from this chapter has been published in the form of a research paper entitled:

CHAPTER 4

FIXED POINT THEOREMS IN M-FUZZY METRIC SPACE.

4.1 - INTRODUCTION

After great success of Zadeh’s [114] work on Fuzzy sets, George and Veeramani’s [17], Kramosil and Michalek’s [118] work on Fuzzy metric space and Sushil Sharma’s [18] work on Fuzzy 2-metric space, many eminent researchers put their efforts to extend the work on new space.

As fuzzy topological spaces induced by fuzzy metric have wide range of applications in quantum particle physics especially related to string and $\varepsilon^{(\infty)}$ theory of Naschie [140-143]. The behaviour of Naschie theory was studied by Tanaka et al. [144].

In 2006, Sedghi and Shobe [20] introduced M-fuzzy metric space. They [20] defined M-fuzzy metric space using their own notion of $D^*$-metric space on which others researchers also (see [21, 145-148]) did the significant work. $D^*$-metric space is a modification of Dhage’s [22] notion of D-metric space. $D^*$-metric space was introduced to study those topological properties which were not valid in D metric space see ([23, 24]). Many mathematicians proved fixed point results in D-metric space, for this see [149-152]. Sedghi and Shobe [20] used $D^*$-metric space analogy to introduce M-fuzzy metric space. M-fuzzy metric space being the new land for researchers to be ploughed, on which Chauhan [45], Sedghi et al. [46], Chauhan and Joshi [48], Sedghi and Shobe [153], Park et al. [154, 155], Veerapandi et al. [156], Rao et al. [157] spread their seeds of success.

The present chapter has two main objectives; the first objective is to obtain some common fixed point theorems for six weakly compatible mappings using implicit relation with property (E). It has brought upon an improvement and has extended the proven results of Chauhan and Joshi [48], Singh and Jain [49], Rathore et al. [158], Pant [159], and Popa [52, 160]. The result finds its justification by
i. Replacing the Fuzzy metric space by M-fuzzy metric space.

ii. Increase the number of self-maps from four to six.

iii. Use the property (E).

The other objective is to prove a common fixed point theorem using Pathak et al. [36] concept of compatible mappings of type (P). Compatibility of type (P) leads to the conclusion that compatibility of type A, B, C and P are equivalent if the self-mappings f and g are continuous. This research work is an extension of Koirer and Rohen’s [161] paper in Fuzzy metric space to M-fuzzy metric space.

4.2 - PRELIMINARIES

Definition 4.2.1- [115] A binary operation \(*: [0, 1] \times [0, 1] \rightarrow [0, 1]\) is a continuous t-norm if it answers the ensuing terms

i. \(*\) is associative and commutative,

ii. \(*\) is continuous,

iii. \(a * 1 = a\) for all \(a \in [0, 1]\),

iv. \(a * b \leq c * d\) whenever \(a \leq c\) and \(b \leq d\) \(\forall a, b, c, d \in [0, 1]\).

Definition 4.2.2-[20] M-fuzzy metric space is known as the triplet space \((Z, M, *)\) with any random set \(Z\), \(*\) as a continuous t-norm, and fuzzy set \(M\) on \(Z^3 \times (0, \infty)\), ensuing the following facts for each \(z_1, z_2, z_3, a \in Z\) and \(t_1, t_2 > 0\),

vii. \(M(z_1, z_2, z_3, t_1) > 0\),

viii. \(M(z_1, z_2, z_3, t_1) = 1\) if and only if \(z_1, z_2, z_3\) are equal,

ix. \(M(z_1, z_2, z_3, t_1) = M(p\{z_1, z_2, z_3\}, t_1)\), (symmetry) where \(p\) is a permutation function,

x. \(M(z_1, z_2, a, t_1) * M(a, z_3, z_3, t_2) \leq M(z_1, z_2, z_3, t_1 + t_2)\),

xi. \(M(z_3, z_2, z_3, t_1): (0, \infty) \rightarrow [0, 1]\) is a continuous,

xii. \(\lim_{t_1 \rightarrow \infty} M(z_1, z_2, z_3, t_1) = 1\).

Example 4.2.1-[20] Consider a non-empty set \(Z\) with a metric \(d\). Denote \(f(x, y) = x \cdot y\) \(\forall x, y \in [0, 1]\). For each \(t \in (0, \infty)\), define \(M(a, b, c, t) = \frac{t}{t + D(a, b, c)}\)
where $D(a, b, c) = \max \{d(a, b), d(b, c), d(c, a)\}$ for all $a, b, c \in Z$. Then $(Z, M, f)$ is a M-fuzzy metric space.

**Definition 4.2.3-[20]** For M-fuzzy metric space $(Z, M, *)$ and $t > 0$, the open ball $B_M(z, s, t)$ with centre $z \in Z$ and radius $s$ in $(0, 1)$ denotes

$B_M(z, s, t) = \{ y \in Z : M(z, y, y, t) > 1 - s \}.$

$A \subset Z$ is open set if for each $z \in A$ there prevails $t > 0$ and $0 < s < 1$ so that $B_M(z, s, t) \subseteq A$.

**Proposition 4.2.1-[20]** Every open ball is an open set in a M-fuzzy metric space.

**Definition 4.2.4-[21]** Let $\{z_n\}$ be a sequence in a M-fuzzy metric space $(Z, M, *)$, then $\{z_n\}$ is convergent if $\lim_{n \to \infty} M(z, z, z_n, t) = 1 \forall t > 0$ converging to $z$ of $Z$ and is Cauchy sequence if $\lim_{n \to \infty} M(z_{n+p}, z_{n+p}, z_n, t) = 1 \forall t > 0$ and $p > 0$. The space is complete iff every Cauchy sequence is convergent.

**Lemma 4.2.1-[21]** In a M-fuzzy metric space $(Z, M, *)$, $M(x, x, y, t) = M(x, y, y, t)$ holds for each $t > 0$ and for every $x, y \in Z$.

**Lemma 4.2.2-[20]** $M(x, y, z, t)$ is non-decreasing w.r.t $t$ in a M-fuzzy metric space $(Z, M, *) \forall x, y, z, a \in Z$.

**Lemma 4.2.3** M is continuous function on $Z^3 \times (0, \infty)$ with M-fuzzy metric space $Z$.

**Lemma 4.2.4-[21]** $\{z_n\}$ is a Cauchy sequence in M-fuzzy metric space $(Z, M, *)$ with the condition (vi) if there survives $q \in (0, 1)$ resembling that

$M(z_n, z_n, z_{n+1}, t) \geq M(z_{n-1}, z_{n-1}, z_n, t/q) \forall t > 0$ and $n = 1, 2, 3 \ldots$

**Lemma 4.2.5-[21]** M-fuzzy metric space $(Z, M, *)$ with the condition (vi) and satisfying $M(x, y, z, qt) \geq M(x, y, z, t)$ for all $x, y, z \in Z, t > 0$ and positive number $q \in (0, 1)$ implies $x = y = z$.  

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**Definition 4.2.5-[46]** Mappings $P_1$ and $P_2$ defined on M-fuzzy metric space into itself are weakly compatible if $P_1 z = P_2 z$ suggests $P_1 P_2 z = P_2 P_1 z$.

To relate the topic under discussion, the prime mover introduces a new class of implicit relation to submit evidence for invariant point assumption.

**Definition 4.2.6-A class of implicit relation.** Let $F$ be the set of all real continuous functions $f$: $(R^+)^4 \rightarrow R$, non-decreasing and satisfying the following conditions

i. for $u, v \geq 0$, $f(u, u, u, u) \geq 0$ this implies $u \geq 1$.

ii. $f(u, 1, 1, 1) \geq 0$ conveying that $u \geq 1$.

**Definition 4.2.7-** In a M-fuzzy metric space $(Z, M, \ast)$ maps $P_1$ and $P_2$ defined on $Z$ meet the property (E), if with prevailing sequence $\{z_n\}$ we have

$$\lim_{n \to \infty} M(P_1 z_n, a, a, t) = \lim_{n \to \infty} M(P_2 z_n, a, a, t) = 1$$

for some $a \in Z$ and for every $t > 0$.

**Example 4.2.2-** Let $Z = R$ and $M(x, y, z, t) = \exp \{t / (t + |x - y| + |y - z| + |x - z|)\}$ for every $x, y, z \in Z$ and $t > 0$. Define $P_1$ and $P_2$ as $P_1 z = z + 1$, $P_2 z = z + 2$.

Consider the sequence $z_n = n + 1$, $n = 1, 2, \ldots$. Thus we have

$$\lim_{n \to \infty} M(P_1 z_n, 4, 4, t) = \lim_{n \to \infty} M(P_2 z_n, 4, 4, t) = 1$$

for every $t > 0$.

Then $P_1$ and $P_2$ satisfy the property (E).

The other aim of stable point hypothesis is to establish result for compatible mappings of type (P) initiated in M-fuzzy metric space as:

**Definition 4.2.8-[20]** Self-maps $P_1$ and $P_2$ on a M-fuzzy metric space $(Z, M, \ast)$ are compatible if for all $t > 0$, $\lim_{n \to \infty} M(P_1 P_2 z_n, P_2 P_1 z_n, t) = 1$, when $\{z_n\}$ is a sequence in $Z$ specifying $\lim_{n \to \infty} P_1 z_n = \lim_{n \to \infty} P_2 z_n = z$ for some $z$ in $Z$.

**Definition 4.2.9-** Maps $P_1$ and $P_2$ from a M-fuzzy metric space $(Z, M, \ast)$ into itself are declared as compatible of class P if for each $t > 0$,

$$\lim_{n \to \infty} M(P_1 P_2 z_n, P_2 P_2 z_n, P_2 P_2 z_n, t) = 1$$

whenever $\{z_n\}$ is a sequence in $Z$ such that for $\lim_{n \to \infty} P_1 z_n = \lim_{n \to \infty} P_2 z_n = z$ for some $z \in Z$.
Pathak, Cho and Chang [36] proved the subsequent prepositions in Fuzzy metric space which too holds in M-fuzzy metric space, that are-

**Preposition 4.2.2**-Two self-maps \( P_1 \) and \( P_2 \) continuous on Fuzzy metric space \((Z, M, *)\) are compatible if and only if \( P_1 \) and \( P_2 \) are compatible of type (P).

**Preposition 4.2.3**- If self-maps \( P_1 \) and \( P_2 \) on a Fuzzy metric space \((Z, M, *)\) are compatible mappings of type (P) with \( P_1 z = P_2 z \) for some \( z \in Z \), then this implies

\[
P_1 P_1 z = P_1 P_2 z = P_2 P_1 z = P_2 P_2 z.
\]

**Preposition 4.2.4**-Let \( P_1 \) and \( P_2 \) be compatible mappings of type (P) on a Fuzzy metric space \( Z \) and let \( P_1 z_n, P_2 z_n \rightarrow z \) as \( n \rightarrow \infty \) for some \( z \in Z \). Then

\[
i. \quad \lim_{n \to \infty} P_2 P_2 z_n = P_1 z \text{ if } P_1 \text{ is continuous at } z.
ii. \quad \lim_{n \to \infty} P_2 P_2 z_n = P_2 z \text{ if } P_2 \text{ is continuous at } z.
iii. \quad P_1 P_2 z = P_2 P_1 z \text{ and } P_1 z = P_2 z \text{ if } P_1 \text{ and } P_2 \text{ are continuous at } z.
\]

**4.3 - MAIN THEOREM**

**THEOREM-(I)**

**Theorem 4.3.1**-Assume six maps \( P_1, P_2, P_3, P_4, P_5 \) and \( P_6 \) on a complete M-fuzzy metric space \((Z, M, *)\) to oneself with \( P_1, P_2 \) as continuous functions gratifying

(1) \( P_3 P_4 (Z) \subseteq P_6 (Z), P_1 P_2 (Z) \subseteq P_5 (Z) \).
(2) \{ P_3 P_4, P_5 \} and \{ P_1 P_2, P_6 \} are weakly compatible pairs, and
(3) \{ P_3 P_4, P_5 \} or \{ P_1 P_2, P_6 \} satisfy the property (E).
(4) for some \( f \in \Phi, \ u, v, w \in Z, \) and \( t > 0 \)

\[
f \{ M(P_3 P_4 u, P_1 P_2 v, P_5 w, t), M(P_5 u, P_6 v, P_6 w, t), M(P_6 v, P_1 P_2 v, P_1 P_2 w, t),
\]

\[
M(P_1 P_2 v, P_6 v, P_6 w, t) \geq 0.
\]

Then \( P_3 P_4, P_1 P_2, P_5 \) and \( P_6 \) have unique common fixed point.

**Proof**- Let the pair \( (P_1 P_2, P_6) \) satisfy the property (E), then there survives a sequence \( \{ u_n \} \) such that

\[
M(P_1 P_2 u_n, y, y, t) = \lim_{n \to \infty} M(P_6 u_n, y, y, t) = 1.
\]
As, $P_1P_2 (Z) \subseteq P_5 (Z)$, then existing sequence $\{v_n\}$ resembles that $P_1P_2u_n = P_5v_n$.

Then $\lim_{n \to \infty} M(P_6u_n, y, y, t) = 1$.

At this stage, put $u = v_n$, $v = u_n$, $w = u_{n+1}$ in (4), we get

$$f\{M(P_3P_4v_n, P_1P_2u_n, P_1P_2u_{n+1}, t), M(P_5v_n, P_6u_n, P_6u_{n+1}, t),
M(P_6u_n, P_1P_2u_n, P_1P_2u_{n+1}, t), M(P_1P_2u_n, P_6u_n, P_6u_{n+1}, t)\} \geq 0,$$

Taking $n \to \infty$

$$f\{M(P_3P_4v_n, y, y, t), M(y, y, y, t), M(y, y, y, t), M(y, y, y, t)\} \geq 0.$$

$$f\{M(P_3P_4v_n, y, y, t), 1, 1, 1\} \geq 0.$$

$$\lim_{n \to \infty} M(P_3P_4v_n, y, y, t) = 1.$$

Therefore, $\lim_{n \to \infty} P_3P_4v_n = \lim_{n \to \infty} P_5v_n = \lim_{n \to \infty} P_1P_2v_n = \lim_{n \to \infty} P_6v_n = y$.

Since $P_3 P_4 (Z) \subseteq P_6 (Z)$, $P_1 P_2 (Z) \subseteq P_5 (Z)$ then there must exist $x, y \in Z$ such that $P_6q = y$, $P_3p = y$.

Put $u = p, v = u_n, w = u_n$ in condition (4)

$$f\{M(P_3P_4p, P_1P_2u_n, P_1P_2u_n, t), M(P_5p, P_6u_n, P_6u_n, t), M(P_6u_n, P_1P_2u_n, P_1P_2u_n, t),
M(P_1P_2u_n, P_6u_n, P_6u_n, t)\} \geq 0,$$

$$f\{M(P_3P_4p, y, y, t), M(P_5p, y, y, t), M(y, y, y, t), M(y, y, y, t)\} \geq 0,$$

$$f\{M(P_3P_4p, y, y, t), M(y, y, y, t), M(y, y, y, t), M(y, y, y, t)\} \geq 0,$$

$P_3P_4p = y$, which suggests $P_3P_4p = y = P_5p$.

Again put $u = p, v = u_n, w = q$ in condition (4)

$$f\{M(P_3P_4p, P_1P_2q, P_1P_2q, t), M(P_5p, P_6u_n, P_6q, t), M(P_6u_n, P_1P_2u_n, P_1P_2q, t),
M(P_1P_2u_n, P_6u_n, P_6q, t)\} \geq 0,$$

$$f\{M(y, y, P_1P_2q, t), M(y, y, P_6q, t), M(y, y, P_1P_2q, t), M(y, y, P_6q, t)\} \geq 0.$$

$$f\{M(y, y, P_1P_2q, t), M(y, y, y, t), M(y, y, y, t), M(y, y, y, t)\} \geq 0.$$

$$f\{M(y, y, P_1P_2q, t), 1, 1, 1\} \geq 0.$$

$$M(y, y, P_1P_2q, t) \geq 1.$$

Therefore, $P_1P_2q = y$, which intimates $P_6q = P_1P_2q = y$.

Thus, $P_3P_4p = P_5p = P_6q = P_1P_2q$.

Since $(P_3P_4, P_5)$ and $(P_1P_2, P_6)$ are weakly compatible, therefore

$P_3P_4P_5p = P_5P_3P_4p$.

This infers
\[ P_3 P_4 y = P_6 y. \]
Also, \( P_1 P_2 P_6 q = P_6 P_1 P_2 q. \)
Which signifies \( P_1 P_2 y = P_6 y. \)
Therefore, \( y \) is a coincident point of \( P_3 P_4, P_1 P_2, P_5, \) and \( P_6. \)
Now, we shall prove that \( y \) is a fixed point of \( P_3 P_4, P_1 P_2, P_5 \) and \( P_6. \)
Put \( u = p, v = u_n, w = y \) in (4), we get
\[ f \{ M(P_3 P_4 p, P_1 P_2 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 u_n, P_6 y, t)\} \geq 0, \]
\[ f \{ M(y, y, P_1 P_2 y, t), M(y, y, P_6 y, t), M(y, y, P_1 P_2 y, t), M(y, y, P_6 y, t)\} \geq 0, \]
\[ f \{ M(y, y, P_6 y, t), M(y, y, P_6 y, t), M(y, y, P_6 y, t), M(y, y, P_6 y, t)\} \geq 0, \]
Thus \( M(y, y, P_6 y, t) \geq 1. \)
Which implies \( P_6 y = y. \)
\[ P_3 P_4 y = P_3 y = P_6 y = P_1 P_2 y = y. \]
Consequently, \( y \) is a common fixed point of \( P_3 P_4, P_1 P_2, P_5 \) and \( P_6. \)

**Uniqueness**
Let \( z \) be a fixed point other than \( y \) of \( P_3 P_4, P_1 P_2, P_5 \) and \( P_6. \)
Then, \( P_3 P_4 z = P_1 P_2 z = P_5 z = P_6 z = z, \)
Put \( u = y, v = y, w = z \) in (4), we get
\[ f \{ M(P_3 P_4 y, P_1 P_2 y, P_1 P_2 z, t), M(P_3 y, P_6 y, P_6 z, t), M(P_6 y, P_1 P_2 y, P_1 P_2 z, t), \]
\[ M(P_1 P_2 y, P_6 y, P_6 z, t)\} \geq 0, \] implies
\[ f \{ M(y, y, z, t), M(y, y, z, t), M(y, y, z, t), M(y, y, z, t)\} \geq 0. \]
Thus we have
\[ M(y, y, z, t) \geq 1. \]
which implies \( y = z. \)
Hence, \( y \) is a unique fixed point of \( P_3 P_4, P_1 P_2, P_5 \) and \( P_6. \)

**Corollary 4.3.2** Let \( P_1 \) and \( P_3 \) be two continuous self-mappings of a complete \( M \)-fuzzy metric space \((Z, M, *)\). Let \( P_5 \) and \( P_6 \) be two self-mappings of \( Z \) satisfying
(1) \( P_3 (Z) \subseteq P_6 (Z), P_1 (Z) \subseteq P_5 (Z). \)
(2) \( \{ P_3, P_5 \} \) and \( \{ P_1, P_6 \} \) are weakly compatible pairs, and
(3) \( \{ P_3, P_5 \} \) or \( \{ P_1, P_6 \} \) satisfy the property (E).
(4) for some \( f \in F, u, v, w \in Z, \) and \( t > 0 \)
\[ f \{ M(P_3u, P_1v, P_5w, t), M(P_3u, P_6v, P_6w, t), M(P_6v, P_1v, P_1w, t), M(P_1v, P_6v, P_6w, t) \} \geq 0. \]

Then \( P_3, P_1, P_5 \) and \( P_6 \) have unique common fixed point.

**Proof**: Taking \( P_4 = P_2 = \text{Id} = \text{Identity mapping} \) in above theorem, we get the required result.

**THEOREM-II**

**Theorem 4.3.3-** Preassume a complete \( M \)-fuzzy metric space \((Z, M, *)\) with six maps to oneself say \( P_1, P_2, P_3, P_4, P_5 \) and \( P_6 \) of \( Z \) fulfilling the succeeding axioms:

i. \( P_3 (Z) \subseteq P_3P_6 (Z) ; P_4 (Z) \subseteq P_1P_2 (Z) \);

ii. The pair \( \{P_3, P_1P_2\} \) and \( \{P_4, P_3P_6\} \) are compatible mappings of type \( P \).

iii. \( P_3P_6 \) is continuous;

iv. \( M(P_3u, P_4w, P_4w, qt) \geq \min \{ M(P_1P_2u, P_3v, P_4v, t), M(P_1P_2u, P_3v, P_3P_6w, t), \\
M(P_4v, P_3P_6w, P_5v, t), M(P_1P_2u, P_4v, P_5P_6w, t) \} \)

then \( P_3, P_4, P_1P_2 \) and \( P_3P_6 \) have a typical decided point in \( Z \).

**Proof**: Let \( z_0 \) be any erratic point in \( Z \). Thus any sequence \( \{v_n\} \) in \( Z \) relates that

\[
v_{2n-1} = P_3P_6u_{2n-1} = P_3u_{2n-2} \text{ and } v_{2n} = P_1P_2u_{2n} = P_4u_{2n-1}
\]

Put \( u = u_{2n-1} \cdot v = u_{2n-1}, \text{ w } = u_{2n} \).

\[
M(P_3u_{2n-1}, P_4u_{2n}, P_4u_{2n}, qt) \geq \min \{ M(P_1P_2u_{2n-1}, P_3u_{2n-1}, P_4u_{2n-1}, t), M(P_1P_2u_{2n-1}, P_3u_{2n-1}, P_3P_6u_{2n}, t), \\
M(P_4u_{2n-1}, P_3P_6u_{2n}, P_3u_{2n-1}, t), M(P_1P_2u_{2n-1}, P_4u_{2n-1}, P_3P_6u_{2n}, t) \}.
\]

This hints \( M(v_{2n}, v_{2n+1}, v_{2n+1}, qt) \)

\[
\geq \min \{ M(v_{2n-1}, v_{2n}, v_{2n}, t), M(v_{2n-1}, v_{2n}, v_{2n}, t), M(v_{2n}, v_{2n}, v_{2n}, t), \\
M(v_{2n-1}, v_{2n}, v_{2n}, t) \}.
\]

Thus \( M(v_{2n}, v_{2n+1}, v_{2n+1}, qt) \geq M(v_{2n-1}, v_{2n}, v_{2n}, t) \).
This suggests that $M(v_{2n}, v_{2n+1}, v_{2n+1}, t)$ is an increasing sequence of positive real numbers.

Now to prove that $M(v_n, v_{n+1}, v_{n+1}, t)$ converges to 1 as $n \to \infty$.

By lemma (4.2.4), $M(v_n, v_{n+1}, v_{n+1}, t) \geq M(v_{n-1}, v_n, v_n, t/q)$

\[ \geq M(v_{n-2}, v_{n-1}, v_{n-1}, t/q^2) \]

\[ \vdots \]

\[ \geq M(v_0, v_1, v_1, t/q^n). \]

Thus $M(v_n, v_{n+1}, v_{n+1}, t) \geq M(v_0, v_1, v_1, t/q^n)$.

Then by axiom (iv) of $M$-fuzzy metric space.

\[ M(v_n, v_{n+p}, v_{n+p}, t) \]

\[ \geq M(v_n, v_{n+1}, v_{n+1}, t/p)^p \ldots p \text{-times} \ldots M(v_{n+p-1}, v_{n+p-1}, v_{n+p}, t/p) \]

\[ \geq M(v_0, v_1, v_1, t/q^n)^p \ldots p \text{-times} \ldots M(v_0, v_1, v_1, t/pq^{n+p-1}) \]

Thus by (vi) axiom of $M$-fuzzy metric space,

\[ \lim_{n \to \infty} M(v_n, v_{n+p}, v_{n+p}, t) = 1. \]

As $Z$ is complete and $\{v_n\}$ is a Cauchy sequence then for a prevailing point $x$ in $Z$, $v_n \to x$.

Thus $\{P_1P_2u_{2n}\}, \{P_4u_{2n-1}\}, \{P_5P_6u_{2n-1}\}, \{P_3u_{2n-2}\}$ are Cauchy sequences converging to $x$.

Put $u = P_1P_2u_{2n}, v = x, w = P_5P_6u_{2n-1}$ in (iv), we get

\[ M(P_3P_1P_2u_{2n}, P_4P_5P_6u_{2n-1}, P_4P_5P_6u_{2n-1}, qt) \]

\[ \geq \min \{M(P_1P_2P_1P_2u_{2n}, P_3x, P_4x, t), M(P_1P_2P_1P_2u_{2n}, P_3x, P_5P_6P_5P_6u_{2n-1}, t), M(P_4x, P_5P_6P_5P_6u_{2n-1}, P_3x, t), M(P_1P_2P_1P_2u_{2n}, P_4x, P_5P_6P_5P_6u_{2n-1}, t)\}. \]
Currently on using the axiom of compatibility of type $P$ and as $n$ tends to $\infty$

\[ M(P_3x, P_4x, P_4x, qt) \geq \min\{ M(P_3x, P_3x, P_4x, t), M(P_3x, P_3x, P_4x, t), M(P_4x, P_4x, P_3x, t), M(P_3x, P_4x, P_4x, t) \} . \]

Then by lemma (4.2.5) we attain,

\[ M(P_3x, P_4x, P_4x, qt) \geq M(P_3x, P_4x, P_4x, t) . \]

For that reason $P_3x = P_4x$.

Now put $u = P_1P_2u_{2n}$, $v = u_{2n-1}$, $w = u_{2n-1}$ in (iv), we get

\[ M(P_3P_1P_2u_{2n}, P_4u_{2n-1}, P_4u_{2n-1}, qt) \geq \min\{ M(P_1P_2P_1P_2u_{2n}, P_3u_{2n-1}, P_4u_{2n-1}, t), M(P_1P_2P_1P_2u_{2n}, P_3u_{2n-1}, P_3P_6u_{2n-1}, t), M(P_4u_{2n-1}, P_5P_6u_{2n-1}, P_3P_6u_{2n-1}, t), M(P_1P_2P_1P_2u_{2n}, P_4u_{2n-1}, P_5P_6u_{2n-1}, t) \} . \]

Now on taking the limit as $n \to \infty$ and on using (ii) we come to have

\[ M(P_3x, x, x, qt) \geq \min\{ M(P_3x, x, x, t), M(P_3x, x, x, t), M(x, x, x, t), M(P_3x, x, x, t) \} . \]

Thus $M(P_3x, x, x, qt) \geq M(P_3x, x, x, t)$.

On that account $P_3x = x$.

This implies $P_3x = P_4x = x$.

Now put $u = P_3u_{2n-2}$, $v = P_3u_{2n-2}$, $w = x$ in (iv) we get

\[ M(P_3P_3u_{2n-2}, P_4x, P_4x, qt) \geq \min\{ M(P_1P_2P_3P_3u_{2n-2}, P_3P_3u_{2n-2}, P_4P_3u_{2n-2}, t), M(P_1P_2P_3u_{2n-2}, P_3P_3u_{2n-2}, P_3P_6x, t), M(P_4P_3u_{2n-2}, P_3P_6x, P_3P_3u_{2n-2}, t), M(P_1P_2P_3P_3u_{2n-2}, P_4P_3u_{2n-2}, P_3P_6x, t) \} . \]

Now on applying the limit $n \to \infty$ and on dealing with (ii) & (iii), the initiator gains

\[ M(P_1P_2x, x, x, qt) \geq \min\{ M(P_1P_2x, P_1P_2x, x, t), M(P_1P_2x, P_1P_2x, x, t), M(P_4x, x, P_1P_2x, t), M(P_1P_2x, P_4x, x, t) \} . \]
Since $P_4x = x$, thus

As a result of lemma (4.2.5) we have $P_1P_2x = x$.

Thus $P_3x = P_4x = P_1P_2x = x$.

Put $u = x$, $v = x$, $w = P_4u_{2n-1}$ in (iv) we get

$M(P_3x, P_4P_4u_{2n-1}, P_4P_4u_{2n-1}, qt) \geq \min \{M(P_1P_2x, P_3x, P_4x, t), M(P_1P_2x, P_3P_6P_4u_{2n-1}, t), M(P_4x, P_5P_6P_4u_{2n-1}, P_3x, t), M(P_1P_2x, P_4x, P_5P_6P_4u_{2n-1}, t)\}.$

Again on applying the limit $n \to \infty$ and working on (ii) & (iii), the prime mover develops

$M(x, P_5P_6x, P_5P_6x, qt)$

$\geq \min \{M(x, x, x, t), M(x, x, P_5P_6x, t), M(x, P_5P_6x, x, t), M(x, P_5P_6x, t)\}.$

On using lemma (4.2.5) we have

$P_5P_6x = x.$

which makes $P_3x = P_4x = P_1P_2x = P_5P_6x = x.$

This shows $x$ is the firm point of $P_3$, $P_4$, $P_1P_2$ and $P_5P_6$.

**Corollary 4.3.4**- Presume a complete $M$-fuzzy metric space $(Z, M, *)$ with four maps to oneself say $P_1$, $P_2$, $P_3$ and $P_4$ of $Z$ fulfilling the succeeding axioms:

i. $P_3(Z) \subseteq P_2(Z); P_4(Z) \subseteq P_1(Z);$

ii. The pair $\{P_3, P_1\}$ and $\{P_4, P_2\}$ are compatible mappings of type P.

iii. $P_2$ is continuous;

iv. $M(P_3u, P_4w, P_4w, qt) \geq \min \{M(P_1u, P_5v, P_4v, t), M(P_1u, P_3v, P_2w, t), M(P_4v, P_2w, P_3v, t), M(P_1u, P_4v, P_2w, t)\}$

then $P_1$, $P_2$, $P_3$ and $P_4$ have a typical decided point in $Z.$
4.4 - CONCLUSIONS

In the denouement while doing work on M-fuzzy metric space, the present author interprets a typical firm statement drawn by fulfilling the condition of weak compatibility. The result is obtained by defining a new class of implicit relations. A corollary is also cited in concern with the first aim showing that the above conditions could be used to acquire a common fixed point theorem for four self-mappings.

On the other hand the author finishes with a fixed point result using compatible mappings of type (P) obtained by considering six self-maps.