CHAPTER 2

Interactions of Nuclear Radiation with matter
2.1: Introduction:

In nuclear science two sources of electromagnetic radiation are of interest, gamma rays and X-rays. The former have their origin in the nucleus and are emitted when energetically excited nuclei decay to a lower energy level. This de-excitation emission of gamma rays is analogous to the emission of X-rays which occurs when a transition takes place between an excited atomic state and one of lower energy. Since nuclei exist in discrete energy levels, a de-excitation process will occur between two such levels and the resulting gamma ray will have definite, discrete energy. X-ray energies are usually on the order of 0 - 50 keV while gamma ray energies range from several keV to several MeV. The mode of interaction with matter will be the same for X-rays as for gamma rays and it is a strong function of the energy. The average specific ionization will be only 1/10 to 1/100 as greater as for electrons and, consequently, practically all the ionization by gamma rays is secondary in nature arising from the ionization produced by the stopping of the primary ion pairs. The energy absorption in a medium can be calculated by well-established formulae if certain constants like
attenuation coefficients, absorption coefficients, effective atomic number etc. are known.

With the increasing use of gamma ray emitting isotopes in industry, medicine, agriculture etc. the study of the absorption of gamma radiation in materials has become an important subject. For scientific interest, the study on the interaction of gamma radiations with matter yields useful informations for designing experimental arrangements for handling the isotopes decaying by gamma emission. The penetration of gamma rays through pure metals is well understood. Gamma rays traversing matter interact with it through separate elementary processes in succession of processes on the over all propagation of gamma rays has been under study in recent years.

There are three mechanisms for interaction of gamma rays or X-rays with matter as discussed in detail below.

2.2 : Compton effect:

Rayleigh elastic scattering and the photoelectric absorption both depend upon the reaction of an atom as a whole. As energy approaches 0.5 MeV, wavelength becomes shorter and there is a greater tendency to interaction to take place with individual electrons. Thomson theory of free electron scattering breaks down and the interaction can
be regarded as a collision of a photon with an electron. This process is known as Compton effect. It is maximum around 1 MeV and is important between 0.5 MeV and 10 MeV. When a photon of energy $h\nu$ strikes the perfectly free electron (at rest), the photon with diminished energy $h\nu'$ is scattered at an angle $\theta$ with a direction of the incident photon and the electron recoils at an angle $\phi$, as shown in fig.(2.1(a)). From the law of conservation of energy and momentum in directions both parallel and perpendicular to the incident photon, the change in the wavelength for the scattered photon is given by the relation

$$\Delta \lambda = \lambda' - \lambda = \frac{h}{m_0c} (1 - \cos \theta) \quad (2.1)$$

where $\Delta \lambda$ is known as Compton shift and the constant $h/m_0c$ the Compton wavelength having value $2.42621 \times 10^{-12}$ m.

Compton shift in wavelength is independent of the incident wavelength and the material of the scatter. It is dependent on the angle $\theta$. Since $\lambda = c/\nu$, Eq. 2.1 gives

$$h\nu' = \frac{h\nu}{1 + (1 - \cos \theta)\zeta}$$

where $\zeta = h\nu/m_0c^2$. For very large incident photon energy ($h\nu \gg m_0c^2$), the energy of the backscattered photon approaches $\frac{1}{2}m_0c^2 = 0.25$ MeV at $\theta=180^\circ$ and 0.51 MeV at $\theta=90^\circ$.

The kinetic energy of the recoil electron is

$$T = h\nu - h\nu' = h\nu \frac{(1 - \cos \theta)\zeta}{1 + (1 - \cos \theta)\zeta} \quad (2.2)$$
The kinetic energy of the electron has its maximum value when θ = 180° and photon is scattered directly backward. The energy is zero for θ = 0 and ν = ν'.

\[ T_{\text{max}} = \frac{h\nu}{1 + 1/2 \zeta} \]

The angle θ and φ can be related as

\[ \cot \phi = (1 + \zeta) \tan \frac{1}{2} \theta \]

The evaluation of the differential cross section is complicated as it requires relativistic wave mechanics. Klein and Nishina (1929) derive a relation for the differential cross-section per electron in the case of a linearly polarised incident plane electromagnetic wave given as

\[ \left[ \sigma_o(\theta) \right]_{\text{pol}} = \left[ \frac{d\sigma_c}{d\Omega} \right]_{\text{pol}} \]

\[ = \frac{1}{4} r_o^2 \left( \frac{\nu}{\nu'} \right)^2 \left( \frac{\nu}{\nu'} + \frac{\nu'}{\nu} + 4\cos^2 \theta - 2 \right) \]

where θ is the angle between the polarisation directions of the incident and emergent ray. If the incident wave is unpolarised, the differential cross section will be the mean value of the above equation averaged over all angles θ, as

\[ \left[ \sigma(\theta) \right]_{\text{unpol}} = \frac{1}{2} r_o^2 \left( \frac{\nu}{\nu'} \right)^2 \left( \frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin \theta^2 \right) \]

\[ = \frac{1}{2} r_o^2 \frac{-\xi \cos \theta + (\xi - 3)(1 + \xi + 1)(1 + \xi - 1) - \xi (2 \xi + 1) \cos \theta}{1 + \xi (1 - \cos \theta)^3} \]
For low energies $\xi \to 0$ and the above relation reduces to the Thomson cross-section. Total cross-section is obtained by integration over all angles ($0 < \theta < \pi$) as

$$
\sigma_c = \int_0^\pi \sigma_c(\theta) \, d\Omega = \int_0^\pi \sigma_c(\theta) \, 2\pi \sin \theta \, d\theta
$$

$$
= 2\pi r_o^2 \left\{ \frac{1+\xi}{\xi^2} \left[ \frac{2(1+\xi)}{1+2\xi} - \frac{1}{\xi} \log_e (1+2\xi) \right] + \frac{1}{2\xi} \log(1+2\xi) - \frac{1 + 3\xi}{(1+2\xi^2)} \right\}
$$

(2.4)

When $\xi$ is small, above relation reduces to

$$
\sigma_c = \pi r_o^2 (1 + 2\xi + 5.2 \xi^2 + 13.2 \xi^3 + \ldots)
$$

as the scattered energy is smaller than the incident energy by a factor $h\nu'/h\nu$. Hence the energy scattering cross section

$$
\sigma_s = (h\nu/h\nu')\sigma_c
$$

(2.5)

The energy absorption cross section $\sigma_a (= \sigma_c - \sigma_s)$ represents the probability for the recoil energy to be imparted to the electron in this process. The atomic Compton cross-section

$$
\sigma_C = Z \sigma_c
$$

The scattering is coherent when $h\nu = h\nu'$ and is incoherent when $h\nu < h\nu'$. In the former case the individual amplitudes for each of the atomic electrons are added and in the latter case the intensities are added. Thus the atomic differential Compton cross-section can be defined as
\[ \sigma(\theta) = \sigma_{\text{coh}}(\theta) + \sigma_{\text{incoh}}(\theta) \] (2.6)

From the above relations it is clear that the total scattering coefficient per electron decreases with increasing photon energy and this decrease is quite slow at low values of energy and for energies above 0.5 MeV, \( \sigma_c \) is roughly proportional to \( (\nu) \)^{-1}. This decrease is much slow than does photoelectric absorption, even in heavy elements.

2.3 : Photoelectric Effect :

Below energies of about 0.1 MeV the predominant mode of gamma ray interaction in all medium and high-Z absorbers is the photoelectric processess.

a) General features of the photoelectric interaction :

An incident photon cannot be totally absorbed by a free electron. Total absorption can takes place with the electron is initially bound in an atom. Then momentum is conserved by the recoil of the entire residual atom. As might be expected, the most tightly bound electrons have the greatest probability of absorbing a photon which is incident upon an atom. It is found both experimentally and theoretically that about 80% of the photoelectric absorption processes take place in the K shell, provided that the incident photon energy \( \nu \) clearly exceeds the K shell binding energy.
Because the entire atom participates, the photoelectric process may be visualised as an interaction of the primary photon with the atomic electron cloud in which the entire photon energy $h\nu$ is absorbed and an electron is ejected from the atom with an energy

$$T = h\nu - B$$

where $B$ is the binding energy of the ejected electrons. The remainder of the energy appears as characteristics X-rays and Auger electrons from the filling of the vacancy in the inner shell. Schematically the photoelectric interaction is illustrated in fig. (2.1(b)).

b) Directional distribution of photoelectrons:

The discrete energy distribution of photoelectrons provides a useful experimental method for the determination of gamma ray energies. In such work the angular distribution of photoelectrons is often relevant. Especially at low photon energies, the photoelectrons tend to be ejected in the direction of the electric vector of the incident radiation, hence at right angles to the direction of incident. At higher energies the angular distribution is more in the forward direction. Of course, the kinetic energy of the photoelectron

$$T = h\nu - B$$
is the same for all directions of emission.

c) Average forward momentum:

Whenever $hv$ clearly exceeds the electron binding energy, the resulting photoelectron will have nearly the same energy as the incident photon. But because of the finite rest mass of the electron, its momentum greatly exceed the momentum of the incident photon. This increase in momentum, combined with the predominantly forward directional distribution of the photoelectrons, means that the residual atom must, on the average, have a finite backward momentum.

Helme (1931) has evaluated by numerical integration the average forward momentum of photoelectrons ejected from atoms for which $Z/137 << 1$.

d) Cross section per atom:

The absolute probability of a photoelectric interaction is described by the atomic cross-section $\sigma$ cm$^{-2}$/atom. Nearly all the theories relates only to interactions with the K electrons and hence give a partial cross section $\sigma^\text{K}$.

e) Linear attenuation coefficient $\tau$ for the photoelectric effect:

The linear attenuation coefficient $\tau$ cm$^{-1}$ is given by
\[ \tau = a^\tau N \]

where \( N \) is the no. of atoms per cubic cm. and \( a^\tau \) is the atomic cross section in square cms per atoms.

2.4: Pair production:

Above incident photon energies of 1.02 MeV, a third type of interaction becomes increasingly important. In this interaction, known as pair production, the photon is completely absorbed and in its place appears a positron–negetron pair whose total energy is just equal to \( h\nu \).

Thus

\[ h\nu = \left( T_- + m_0c^2 \right) + \left( T_+ + m_0c^2 \right) \]  

(2.7)

where \( T_- \) and \( T_+ \) are the kinetic energy of the negatron and positron, respectively, and \( m_0c^2 = 0.51 \text{ MeV} \) is the electronic rest energy. The process occurs only in the field of the charge particles, mainly in the nuclear field but also to some degree in the field of an electron. The presence of this particle is necessary for momentum conservation. Schematically the over-all process is shown in Fig.(2.1(c)).

a) Basis of the theory of pair production:

The pair production process lends itself nicely to a distribution in terms of the Dirac electron theory, which
considers positron as a holes in an otherwise completely filled sea of negative energy states of negatrons. This feature of Dirac theory is illustrated in fig (2.2), where we see that to lift a negatron out of an negative energy state requires crossing an energy barrier of \(2m_e c^2\). The negatron is created in a positive energy state and so becomes observable, having the same properties as a negatron except that its charge has the opposite sign. The total energy between the hole and an electron is just equal to \(hv\).

The pair production process is intimately related to the Bremsstrahlung process. In Bremsstrahlung, an electron undergoes a transition between two states, both of positive energy, and a photon is emitted instead of being absorbed. Mathematically, the theories of the two processes are nearly identical and are usually treated together.

b) Angular distribution of pair electrons:

The distribution of the positron and negatron is mainly forward for incident quanta of very high energy. The average energy between the incident quantum and the created electrons is Heitler (1934) of the order of \(m_e c^2 / T\), for \(T >> m_e c^2\). For incident photon energies of the order of \(2m_o c^2\) the angular distribution is much more complicated, and the emphasis on the forward direction is much less marked.
c) Energy distribution of pair electrons:

The differential cross section \( (\alpha_K) \) cm\(^2\) per nucleus, for the creation positron of the kinetic energy \( T_+ \) (and a negatron of a kinetic energy \( \nu - 2m_o c^2 - T_+ \)), can be written as

\[
d(\alpha_K) = \frac{\sigma_0 Z^2 P}{\nu - 2m_o c^2} dT_+ \quad (2.8)
\]

where

\[
\sigma_0 = 5.8 \times 10^{-28} \text{ cm}^2/\text{nucleus} \quad (2.9)
\]

and the dimensionless quantity \( P \) is complicated function of \( \nu \) and \( Z \), which varies only between 0 (for \( \nu < 2m_o c^2 \)) and about 20 (for \( \nu = \infty \)), for all values of \( Z \).

d) Screening corrections:

For very high energy photons an appreciable contribution to the pair production cross section may come from a distance \( r \) from the nucleus which is greater than a radius of the K electron shell. Then the effective nuclear charge is reduced, because of screening by the charge of the atomic electrons. In the Bethe-Heitler theory of pair production the effect of the atomic electrons is approximated by using the Thomas-Fermi statistical model of the atom. Then the electrostatic potential \( U(r) \) at a distance \( r \) from the centre of the nucleus can be represented as approximately
\[ U(r) = \frac{Ze}{r} e^{-r/a} \quad (2.10) \]

where \( a = \frac{\hbar^2}{m_0 c^2 z^{1/3}} = \frac{a_H}{z^{1/3}} \)

is the radius of a sphere which encloses a fixed fraction of the atomic electrons, or, more closely, the radius of a atomic electron cloud which screens the nucleus Schiff (1949). The Thomas Fermi model should be reasonably good for heavy elements and poorest for the light elements which contain too few atomic electrons to justify statistical averaging.

e) Total pair production cross section per nucleus:

The total nuclear pair production is given by the equation

\[ a_k = \int d(a_k) = \sigma Z_o^2 \int_0^{\frac{h\nu-2m_o c^2}{h\nu-2m_o c^2}} P \frac{dT^+}{h\nu-2m_o c^2} \quad (2.11) \]

\[ a_k = \sigma_o Z_o^2 \int_0^1 P \left( \frac{T^+}{h\nu-2m_o c^2} \right) \]

\[ = \sigma_o Z_o^2 \bar{P} \text{ cm}^2/\text{nucleus} \]

where \( \bar{P} \) can be regarded as a average value of \( P \). \( \bar{P} \), and hence \( a_k \), increases approximately logarithmically with \( h\nu \).
Analytical integration of integration (2.11) is possible only for extremely relativistic cases and gives, when screening is neglected Bethe (1949),

\[ a_k = \sigma_\infty^2 \left( \frac{28}{9} \ln \frac{2h\nu}{m_0c^2} - \frac{218}{27} \right) \]  

(2.12)

In the case of complete screening one obtains

\[ a_k = \sigma_\infty^2 \left( \frac{28}{9} \ln \left( \frac{183}{27} \right)^{-1/3} \right) \]  

(2.13)

For \( h\nu \gg 137m_0c^2 Z^{-1/3} \). It can be seen from this equation for very high energies ( \( \sim 10^4 \) MeV ) the pair production cross section depends only on screening and is independent of photon energies.

f) Pair production linear attenuation coefficient:

The linear attenuation \( k \) for pair production is simply

\[ k = \frac{a_k N}{\text{cm}} \]

where \( N \) is the no. of atoms per cubic cm.

g) Energy absorption:

When a photon is absorbed in a pair production encounter only a portion of its energy appears at once as a kinetic energy of the electron pair. Thus, by analogy with our considerations of the compton and photoelectric processes the true primary absorption coefficient may be written as
The remaining $2m_0c^2$ of the total photon energy $h\nu$ resides in the rest masses of the electron pair. This energy is given up after the positron has been slowed down by ionising and radiative collision and has annihilated itself by combining with some negatron. The resulting two 0.51 MeV annihilation photons are then emitted from the scene of the annihilation. Their directional distribution is random, hence isotropic with respect to the direction of the original primary photon. This annihilation photons play the role of scattered radiation, when the overall energy absorption process is considered. A pair production scattering coefficient $k_s$ is given by

$$k_s = k \left(1 - \frac{2m_0c^2}{h\nu}\right)$$

(2.14)

The total pair production attenuation coefficient $k$ is then made up of a true absorption $k_a$ and a scattering coefficient $k_s$, or

$$k = k_a + k_s$$

At energies below about 3 MeV, the Compton effect dominant, so that this scattering correction has a very small effect on the total scattering. At high energies, the
fraction $2m_0c^2/\hbar\nu$ becomes small, and again the scattering has only a small effect on any total absorption.

2.5: Definitions of quantities:

a) The mass attenuation coefficient ($\mu_m/\rho$):

The mass attenuation coefficient is defined as

$$\frac{\mu_m}{\rho} = \frac{N_A}{u\ A} \cdot \sum_j \sigma_j = \frac{N_A}{u\ A} \cdot \sigma = \sum_j \frac{\mu_j}{\rho} \quad (2.16)$$

where $\rho$ is the density of the target, $N_A$ is Avogadro's number, $u$ is the atomic mass unit ($1/12^{th}$ of the mass of an atom of nuclide $^{12}\text{C}$), $A$ is the relative atomic mass of the target element, and $\sigma$ is the total cross section for an interaction by a photon with energy $E$. The above Eq. (2.17) is written to indicate that total interaction cross-section (and mass attenuation coefficients) for interaction of the $j^{th}$ type.

b) The mass energy transfer coefficient ($\mu_{tr}/\rho$):

The transfer of energy from photons to matter is a two-step process, involving first the conversion of photon energy into kinetic energy of electrons and to rest energy of electron pairs and second, dissipation of the kinetic energy of secondary electrons along their tracks in
collision with other electrons, to produce excitation and ionisation of the atoms of the medium.

The product of average fraction of incident photon energy given to the charged particles and probability of number of interactions between incident photon and matter per unit thickness for interactions with matter \((\mu_m/\rho)\) is called mass energy transfer coefficient. It depends upon photon energy and nature of interaction involved.

It is defined as:

\[
\frac{\mu_{tr}}{\rho} = \frac{N_A}{u} \cdot \sum_j f_j \sigma_j = \sum_j f_j \left( \frac{\mu_j}{\rho} \right)
\]

\[(2.17)\]

where \(f_j\) is the average fraction of photon energy \(E\) that is transferred to kinetic energy of charged particles in interactions of type \(j\), and \(f\) is the weighted average of \(f_j\).

c> Mass energy transfer coefficient for compton effect:

Mass energy transfer coefficient for compton effect is given by

\[
\frac{\sigma_{tr}}{\rho} = \frac{N_A}{A} \cdot e^{\sigma_{KN}} \text{ (cm}^2/\text{gm)}
\]

where \(e^{\sigma_{KN}} = \text{Klein Nishina cross section for electron.}\)
d) Mass energy transfer coefficient for photoelectric effect:

According to law of conservation of energy, fraction of incident photon energy transferred to photoelectrons is given by

$$\frac{T}{h_\nu} = \left( \frac{h_\nu - E_b}{h_\nu} \right)$$  \hspace{1cm} (2.18)

where $E_b$ = Binding energy of electron and $T$ is the kinetic energy given to the electron.

When an electron is removed from the atomic shell, the resulting vacancy is filled by an electron from less tightly bound shell, accompanied by emission of fluorescence x-ray energy $h_\nu_k$ or $h_\nu_L$ for K or L shell vacancies. The probability of this happening is called fluorescence yield $Y_k$ or $Y_L$.

In addition to fluorescence yield, another function $P_k$ or $P_L$ is defined as a fraction of all photoelectric interactions that occur in K shell or L shell. The product $P_k Y_k$ or $P_L Y_L$ is the fraction of all photoelectric events in which K or L fluorescence x-ray is emitted in the atom. Then $P_k Y_k h_\nu_k$ or $P_L Y_L h_\nu_L$ is mean energy carried away from the atom by K or L fluorescence x-ray per photoelectric in all shells. The mean energy transferred to charged particles per photoelectric events for K shell is given by
\[ h\nu - E_b = h\nu - P_{YK}^{(Y)} h\nu_K \]

The photoelectric mass energy transferred coefficient is given by

\[ \frac{\tau_{tr}}{\rho} = \frac{\tau_{tr}}{\sigma} \left( \frac{h\nu - P_{YK}^{(Y)} h\nu_K}{h\nu} \right) \]

\[ (2.19) \]

\[ (2.20) \]

e> Mass energy transfer coefficient for pair production:

The fraction of the incident photon's energy that is transferred to kinetic energy of charged particles for nuclear and electron pair production, is \((h\nu - m_0c^2)/h\nu\)

So, mass energy transfer coefficient for pair production is given by

\[ \frac{K_{tr}}{\rho} = \frac{K}{\rho} \left( \frac{h\nu - 2m_0c^2}{h\nu} \right) \]

\[ (2.21) \]

f> Total mass energy transfer coefficient \((\mu_{tr}/\rho)\):

The total mass energy transfer coefficient for gamma ray interactions is given by

\[ \frac{\mu_{tr}}{\rho} = \frac{\tau_{tr}}{\rho} + \frac{\sigma_{tr}}{\rho} + \frac{K_{tr}}{\rho} \]

\[ = \frac{\tau}{\rho} \left( \frac{h\nu - P_{YK}^{(Y)} h\nu_K}{h\nu} \right) + \frac{\sigma}{\rho} \left( \frac{\tau}{h\nu} \right) + \frac{K}{\rho} \left( \frac{h\nu - 2m_0c^2}{h\nu} \right) \]
g> Mass energy absorption coefficient:

The mass energy absorption coefficient is a measure of the average fraction amount of the incident photon energy transferred to the kinetic energy of charged particles as a result of these interactions. This imparted charged particle kinetic energy is the amount of the energy available for the production of chemical, biological and other effects associated with exposure to ionizing radiations. It is defined as

\[
\frac{\mu_{en}}{\rho} = \frac{N_A}{\mu \cdot A} \sum_j f_j (1 - g_j) \sigma_j
\]

\[
= f \sum_j (1 - g_j) \left( \frac{\mu_j}{\rho} \right)
\]

\[
= f(1-g)\mu = (1-g)(\mu_{tr}/\rho)
\]

where \( g_j \) is the average fraction of the kinetic energy of secondary charged particles produced in an interaction of type \( j \) that is subsequently lost in radiative processes, and \( g \) is the weighted average of \( g_j \). Evaluation of \( g \) takes into explicit account.

a) the emission of bremsstrahlung,
b) positron annihilation in flight,
c) fluorescence emission following electron and positron impact ionisation and
d) the effect on the processes of energy loss straggling and knockon electron production as secondary particles slow down (i.e. of going beyond the continuous slowing down approximation).

For low $Z$ and $h$, $g \longrightarrow \theta$

\[
\frac{\mu_{rn}}{\rho} = \frac{\mu_{tr}}{\rho}
\]

For homogeneous mixtures and compounds, the mass attenuation coefficient can be expressed as

\[
\frac{\mu_m}{\rho} = \sum_j \frac{W_j}{\rho} \frac{N_A}{\rho} \sum_i \sigma_{jl}
\]

\[
= \sum_i W_i \left( \frac{\mu_m}{\rho} \right)_i
\]

Where $W_i$ is the fraction by weight of the $i^{th}$ atomic constituent. Similarly, the mass energy transfer coefficient can be written as

\[
\frac{\mu_{tr}}{\rho} = \sum_i W_i \frac{N_A}{\rho} \sum_j \sigma_{ji} f_{ji}
\]

\[
= \sum_i W_i \left( \frac{\mu_{tr}}{\rho} \right)_i
\]

Because it includes factors which depends upon the slowing down of the secondary charged particles in the medium as a whole.
Fig. 2: a) Photoelectric effect
b) Compton scattering
c) Pair production
Fig. 2.2: Schematic representation of the positive and negative energy states of the electron.