CHAPTER 3

SELECTION OF GEARS BASED ON FEA

3.1 INTRODUCTION

With the continued increase in the load and speed of gear used in modern service conditions, the requirement for adequate design procedures to accurately evaluate the strength of gear teeth becomes vital. Consideration of the strength of teeth is normally divided into two parts, namely bending strength and strength under contact load conditions. Though a large number of methods are available, the Finite Element Analysis (FEA) a numerical technique is used for obtaining an approximate solution for a wide variety of engineering problems. It has two characteristics that differentiate it from other numerical methods.

- This method utilizes an integral formula to generate a system of algebraic equation
- It uses continuous piecewise smooth functions for approximating the unknown quantities

The static analysis aims at the study of the variation of stresses and deflections on the load surface. Before going for fabrication, it is necessary to study the performance of this type of gear using Finite Element Method (FEM). To analyse geometry complicated elastic body, like gears FEM is used very often. The analysis is conducted for the following gear concepts and based on the results; gears which perform better are selected for further FEA and
experimental analysis. Here ANSYS 14.5 workbench is used to conduct Finite Element Analysis.

3.2 SELECTION OF GEARS FOR INVESTIGATION

3.2.1 Gear Concepts Chosen for the Investigation

- **Solid spur gear made of 20MnCr5 steel**

  This gear is made of 20MnCr5 steel, which is of conventional type spur gear as shown in Figure 3.1a.

- **Solid spur gear made of cast iron grade-35**

  This is made of cast iron grade-35, which is also a type of conventional spur gear as shown in Figure 3.1b.

- **Solid spur gear made of hylam**

  This is made of hylam material; this is also a type of conventional spur gear which is shown in Figure 3.1c.

![Figure 3.1 Solid gear (Conventional)](image-url)
• **Laminated 20MnCr5 spur gear**

Here the entire face width of the gear is split into five equal divisions called as plies, and they are assembled to form a full gear. Each ply is made of 20MnCr5 steel (Figure 3.2a).

• **Laminated cast iron grade-35 spur gear**

In this, each ply is made of cast iron grade-35 (Figure 3.2b).

• **Laminated hylam spur gear**

To make laminated hylam spur gear, plies made of hylam material were assembled together (Figure 3.2c).

![Figure 3.2 Laminated gear](image)

• **Bimetallic spur gear type 1**

Here first, third and fifth plies are of 20MnCr5 steel plies, second and fourth plies are of cast iron grade-35. These plies were assembled to form a bimetallic spur gear of type 1 as shown in Figure 3.3a.
• **Bimetallic spur gear type 2**

In this type, plies made of 20MnCr5 steel with 5mm thickness will be the first and fifth ply. The second, third and fourth plies are made of cast iron grade-35 (Figure 3.3b).

![Figure 3.3 Bimetallic gear](image)

• **Sandwich spur gear type 1**

Here first, third and fifth plies are of 20MnCr5 steel plies. The second and fourth plies are of hylam. These plies were assembled to form a sandwich spur gear of type 1 as shown in Figure 3.4a.

• **Sandwich spur gear type 2**

In this type, plies made of 20MnCr5 steel with 5 mm thickness will be the first and fifth ply. The second, third and fourth plies are made of hylam material having a face width of 12 mm (Figure 3.4b).
3.2.2 Three Dimensional Model of Spur Gear

Three dimensional model of spur gear was created using the modeling software SOLIDWORKS 2014. Spur gear model was created as per the parameters given in the Table 3.1. The parameters are as per IS: 2535 – 1963. From the tool box available in design library of Solidworks, ISO standard spur gear was chosen.

<table>
<thead>
<tr>
<th>Description</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>Module</td>
<td>3.89 ≈ 4</td>
<td>3.89 ≈ 4</td>
</tr>
<tr>
<td>Face width</td>
<td>20 mm</td>
<td>20 mm</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>87.8 mm</td>
<td>107.3 mm</td>
</tr>
<tr>
<td>Pitch circle diameter</td>
<td>77.96 mm</td>
<td>97.45 mm</td>
</tr>
<tr>
<td>Base diameter</td>
<td>73.25 mm</td>
<td>91.57 mm</td>
</tr>
<tr>
<td>Root diameter</td>
<td>69.85 mm</td>
<td>89.39 mm</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>
For creating conventional solid gear face width was given as 20 mm. After giving other parameters like module, the number of teeth and pressure angle spur gear part model was created. Figure 3.5 shows the conventional solid gear model.

![Conventional spur gear - 3D model](image1)

**Figure 3.5** Conventional spur gear - 3D model

![Single ply of spur gear - 3D model](image2)

**Figure 3.6** Single ply of spur gear - 3D model
To create laminated, bimetallic and sandwich spur gears, ISO standard spur gear was chosen. Here face width was given as 4 mm to create a single ply (Figure 3.6) and other parameters remains same as conventional solid gear. Using the assembly module five plies was assembled and the laminated gear was created as shown in Figure 3.7.

![Laminated spur gear - 3D model](image)

**Figure 3.7** Laminated spur gear - 3D model

### 3.2.3 Structural Analysis Procedure

Analysis of gear is accomplished by the partial differential equation solution which describes the given model. Figure 3.8 shows the process involved in this structural analysis.
The gear tooth is meshed with 10 nodes Solid 92 element with fine mesh (Shuting 2008). SOLID92 is highly suited for irregular mesh models because of its quadratic displacement behaviour. This element has 10 nodes, each with 3 degrees of freedom, i.e. translations in x, y and z directions. It has creep, plasticity, stress stiffening, swelling, large strain and large deflection
capabilities (ANSYS Academic Teaching 2013). Figure 3.9 shows the sketch of the element used in the Finite Element model of gear tooth sector and the meshed gear model is shown in Figure 3.10

Figure 3.9 SOLID92 3-D 10-node tetrahedral structural solid element

Figure 3.10 Meshed gear model
3.2.4 Loading of Gear

Load is applied as a force along the pitch line of the gear tooth as shown in Figure 3.11. The Load was calculated for the worst condition of 1000 rpm recommended by the manufacturer. From the Equation 3.1, torque (T) is calculated and from the calculated torque force (F) is calculated using the Equation 3.2.

\[ P = \frac{2\pi NT}{60} \]  
\[ T = F \times R \]

Figure 3.11 Gear tooth pitch line

3.3 STRUCTURAL ANALYSIS OF GEARS

The results of deformation and stress value of the above said concept gears are discussed in this section. Based on the FEA results, gears were selected for further investigation. Since the load is equally distributed along the face width of the gear tooth (line of contact), the type of bonding was not considered in the case of stacked gear concept. While going for real time
working plies will be fastened using fasteners and the relative motion between adjacent plies will be arrested using dowel pins. As discussed in Chapter 1.7 the thickness of the plies (4 mm) were selected based on the market availability of 20MnCr5 sheets and hylam sheets.

3.3.1 FEA result of solid and laminated gear

Figure 3.12 and Figure 3.13 shows the equivalent (Von-Mises) stress profile for both solid spur gear and laminated spur gear for the same applied load.

![Figure 3.12 Solid spur gear – stress distribution]

In the case of solid spur gear the applied load is uniformly distributed along the line of contact and in the case of laminated spur gear, even though the load is applied at the line of contact the total applied load is shared uniformly by the individual plies. This minimizes the stress concentration level on the gear tooth of laminated gear.
Figure 3.13  Laminated spur gear – stress distribution

Figure 3.14  Solid spur gear – safety factor
From the safety profile shown in Figure 3.14 and Figure 3.15, it is understood that the factor of safety can be improved by implementing the laminated gear concept. The growth of the weak area from the flank of tooth root was restricted because of the lamination. Whereas, in the case of solid spur gear, the profile of the weak area is the maximum which connects both flank and root area.

![Laminated gear - Safety Factor](image)

**Figure 3.15  Laminated spur gear – safety factor**

When compared with solid spur gear the maximum Von-Mises stress value was reduced by 29% and the value of the minimum factor of safety was increased by 42.7%. The result gives good recommendation for further investigation in this area.
Figure 3.16 Solid 20MnCr5 spur gear - deformation pattern

Deformation of both solid and laminated 20MnCr5 gears is shown in Figure 3.16 and Figure 3.17.

Figure 3.17 Laminated 20MnCr5 spur gear - deformation pattern
The maximum deformation value of solid 20MnCr5 spur gear tooth is 0.0242 mm and for the laminated 20MnCr5 spur gear tooth is 0.0238 mm, which is 1.6 % less when compared with solid spur gear. Even though the percentage of deformation reduced by laminated gear is not a soaring value, the result assures that laminated gears can be a replacement for conventional solid gears.

![Figure 3.18](image)

**Figure 3.18  Solid cast iron grade-35 spur gear - deformation pattern**

Similarly the deformation of laminated cast iron grade-35 gear (Figure 3.18) is 1.6% less when compared with the solid cast iron grade-35 gear (Figure A1.1). In the case of laminated hylam gear (Figure A1.3) the maximum deformation value is reduced by 1.6% when compared with solid hylam gear (Figure A1.2). The overall result of these three materials shows that deformation can be reduced to 1.6% when going for laminated gears.
Even though the percentage reduction in deformation is low, it is confirmed that, the deformation will not increase when going for laminated gear concept.

### 3.3.2 FEA Results of Bimetallic Gear

The maximum stress value of bimetallic type 1 gear (Figure 3.19) is 467.5 MPa, which is 26.39% less when compared to conventional solid gear. In the case of bimetallic type 2 gear (Figure A1.5) the maximum stress value is 474.34 MPa, which is 25.31% less when compared to conventional solid gear.

When compared to type 2 bimetallic gear (Figure A1.6) deformation of type 1 bimetallic gear (Figure A1.4) is 17.14% lower because of three 20MnCr5 plies i.e. in type 1 bimetallic gear middle ply is also 20MnCr5.

![Figure 3.19 Type 1 bimetallic gear – stress distribution](image_url)
3.3.3 FEA Results of Sandwich Gear

In type 1 sandwich gear, the maximum stress value is 527.68 MPa (Figure 3.20), which is 16.92% less when compared to conventional solid gear. In the case of sandwich type 2 gear the maximum stress value is 541.78 MPa (Figure A1.8), which is 14.7% less when compared to conventional solid gear. When compared to type 2 sandwich gear (Figure A1.9) deformation of type 1 sandwich gear (Figure A1.7) is 21.93% lower because, here the middle ply is also replaced with 20MnCr5 ply.

![Type 1 sandwich gear - stress distribution](image)

**Figure 3.20** Type 1 sandwich gear - stress distribution

From Table 3.2, it is inferred that laminated cast iron and laminated hylam gears are having high deformation because of their low strength. However, they may be applicable for some low load application. However, they are not going to be subjected to further investigation because; here the gears will be investigated in the high load test rig.
<table>
<thead>
<tr>
<th>S.No.</th>
<th>Gear Set Details</th>
<th>Maximum Deformation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solid 20MnCr5 spur gear</td>
<td>2.4224e-002</td>
</tr>
<tr>
<td>2</td>
<td>Laminated 20MnCr5 spur gear</td>
<td>2.3834e-002</td>
</tr>
<tr>
<td>3</td>
<td>Solid cast iron grade-35 spur gear</td>
<td>4.409e-002</td>
</tr>
<tr>
<td>4</td>
<td>Laminated cast iron grade-35 spur gear</td>
<td>4.3368e-002</td>
</tr>
<tr>
<td>5</td>
<td>Bimetallic laminated spur gear type 1</td>
<td>2.9783e-002</td>
</tr>
<tr>
<td>6</td>
<td>Bimetallic spur gear type 2</td>
<td>3.5025e-002</td>
</tr>
<tr>
<td>7</td>
<td>Solid Hylam spur gear</td>
<td>5.9083e-002</td>
</tr>
<tr>
<td>8</td>
<td>Laminated hylam spur gear</td>
<td>5.8132e-002</td>
</tr>
<tr>
<td>9</td>
<td>Sandwich spur gear type 1</td>
<td>3.2394e-002</td>
</tr>
<tr>
<td>10</td>
<td>Sandwich spur gear type 2</td>
<td>3.5841e-002</td>
</tr>
</tbody>
</table>

Solid cast iron and solid hylam gears also have a low strength. However, for comparison purpose they are also subjected to further investigation. Similarly, type 1 bimetallic and sandwich gears perform better in loading condition when compared to type 2 bimetallic and sandwich gears. Therefore, type 1 bimetallic and type 1 sandwich gears are also subjected to further investigation. For the rest of the study type 1 bimetallic gear will be
simply called as bimetallic gear and type 1 sandwich gear will be called as sandwich gear. However, the safety factor in case of bimetallic gear is 0.89 and for sandwich gears is 0.26. Which is suggested for reduced loading application. Gears subjected for further investigations are given below:

- Solid 20MnCr5 spur gear
- Laminated 20MnCr5 spur gear
- Solid cast iron grade-35 spur gear
- Bimetallic laminated spur gear (1\textsuperscript{st}, 3\textsuperscript{rd} and 5\textsuperscript{th} - 20MnCr5, 2\textsuperscript{nd} & 4\textsuperscript{th} ply - CI 35)
- Solid Hylam spur gear
- Sandwich spur gear (1\textsuperscript{st}, 3\textsuperscript{rd} and 5\textsuperscript{th} - 20MnCr5, 2\textsuperscript{nd} & 4\textsuperscript{th} ply - Hylam)

3.4 ANALYSIS OF ROOT BENDING STRESS

An analysis of stresses responsible for tooth breakage type of failure is dealt with in this chapter. Gear tooth breakage occurs due to two reasons

- When the induced bending stress at the root exceeds the tooth strength
- When a sudden static overload is encountered in service

The maximum tensile stress at the root is responsible for the catastrophic failure of a gear tooth. For the above reasons, determining the bending stresses at the root becomes important conventionally, the gear tooth is assumed as a cantilever beam subjected to point load.
The capability of transmitting load by the gear tooth can be analysed using FEM. The circumferential force on the pitch circle during the meshing of gear tooth is estimated. While transmitting power between two shafts, gear teeth are commonly subjected to two types of stress. One is bending stress, which is induced on gear teeth caused by tangential force and another one is surface contact stress, which is addressed in international standards such as AGMA Standard 2003 A86 and DIN 3990 standards. The applied load is assumed to be distributed along the face width of the tooth uniformly.

3.4.1 Theoretical Calculation of Tooth Root Bending Stress

Theoretical bending stress is calculated using the modified Lewis Equation (3.3).

\[
\sigma_v = \frac{F_r}{K_v B Y m}
\]  

(3.3)

Referring to Figure 3.21, the normal force ‘\(F_n\)’ is resolved into two components, i.e. Radial force \(F_r\) and Tangential force \(F_t\). Equal and opposite force were produced due to this normal force. The tangential force ‘\(F_t\)’ was derived from the Equation (3.6). \(K_v\) is the velocity factor or Lewis form factor.
and is specified by Barth’s Equation (3.4) for known pitch line velocity in m/s. Here $K_v$ is chosen for spur gear not carefully generated (worst condition) because the objective is to make the gear plies by press working operation.

$$K_v = \frac{50}{50 + (200V)^{0.5}} \quad (3.4)$$

Torque is produced by the tangential force ‘$F_t$’ which drives the pinion. The tangential force is the one which bends the tooth and it is compressed by the radial force ($F_r$). Radial force ($F_r$) is arrived using the Equation (3.5).

$$F_r = F_t \times \tan \alpha \quad (3.5)$$

where

$$F_r = \frac{M_r}{d/2} \quad (3.6)$$

From the Lewis Equation (3.3), it is found that the maximum bending stress on gear tooth root is 556.14 MPa while, the gear is running at 1000 rpm which is referred as the worst condition by the manufacturer.

### 3.4.2 Finite Element Modeling of the Gear Tooth Sector

As the three dimensions of gear tooth, namely tooth thickness, tooth height and face width in three mutually perpendicular directions are comparable with one another, the 3D modeling is the most suitable. Again for the stress analysis to be reliable, it should be made only using a three dimensional model and finite element method is used for such an analysis. The surface of the gear tooth is nonlinear in this analysis.
Figure 3.22 shows the Finite Element model of involute spur gears tooth. The boundary conditions are applied to the meshed tooth gear. The sides and the bottom portion of teeth sector are arrested for all six degrees of freedom, and the gear sector is considered as a cantilever beam.

Figure 3.22  Meshed gear tooth model

Figure 3.23  Force acting at tooth tip
During the engagement of mating gears, the ‘line of contact’ begins from the bottom and moves to the tip of the gear tooth along the profile. When the force acts at the tooth tip, it acts like a cantilever beam and maximum stress was induced at the tooth root because of force acting at long distance. Therefore, tangential force is applied at the tooth tip throughout the face width as shown in Figure 3.23.

The gear forces are considered to be effective on a single pair of teeth in the mesh. The force components, i.e. $F_t$, $F_r$ and $F_n$ are calculated from the power ($P$) value of 50 hp at gear speed ($n$) of 1000 rpm. Assumptions made on calculation and analysis is given below:

- Force is applied along the tip of a tooth while it is in a static condition
- The radial force is negligible
- The load is assumed to be uniformly distributed across face width
- The forces caused by sliding friction of the tooth are negligible
- Stress concentration is negligible in the tooth fillet

### 3.5 TOOTH CONTACT STRESS

Contact analysis was conducted to study the compressive stress at the gear mesh interface. In Finite Element Analysis, contact analysis is used to simulate the interface where surface touch or slide against each other. If the material is less stiff, then there will be local deformation due to the compressive stress, and area contacts become more pronounced. For performing contact analysis the generalized Finite Element software ANSYS 14.5 Workbench is used.
When two gears are in meshing condition, the tangent line which connects both base circles is known as the line of action in the case of involute gears. In Figure 3.24 line $CD$ is the line of action. For completing one tooth mesh cycle, gear tooth contact starts from the point A and ends at the point B. When the point E gets meshed point A disengages and there will be only single tooth contact at the point B. Here two gear teeth were arranged in their contact position where the single tooth contact begins because, stress will be more when the single tooth takes the entire load.

### 3.5.1 Theoretical Calculation of Contact Stress

The Hertz contact stress Equation (3.7) is used for the calculation of contact stresses between the mating teeth. For the applied force ($F$) of 9069 N, contact stress was calculated for the three gears made of three different materials. Here, $R_1$ and $R_2$ are pitch radii of pinion and gear respectively, face width ($B$) is 20 mm and $E_1$ and $E_2$ are young’s modulus of pinion and gear material respectively, which is discussed in Section 1.6 of Chapter 1. Table 3.3 shows the theoretical result of contact stress for 20MnCr5, Cast iron grade-35 and hylam gears.
\[
\sigma_c = \frac{F \left(1 + \frac{R_1}{R_2}\right)}{R B \pi \left[\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right] \sin \phi}
\]  

(3.7)

<table>
<thead>
<tr>
<th>Description</th>
<th>20MnCr5</th>
<th>CI -35</th>
<th>Hylam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact stress ((\sigma_c)) N/mm(^2)</td>
<td>1499.69</td>
<td>1265.80</td>
<td>1083.83</td>
</tr>
</tbody>
</table>

3.5.2 Finite Element Modeling for Contact Analysis

As the contact conditions are sensitive, the elements nearer to the contact area to be refined. If the whole body of the gear is fine meshed then the computation time will be increased. To reduce the computational time, fine mesh should be done nearer to the contact zone. The meshed gear model is shown in Figure 3.25.

Figure 3.25 Meshed gear model for contact analysis
For conducting this nonlinear contact analysis between the involute profiles of mating gear, CONTA174 element is used to represent contact and sliding between target surfaces (TARGE170). These elements are used for structural and coupled field contact analysis (Ali 2011). It has three degrees of freedom at each node; translations in the nodal x, y and z directions.

As a boundary condition, fixed support is applied on the surface of the inner rim of the lower gear. Frictionless support is applied to the inner rim of the upper gear to allow its tangential rotation but restrict from radial translation. Torque is applied to the inner rim of the upper gear in a clockwise direction which acts as a driving torque.

3.6 CONCLUSION

From the structural analysis, basic study was carried out for various gear concepts. Based on the results, certain gears were selected for further investigations. The procedure and assumptions for computation and analysis of tooth bending stress and contact stress were discussed. Theoretical results for both bending stress and contact stress were found out with the modified Lewis equation and Hertzian contact stress equation respectively. The results of tooth bending stress and contact stress for all selected gears were discussed in Chapter 7 and compared with theoretical results.