CHAPTER 1

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The classic fixed point theorem of Banach is an important landmark in the history of functional analysis and has had applications in various branches of pure and applied mathematics. With the help of this theorem it has been possible to discuss many apparently diverse questions from algebra and analysis on the same platform establishing thereby a unifying approach to those problems. No systematic study has been undertaken so far to apply this theorem in applied mathematics. The contraction principle has a great many applications and they are scattered throughout almost all branches of mathematics. For instructive and interesting applications we refer to Graffi (1951), (1954), Hayden and Suffridge (1976) and Earle and Hamilton (1970).

Though Banach established his theorem as early as 1922, its extensions and generalizations started only at the beginning of last decade. Today these generalizations occupy a considerable place in literature and form a branch of study on its own right. A minor short coming of Banach's theorem is that the mapping here is necessarily continuous and hence it is not applicable to the problem where the mapping is discontinuous. The generalizations, which appeared till 1968-1969, suffers from the same defect.
A break through was made in 1968 when Kannan published a fixed point theorem in the Bulletin of Calcutta Mathematical Society generalizing Banach's theorem. In Kannan's theorem the mapping, though contractive type, need not be continuous. After the publication of Kannan's work, there appeared a number of papers generalizing Kannan's work, where the mappings are contractive type and are not necessarily continuous. Among all the generalizations Cirić's quasi contractions is the most effective one and Cirić's theorem on quasi contraction mappings generalizes all the fixed point theorems including Banach's. Also some generalizations had been done by Pittnauer in which he had considered four points of the space under consideration instead of two points considered earlier.

An attempt has been made in the present thesis to extend some of these contractive type mappings and to establish their fixed points. The thesis is divided into eight chapters including this one which gives the general introduction.

In Chapter 2 we have studied a fixed point theorem for pair of mappings involving four points of the space under consideration, and which satisfy a non-linear contraction type inequality. This result improves the
result obtained by Pittnauer and also contains as special cases some well-known results. As application of this result we have proved first a theorem for family of mappings which are commuting. Also we have proved three theorems for sequence of maps which are not necessarily continuous and commuting.

Chapter 3 is devoted to a discussion on non-unique fixed point theorem. From this result the theorem of Pachpatte has been derived as a special case.

Chapter 4 deals with existence of fixed points for operators mapping a Banach space into itself. In doing so we have not assumed the space under consideration to be uniformly convex or reflexive. We have also established the existence of fixed points for star-shaped space and Hilbert space.

Chapter 5 deals with the existence of fixed points for generalized multi valued contraction mappings. First we derive a fixed point theorem for such mappings, extending the result of Ćirić and then prove two theorems which are the direct applications of the first theorem. In doing so we have indeed generalized some well-known results for multi valued contractions.
In chapter 6 we have established some conditions which guarantee not only the uniqueness of solution of the equation in the Darboux problem but also the convergence of successive approximations. These conditions are obtained without following the usual method of proving the convergence of successive approximations, but instead it has been shown that those results all follow as a consequence of certain general theorem concerning mappings defined on some appropriate function spaces. In this process we have seen that the observations made by Patczewski and afterwards by Wong for the uniqueness of solutions and convergence of successive approximations in the Darboux problem for partial differential equations hold also in the case of hyperbolic partial differential equation with certain modifications.

Chapter 7 deals with the extension of contraction type mapping to non archimedean probabilistic metric spaces.

The idea of fixed point plays a very important role in solving deterministic operator equations. Recently the idea of random fixed point theorems which are the stochastic generalization of the classical fixed point theorems has become a very important part of the theory of some operator equations which can be regarded as random
operator equations. Chapter 8 is devoted to establish a fixed point theorem for a pair of random generalized non-linear contraction mappings involving four points of the space under consideration. It is interesting to note that with suitable modification of the conditions of the theorem we can easily obtain stochastic generalizations of the results of different classical fixed points.