CHAPTER 4

Mixture Rule for Liquid
4.1 Mixtures of material:

The primary attenuation of gamma rays in chemical compounds or other mixtures of elements is assumed to depend only upon the sum of the cross sections presented by all the atoms in the mixture. Because chemical bonds are only of the order of a few electron volts, there have no significant effects on the Compton, photo or pair interactions.

Mass attenuation coefficient for mixtures is given by

\[
\frac{\mu}{\rho} = \sum W_i \frac{\mu_i}{\rho_i}
\]  \hspace{1cm} (1)

where \( \rho \) is the density, and which is made up of mixture of elements whose mass attenuation coefficients are \( \mu_i/\rho_i \). \( W_i \) is the proportion by weight of the \( i \)th element constituent with mass attenuation coefficients \( \mu_i/\rho_i \).

By using mixture rule Hubbell (1982) has calculated mass attenuation \( \frac{\mu}{\rho} \) for photon energies 1 keV to 20 MeV for 40 elements ranging from hydrogen \( (Z=1) \) to Uranium \( (Z=92) \) for mixtures and compounds.

We develop here mixture rule for liquid as follows.
Let $M_1$ be the mass of soluble salt in gm, $V_1$ is its volume in solid form, $M_2$ mass of water which is equal to its volume $V_2$.

The density of the solution $\rho$ is given by

$$\rho = \frac{M}{V} = \frac{M_1 + M_2}{V_1 + V_2} \quad \text{(2)}$$

If $\rho_s$ is the density of the solid soluble (crystal), then $M_1 = \rho_s V_1$. The concentration $c$ of the solution is $M_1/V_2$. The density of the solution in terms of concentration $c$ then becomes

$$\rho = (1 + c)/(1 + \frac{c}{\rho_s}) \quad \text{(3)}$$

Thus in the absence of solute $c = 0$, $\rho = \rho_w = 1$. Also as $c \to \infty$, $\rho \to \rho_s$. The mixture for salt solution in water read as

$$\frac{\mu}{\rho} = \frac{\mu_s M_1}{\rho_s M} + \frac{\mu_w M_2}{\rho_w M} \quad \text{(4)}$$

In terms of concentration $c$ this becomes

$$\frac{\mu}{\rho} = \frac{\mu_s c}{\rho_s (1+c)} + \frac{\mu_w c}{(1+c)} \quad \text{(5)}$$

or

$$\frac{\mu}{\rho} = \left(\frac{\mu_s}{\rho_s} - \mu_w\right) \frac{c}{(1+c)} + \mu_w \quad \text{(6)}$$
Multiplying Eq. (5) by \( \rho \) using eqn. (3) and simplifying, we get

\[
\mu = \left( \frac{\mu_s - \mu_w}{\rho_s} \right) \frac{c}{1 + \frac{c}{\rho_s}} + \mu_w \tag{7}
\]

For dilute solutions (low concentration) \( c/\rho_s \ll 1 \) and eqn. (7) reduces to

\[
\mu = \left( \frac{\mu_s - \mu_w}{\rho_s} \right) c + \mu_w \tag{8}
\]

This is the equation of straight line giving linear dependence of attenuation coefficient on concentration for dilute solutions. The graph of \( \mu \) against \( c \) thus provides the linear relationship.

\[
\mu = mc + a \tag{9}
\]

with the slope \( m \) having dimensions of linear absorption coefficient and the intercept \( a \) corresponding to \( \mu_w \) (since in the absence of solute, \( c = 0 \)). Comparing eqn. (9) with (8) we get,

\[
m = \frac{\mu_s - \mu_w}{\rho_s} \tag{10}
\]

this gives \( \mu_s = m \rho_s + \mu_w \tag{11} \)
Thus measurement of $m$ enables us to calculate $\mu_s$ and hence $\mu_s/\rho_s$. The intercepts of the lines corresponding to $c = 0$ give values of $\mu_w$. 