CHAPTER -2

REVIEW OF RELATED LITERATURE

The previous chapter set the broad view of the study; its origins, what it purports to explore and the theories that will guide this study. The study is concerned with epistemology of common mathematical knowledge as it relates to teaching and learning of mathematics in school. The present chapter reviews available literature related to mathematics education which makes reference to epistemology in the classroom learning and teaching context within the socio-cultural framework. There is a considerable variation in the methods used for framing investigations of mathematical epistemological perspectives by learners and/or teachers; nonetheless the review is done according to the following sub headings, epistemological basis of everyday mathematics, epistemological basis of school mathematics, learners mathematical practices in and out-of-school and other studies focusing on epistemology in the classroom.

2.1: Epistemological basis of everyday mathematics

Most cultures do not have a category called mathematics, let alone refer to some of the mathematical concepts in isolation, rather they are integrated into the contexts in which they arise, in a complex of other ideas that surround them. But what are these everyday concepts? What is their nature and how are they learned? Some studies highlighting these aspects of everyday or community mathematical practices are reviewed, they are as follows:

2.1.1: Mathematics in cultural practice

A number of studies focusing on social and cultural aspects of mathematics have been done by researchers and scholars such as Ascher (1990), D’Ambrosio (1990), Gerdes (1991), Zaslavsky (1973), etc focusing on mathematics in different cultures.

Gerdes (1998) looks at Southern African women cultural activities, such as ceramics, beading, mural decoration, basket weaving, hair braiding, tattooing, string figures, which bore a strong artistic and mathematical character. He analysed these for the mathematical concepts and ideas embedded in them. He studied different types of rotational symmetry, axial, double axial and rotational symmetry emerged quite naturally. The basket weaving was studied amongst the Mozambican cultures. Gerdes has
proposed a procedure for uncovering the mathematics in the traditional objects. He has given this in a form of a question that a researcher poses, “Why do these material products possess the form that they have?” From this the researcher concluded that the adopted pattern or approach provides the optimal solution to the task. The properties and relations of circles, angles, rectangles, squares, regular pentagons and hexagons, cones, pyramids, cylinders symmetry were studied. He demonstrated these approaches through studying alternate axiomatic constructions of Euclidean geometry based on how houses are built; an alternate construction of regular polygons by considering how artisans weaved funnels; rediscovering the theorem of Pythagoras by studying the technique used in weaving square buttons, and finally by considering traditional fish traps which led to alternate circular functions, soccer balls and (semi) regular polyhedra.

Further Gerdes studied the geometry of the ‘sona’ of eastern Angolan people, a non-Euclidean geometry. He analyzed the symmetry and monolinearity of the figure(s) and corresponding geometrical algorithms for their construction, etc. He experimented with possibilities for use with sona in mathematics education as well as initiated a mathematical exploration of the properties of some classes of ‘sona’. Ascher (1988) also conducted studies on the sona independently focusing on the geometry and topological aspects of the ‘sona’, that is, symmetry, extensions, enlargement through repetition and isomorphism.

Ascher (2002) also carried out studies elaborating several specific cases of mathematical ideas and their cultural embedding. In her book “Mathematics Elsewhere: An exploration of ideas across cultures”, she has elaborated on, the logic of divination, marking of time, cycles of time, models and maps, systems of relationships and figures on the threshold. The ideas for this volume came from the Mayas of South America, the Marshall Islanders, Tongan, and Trobriand Islanders of Oceania, the Borano and Malagasy of Africa, the Basque of Europe, the Tamil of Southern India, and the Balinese and Kodi of Indonesia. The mathematical ideas in the cultural aspects that she looked at involve number, logic, spatial configuration, and, more significantly the combination or organization of these into systems and structures.

Mosimege (2000) looked at culturally specific games and showed how such games could be mathematised to reveal a variety of mathematical concepts that could be
used at primary, secondary and tertiary levels of education. He looked at such games as 'Malepa', 'Morabaraba' and 'Moruba' in South Africa among the Barolong Community in Mafikeng, North West Province. Looking at the Malepa, he identified various mathematical concepts such as geometric figures, relationships between the various figures and generations drawn from these relationships. In Morabaraba, he identified various quadrilaterals and the similarities and differences between them. Symmetry and counting were the other aspects noted. In the Moruba game, he identified counting, symmetry, probability (exploring chances of each player in winning the game), estimation and decision making.

Mosigege (2004) also explored the indigenous mathematical knowledge of the Ndebele community at the Lesedi Cultural village. He looked at the traditional baskets, traditional hats, beadwork and other items. He found that the women involved in the practices dealt with mathematical concepts like measurement of lines and angles, estimation, counting, patterns (as in repetitive cycles), geometry (similar figures), symmetry.

Cherinda (2002) explored the mat weaving technique called 'twill' using a weaving board in Mozambique. Twill weaving he writes, by its nature involve mathematical ideas such as translation, rotation, symmetry, sequences and series, combinatorics, algebraic groups etc.

Panda (2004) did an ethnographic study of the agricultural tool making and house construction by the adult Saora tribal villagers, in India. She found that the Saoras engaged in multivariate judgments in designing agricultural tools and houses. For the agricultural tool there were two types of 'Khasada' (wooden tool), one used for tilling terraced land in the hill slopes: The curvature and the angle of the 'Khasada depended on the slopes, length, and the width of each terraced bed in the hill tops. The other tool by the same name was used for ploughing land in the steep sloped hills. The angle of this tool depended on the gradient of the slope and the height of the tool user.

For houses, the structured designs of the houses were decided depending on complex consideration of various topographical features of the house plot, social status of the family and the specific load bearing requirements (Panda, 2004; Pasbola, 2006). They found that the Saoras have the notional understanding of the distribution of force
and the conversion of horizontal force into vertical force and vice versa, and they also
demonstrated a good intuitive knowledge of the relationship between angle and the
force/weight.

D'Ambrosio (1984) has written extensively on the idea of social and cultural
mathematics. He has investigated how cultural groups vary in their use of mathematics,
that is, how they ‘mathematise’. For example how they count, measure, relate, classify,
and infer. According to him, “... mathematical practices differ from one cultural group
to another. At this level, mathematics comes close to being a variant of common
language associated with the concept of codifying popular practices and daily needs.
These would include uses of numbers of quantities, the capability of qualifying and
quantifying and some patterns of inference” (p.44). Each social group has its own
mathematical practices, for example, children, farmers, engineers and professional groups
of people develop their own pattern of doing mathematics with their own symbols, codes,
etc.

Zaslavsky’s (1973) in her book “Africa counts: Number and Pattern in African
Culture” pioneered the study of the history of mathematics in southern part of the Sahara
desert. Zaslavsky gives early evidence of mathematical activity in Africa, south of the
Sahara, by looking at a bone dated 9000 – 6500 B.C, dug up at Ishango in Zaire (Now
Democratic Republic of the Congo DRC). The marks on the bone were interpreted as
arithmetical game of some sort based on base 10 as well as having knowledge of
duplication and of prime numbers. She also presented numeration systems as she
discusses written, spoken and gesture counting practices/systems. In her study she also
looks at number symbolism, superstition and taboos on counting. In the same study
Zaslavsky looked at riddles and puzzles. She further explored mathematics in games. In
all these she showed early works of counting, measuring, designing, playing, classifying,
sorting etc. And she concludes that the communities in Africa, South of the Sahara as
elsewhere have been involved in and have used mathematics concepts time in
immemorial.

function of activity in the environment found that landmarks in an environment facilitated
the construction of spatial representation. The theme provided a relevant context in
which to coordinate and consolidate spatial information. Generally they asserted that landmarks which had functional value aid in the consolidation and coordination of an overall cognitive representation. The study involved a comparison of first and sixth graders – the average age of the first graders was six (6) while that of the sixth graders was ten (10) years old.

These studies look at the mathematics knowledge in and out of school contexts underscoring/highlighting the point that mathematical concepts cut across different cultures and also that mathematics is a human activity that all cultures endeavour in as they solve problems in their everyday lives. The mathematics within communities is understood as the technique of understanding, explaining, learning about, coping with and managing the natural, social and political environment by relying on processes like counting, measuring, sorting ordering and inferring which result from well identified cultural groups (D'Ambrosio, 1988). These studies in social and cultural mathematics knowledge mentioned above show that there is a wide spectrum of common mathematical knowledge between the everyday mathematical knowledge and school mathematical knowledge (for example, circles, angles, probability, symmetry, rotation, rectangles, number etc). However, the mathematical practices differ from one cultural group to another. From these studies of mathematics in context, what comes through is that the practitioners do not refer to it as mathematics themselves, and it is not mathematized the way it is projected. The practices which are analyzed as mathematics activities/concepts are embedded in on-going practices/activities.

According to Bernstein (1999) everyday knowledge belongs to what he termed as 'horizontal discourse' type. The knowledges of horizontal discourses are embedded in on-going practices. These knowledges are accompanied with some emotions, and also they are directed towards specific, immediate goals, and relevant to the context of the learner/ or community. The everyday knowledge discourses comprise strategies that are local and they are segmentally organized, that is, there is no necessary connection between the solution to one task and the next. Thus, it is context specific and dependent, designed to both draw on and respond to encounters with people and the local environment (Bernstein, 1999). The mathematical knowledge developed in these practices are by individuals and communities over a lifetime of personal as well as
collective experience(s) and enculturation which relies on common-sense and is context specific and dependent, directed towards the achievement of specific, immediate, and relevant goals (e.g. Bernstein, 1999; FitzSimons, 2006; Nunes, 1993 etc). It appears there is a particular way this knowledge is passed on from one generation to the next as elaborated in the next section.

2.1.2: Learning in the community (learning of everyday concepts)

Learning of these everyday concepts is informal and segmented as the studies on learning in these contexts highlight. Lave J. studied adults in contexts to see and understand how they came to learn their trade/craft. She studied midwives, tailors, quarter-masters, butchers and non-drinking alcoholics (Lave & Wenger 1991). On her work among the tailors of Liberia, Lave (1977) revealed that they utilized apprenticeship method. The tailoring apprentices were immersed in the practice of tailoring and learning to be tailors. The apprentices lived with the experienced tailors. They operated their trade in a street which was full of tailors’ workshops. In this environment without being taught formally, they observed the master and imitated them. That is, apart from observing the full process of garment production; they practiced (in their imitation) a few basic skills, on a few tasks related to the process of making a garment for example cutting or just using a sewing machine. As they gained confidence in these skills, they attempted doing a complete garment on their own. The completed garments apart from receiving comments from masters and peers were also sold albeit at a cheaper price compared to the master’s works. The apprentices worked through from less complex garments to more complex types.

The learning was organized through participation in a sequence of work activities that were pedagogic in their organization. For example when the master asked the apprentice to do a piece of a garment or a routine task(s) for him, the complexity of the tasks given would vary according to the level of competence of the apprentice in the eyes of the master. Thus, the apprentices moved from tasks of low accountability to those where consequences of errors were great. There was no formal teaching, only occasional instructions or pointing out errors. The apprentices learnt as they self-corrected, listened and observed the masters’ work.
Thus, the apprentices were engaged in work tasks with close guidance of peers and masters, but also there was indirect guidance provided by the setting and the practice within that setting. Lave & Wenger (1991) put it this way, "... legitimate peripheral participation related to a 'way of being in social world, not a way of coming to know about it' and thus denoted the process of moving from being a novice to an expert. Language was central to it in that 'language was part of practice, and it was in practice that people learnt'" (Lave & Wenger, 1991, p.85 cited in Daniels, 2001, p.72).

The social circumstances of the learner (apprentice) shaped the ways in which knowledge was acquired and used. The workplace learning as shown above was informal and incidental. Learning was part of social practice and for new learners to participate in it entailed that they had broad access to arenas of mature practice (Lave & Wenger 1991). As Lave & Wenger (1991) further explained,

"An apprentice's contributions to ongoing activity gain value in practice ... a value which increases as the apprentice becomes more adept. As opportunities for understanding how well or poorly one's effort contribute are evident in practice, legitimate participation of a peripheral kind provides an immediate ground for self evaluation. The sparsity of tests, praise or blame typical of apprenticeship follows from the apprentice's legitimacy as a participant."


While the social and the context were important, it is not only this that was relevant, but also the context of culture when an analysis of meaning was undertaken. There was need to connect to other texts and situations in the community or how individual history made it known as relevant to the meaning of the present case (Lemke 1997). Lemke (1997) himself put it this way, people,

"... function in micro-ecologies, material environments endowed with cultural meanings; acting and being acted on directly or with the mediation of typical cultural tools and cultural-material systems of words, signs and other symbolic values. In these activities 'things' contribute to solutions every bit as such as "minds" do; information and meaning is coded into configurations of objects, material constraints and possible environmental options as well as in verbal routines and formulas or "mental" operations. (Lemke, 1997, p.38 quoted in Daniels, 2001, p.71).

Greenfield (2004) studied the Zinacantec people who were learning weaving using the back strap loom in their natural environment. Greenfield work was amongst the
Mayan of Chiapas, the Zinacantec peoples. She lived amongst these people and studied how the 'weaving using the back strap loom' was passed on from generation to generation. She observed that once the girls (Zinacantec) were born, the parents placed a number of items within their vicinity. These included the weaving tools, cooking utensils and other artifacts. This practice was performed over many generations. Thus, as the children (the Zinacantec -infants) matured they watched their mothers and other older females utilizing these items. According to Greenfield (2004) apart from utilizing the items, there was a particular position/posture that was adopted that was essential for back-strap loom weaving. This position (a kneeling position) was also adopted for cooking 'tortillas', washing clothes, changing babies etc.

Thus, Greenfield explained in her book 'Weaving Generations Together. Evolving Creativity in the Maya of Chiapas', that being born into and growing up in Zinacantec culture provided certain basic body skills fundamental to weaving on a back-strap loom. That is, restrained movement, the ability to kneel comfortably for long periods and the ability to maintain one's balance while leaning easily forward or backward. (Greenfield, 2004)

Observation was one of the major learning strategies; there were many opportunities for observing adults and she also noted that the time-visual attention span was good among the Zinacantec people. She found that the learners used 53 percent on observation and the remainder (47 percent) on participation. However, observation as a technique for learning was not an isolated activity, it was more than just 'observing' - learners were part of a 'community of practice'. Thus, she wrote that, in the Mayan case, the non-native weavers experienced difficulties using observation only to learn. Thus, in many instances an instructor took time to work on the work on the learner's tasks/item. The quality of observation improved tremendously because the learners were genuinely interested in seeing what happened or really wished to learn/observe. The instructor provided a model for the learners to observe. This type of apprenticeship was non-verbal. The instructor took over what the learner was doing and let them watch or alternatively the instructor worked cooperatively with the learners on the task. The kind of assistance depended on the level of competence or skill of the learner(s).
In cases where verbal instructions were given, the verbal communications were short or brief instructions of action words, for example, "don't put it like that". Explanations were rarely given—but action or speech facilitating action was used in communicating ideas, ways of doing something etc. Greenfield (2004) also observed that apprenticeship involved bodily movements—
in the habitual aspect of learning. Apart from observing, learners were instructed by competent others in the community on how to position themselves.

In learning by doing or participating in the activity, young learner's used/adopted play as part of their learning. She observed that young learners' imitated adult activities by having or using miniature items for example 'play weaving'. Greenfield noted that by the time the young learners came to a real thing (back-strap loom), they had the basics—they knew what to do. She noticed that play was widespread in the community. Even in the play young learners received careful instructions by adults. Peers and adults helped in scaffolding for the young learners.

Greenfield (2004) thus concluded that the salient features of the learning was that it was part of (the) heart and soul of the people and it was habitual practice. It offered learners identity characteristics. She wrote that 'in sum, the weaving apprenticeship was a vehicle for the social reproduction of their cultural theme(s), which she identified as, a notion of a single 'true' way of weaving, the interdependence of family members and the age-graded flow of authority from older to younger.

In looking at strategies for learning in context, be it in work or cultural setting, the question that arises is whether there are common ways. Rogoff (2003) in her book 'The Cultural Nature of Human Development' says that learning should be seen as supported by a chain of social and cultural circumstances surrounding a learner, and further that the interactions of learners are not static, but take place in changing organization in a changing environment.

"... the progressive, mutual accommodation between an active, growing human being and the changing properties of the immediate settings in which the developing person lives, as this process is affected by relations between these settings, and by the larger contexts in which the settings are embedded. (Bronfenbrenner, 1979, p.21 quoted in Rogoff, 2003, p.45)
Thus, within this framework Rogoff analyzed studies of learning in a cultural context, that is, everyday learning in its natural environment. She did not study one particular community, but analyzed several studies in cultural context. The aim was

"to integrate the available ideas and research to contribute to a greater understanding of how culture matters in human development. What regularities can help us make sense of the cultural aspects of human development? To understand the processes that characterize the dynamic development of individual people as well as their changing cultural communities, we need to identify regularities that make sense of the variations across communities as well as the impressive commonalities across our human species." (Rogoff, 2003, p.7).

She came up with a way that learning could be conceptualized. She referred to type of learning as 'Intent Participation' in Community activities, "... children are legitimate peripheral participants (Lave & Wenger 1991) in the mature activities of their community, watching what is going on and becoming involved. The novices learn largely through their engagement with other apprentices and the master, in real production, observing their peers and the master and learning through their own involvement." (Coy, 1989a, Lave & Wenger 1991 in Rogoff, 2003, p.323) She indicated that the process was like 'osmosis', learners developed in participation from legitimate peripheral towards full participation. In this transition, learners picked up values, skills and mannerisms in incidental fashion through close involvement with a socializing agent.

The learning was through guided participation in cultural activities – by this she meant that as children participated in the activities, they were guided by the values and practices of their cultural communities. The guidance took many forms; explanation was part of it, teasing and shaming or even when one’s behaviour was held up for others to evaluate were all part of guidance. It also included efforts by social partners and by the peers. Thus, in talking about guided participation she put it this way,

"In addition to instructional interactions, ... focuses on the side-by-side or distal arrangements in which children participate in the values, skills and practices of their communities without intentional instruction or even necessarily being together at the same time. It includes varying forms of participation in culturally guided activities through the use of particular tools and involvement with cultural institutions." (Ibid, p. 284)

From what Rogoff has put forward, there are many opportunities for learning in the cultural context, some involving subtle, tacit taken for granted events and ways of
doing things, which required learners to notice and understand as they participated in the community activities, while others were cultural tools designed to teach desired aspects of culture for its continued existence for example narratives, folk songs etc. In this regard she wrote that children’s learning opportunities were structured,

“In some cultural systems (where) children have (had) the opportunity to learn by observing and pitching in to mature activities of the community. Children watch ongoing events keenly and listen closely to narratives and nearby conversations and contribute as they are ready. Their caregivers and companions offer access and often provide support and pointers in the context of shared community activities.(Ibid p. 366).

Thus, learning in the community was facilitated by the availability of many learning situations which offered encounters between learners and their milieu. Learning was not constrained by time, place or people. It was open to all at any time. Learners demonstrated positive commitment supported by appropriate ‘attitudes’ and ‘values’.

From Rogoff’s learning in the community, a number of strategies of learning come up though many are unconscious efforts.

- keen observation and collaboration in groups
- listening closely and contributing as and when ready
- involves complex group relationships involving learners who learn to take responsibility for contributing to their own learning and to the community’s goals
- within the zone of proximal development, caregivers and others companions offer access and often provide support and pointers in the context of shared community activities.

Thus, learning comprised of a learning situation, exposure to the learning situation, involvement in the learning activities, that is, interaction with learning situation, the product or the outcome of the learning activities reflected in the behaviours of the learners.

Using Rogoff’s ideas on learning Jane and Robbins (2007) carried out an empirical study of grandparents teaching everyday concepts in science and technology. They investigated what science and technological experiences grandparents from a range of cultural heritages engage in with their grandchildren. How the grandparents support and extend their grandchildren’s learning in science and technology and what the benefits of intergenerational conversations are. The study is located in the Vygotsky’s cultural-
historical theory utilizing Rogoff's (2003) ideas on development as a cultural process and that people develop through their varied participation in the changing cultural activities of their communities. Vygotsky's concepts are utilized in their study such as 'mediated action', intermental and intramental functioning and everyday and scientific thinking/concepts.

The study involved twelve sets of grandparents and grandchildren as participants. The study focused on the shared, spontaneous, cooperative activities that were recognized as science and technology in nature. Using Rogoff's (1998) three foci of analysis they analyzed photographs, grandparents' journal entries and discussions to determine how,

- the children learning occurred through shared activities particularly learning of scientific and technological concepts.
- the psychological processes (intramental to intermental functioning) and the nature of the conversations between grandparents and their grandchildren occurred.

They found that through participation in simple but meaningful everyday activities with their grandparents, children's technological and scientific understanding developed and transformed. They found that children come into classrooms with some skills and knowledge in science and technology. The children develop many spontaneous concepts through their engagement in activities in informal contexts. They also found that the children's understanding were actively supported and scaffolded by others, which developed through mutual involvement in culturally appropriate activities.

Thus, epistemologically there is a distinct pattern in the strategies and pedagogy of knowledge in everyday contexts. According to Bernstein the pedagogy in the everyday context is exhausted in the context of its enactment, or is repeated until the particular competence is acquired. It is directed towards acquiring a common competence rather than a graded performance. This is in line with the nature of the knowledges, competences and literacies. These are segmented, contextually specific, context dependent, and embedded in on-going practices, usually with strong affective loading, and directed towards specific, immediate goals, highly relevant to the acquirer in the context of his/her life” (Bernstein, 1999). The learning nonetheless fits in the socio-cultural theory as espoused by Vygotsky as follows:
2.1.3: Socio-cultural perspectives on learning and teaching

The sociocultural perspective suggests that learning is a process of appropriating 'tools for thinking', that are made available by social agents. Vygotsky explained what happens in a child's development in terms of socio cognition. He saw development as a process of participation with others in activities that are mediated by cultural tools, and are constituted with and by interpersonal and community or contextual factors. Individual development cannot be understood without reference to the social and cultural context within which it is embedded. Higher mental processes in the individual have their origin in social processes. Mental processes can be understood only if we understand the tools and signs that mediate them. In higher forms of human behavior, the individual actively modifies the stimulus situation as a part of the process of responding to it.

Social context is so important to Vygostky that it is not simply one more variable to be accounted for, rather, social activity that is the interaction between individual and context. Thus, for Vygotsky development is the conversion of social relations into mental functions. "Every function in the child's cultural development appears twice; first, on the social level, and later on the individual level; first, between people, then inside the child. All the higher functions originate as actual relations between individuals." (Vygosky, 1978 cited in Kozulin, 2003, p.24) The child converts social relations into psychological functions through mediation. Mediation occurs through a linking tool or sign. A tool is something that can be used in the service of something else. Vygotsky theorizes about concept of mediation, he argues that higher mental actions are mediated within activities by tools, artifacts and cultural inventions (Vygotsky, 1997). Another important Vygotskian theory is the notion that thinking progresses through intermental functioning. That is thinking occurs first on the social plane, and later on the individual plane (within the child). Thus, learning is a social process that takes place between people. He conceptualized learning as internalization of social interactions in which communication is central. Learning takes place in social interactions in a specific context which comes internalized by a person (Hedegaard, 2001, p. 16 – 17).

Thus, from the socio-cultural perspective on learning, everyday knowledge supports the development of children. Epistemology of school mathematics, however, is
different from the everyday type. Section 2.2 looks at the nature and structure/domain of school mathematics.

2.2: Epistemological basis of school mathematics

School mathematics operates in a domain which is far removed from material experience of the learners and it exits within a hierarchical network of related concepts. According to Bernstein, it is coherent, explicit and it is a systematically principled structure. But how are the school mathematics concepts learned and how do they interact with everyday concepts, section 2.2.1 looks at this aspect.

2.2.1: Scientific concepts

Through a series of experiments Vygotsky demonstrated that children acquire important concepts inside school (scientific concepts) and also outside school (everyday concepts). Vygotsky used the term scientific concepts to refer to ideas that have explicitly been introduced by adults in school. These concepts begin in the domain of conscious awareness and volition, and form a logical system in a particular discipline. They are generalisable, removed from material experience and exist within a hierarchical network of related concepts (Vygotsky, 1987). Spontaneous (everyday) concepts as shown in section 2.1 on the other hand develop within children’s daily lives as a result of their interaction with adults, peers and non-social environment. Vygotsky saw the two types of concepts as interdependent. Vygotsky wrote that “Scientific concepts grow downward through spontaneous concepts; spontaneous concepts grow upward through scientific concepts” (Vygotsky, 1986, p.194 cited in Renshaw, 1992). The relationship between the spontaneous and scientific concepts provides a way of looking at how a child develops. At any given developmental moment, Kozulin (1990) suggests that there is a proportion between scientific and spontaneous concepts. Nonetheless a special process is required to bring together a child’s everyday representations with the more abstract scientific representations. From Vygotsky perspective, learners access scientific concepts through assistance from knowledgeable adults/peers through instruction which enables learners to reconceptualise their everyday experiences. On the learning of scientific concepts Vygotsky says, “Words take over the function of concepts and may serve as means of communication long before they reach the level of concepts characteristic of fully developed thought” (Uznadze, cited in Vygotsky, 1986, p.101). According to
Vygotsky, "Verbal communication with adults ... become a powerful factor in the development of the child’s concepts. The transmission from thinking in complexes to thinking in concepts pass unnoticed by the child because his pseudo concepts already coincide in content with adult concepts" (Vygotsky, 1986, p.123). For Vygotsky pseudo concepts enable children to communicate effectively with adults and that this communication (the intermental aspect) is necessary for the transformation of the complex into a genuine concept (the intramental aspect) for the learner.

According to Vygotsky appropriation of scientific concepts is not limited to structuring and raising of spontaneous concepts to a higher level, but the scientific concepts begin to mediate their thinking and problem-solving, that is, "reflective consciousness comes to the child through the portals of scientific concepts" (Vygotsky, 1986, 1987 cited in Karpov, 2003, p.66).

Spontaneous concepts as shown in the previous section are the result of generalization of everyday personal experience in the absence of systematic instruction. These concepts are unsystematic, not conscious and often wrong (Karpov, 2003). However spontaneous concepts play an important role in children’s learning as a foundation for the acquisition of scientific concepts, for example, “historical concepts can begin to develop only when the child’s everyday concept of the past is sufficiently differentiated” (Vygotsky, 1986, p.194, cited in Karpov, 2003, p.66).

Research in the Vygotskian tradition has shown that the appropriation of spontaneous and scientific concepts are the result of fundamentally different types of learning. Spontaneous concepts are as a result of empirical learning while scientific concepts are due to theoretical learning. Empirical learning is explained as being based on children’s comparison of several different objects or events, picking out their common salient characteristics, and formulating, on this basis, a ‘general concept’ about this class of objects or events (Karpov, 2003). The strategy, however, does not work all the time, and thus it might lead to misconceptions. It works in instances where the common salient characteristics of objects or events reflect their significant, essential characteristics. Karpov (2003) gives an example where an erroneous conclusion is made, “A tail and fins are common, but not essential characteristics of fish. Therefore, the child’s spontaneous concept of fish developed as the result of empirical learning would be a misconception.”
He thus, saw empirical learning as reflecting students' attempts to compensate for the deficiencies of the traditional system of schooling instruction by trying to discover for themselves scientific knowledge.

Theoretical learning, on the other hand, is based on students' acquisition of methods for scientific analysis of objects or events in different subject domains (Karpov, 2003). The method just as the other one on empirical learning is aimed at selecting the essential characteristics of objects or events of a certain class and presenting these characteristics in the form of symbolic and graphic models. The role of the teachers is to teach these methods of scientific analysis and students' task is to master and internalize these methods as they practice using them. As students increase these methods, they form/serve as cognitive tools that mediate the students' further problem solving.

Vygotskian ideas of the differences between empirical and theoretical learning and in particular the advantages of theoretical learning has influenced a number of psychologists especially in Russia. Didactic analysis is needed to create instructional procedures powerful enough to build connections between scientific concepts (that is, the fundamental mathematical concepts) and the learner's everyday concepts (Renshaw, 1992). Davydov (1975) conducted a logical analysis aimed at supporting the view that relations of quantity are the fundamental concepts upon which the whole of the number curriculum could be built. He builds on Vygotsky's account of the process of development. According to Davydov the sociocultural approach to learning suggests that the general and abstract come before the specific and particular. “Succeeding in making the particular visible through the general is a characteristic feature of the kind of academic subject which awakens and develops the child’s ability to think theoretically…” (Davydov, 1975b, p.204 cited in Renshaw, 1992).

As noted above, everyday concepts tend to be personal but are tied to concrete experiences and resist systematicity; scientific concepts on the other hand are systematic and general and are initially empty of personal meaning. Therefore, an appropriation of the scientific concepts cannot occur automatically, a teaching process that builds connections between the everyday and scientific concepts are needed (Renshaw, 1992). Davydov curriculum achieved this by engaging children in a series of activities that required comparison of quantities, or the construction of quantitative relations of equality.
and inequality. The connection between everyday concepts and the mathematical concepts is accomplished in the context of activities where objects are manipulated, selected, grouped, and drawn. As the children perform/manipulate the objects they are assisted to talk in mathematical ways about the visual representations of various quantitative relations. The everyday terms used by the children to describe quantities are elaborated and made more precise and general. Thus, within the context of the instructional dialogue, the teacher is able to move the children progressively toward greater abstraction and generality (Davydov, 1975).

The Davydov curriculum is discourse-oriented encouraging collaborative learning which offers teachers and learners a chance to keep a continuous check on what they are doing. However, apart from being a discourse-oriented curriculum, Davydov’s curriculum is also referred to as a ‘thinking curriculum’, which adds social learning in terms of value orientations and identity formation. Davydov’s curriculum was inspired by the idea of designing a genuine Vygotskian-inspired thinking curriculum. For Davydov cognitive development and the formation of a personal and social identity are intertwined processes. He thus, argued that curriculum content should be ‘knowledge-oriented’ but coupled with social and communicative competences and the related intellectual and moral attitudes and dispositions of the students. The emphasis was on use of exercises in the negotiation of meaning by means of discursive reasoning and arguing, not just in terms of the data given but also in terms of the epistemic claims involved. In his “Ascending from the abstract to the concrete (A/C) teaching format he sought to teach teachers how to ‘orchestrate’ the conjoint making of meaning and personal sense in a regular classroom.” (Davydov, 1975, p.5)

Davydov observed that “three components exist in each arithmetic calculation; numbers, arithmetic rules and relations. Usually when children are executing an arithmetic procedure, their attention is principally turned to numbers and arithmetic rules, but not to relations” (Davydov, 1975, cited in Renshaw, 1992). As children do not pay attention to relations, they lack a base for inductive generalization. Thus, Davydov’s curriculum starts with introduction of direct and indirect comparisons of quantities, without using numbers. Students examine the role of measurement in everyday life, making direct comparison of quantities focusing on specific quantitative attributes, and
also learning to make indirect comparison of quantities. The students are introduced to concepts of ‘measure’, ‘unit of measure’, and ‘unit’. A ‘measure’ is introduced as any quantity that is deliberately isolated as a means of measuring other quantities. Students find a relationship expressed as a number between a quantity as a whole and some part of it (equal in size to the ‘measure’). Students use continuous quantities and sets of discrete objects as well. The concept of unit is developed on the basis of the relationship of size to ‘measure’ and is taught by its definition (that which is measured off and is equal to the unit of measure is a unit, or ‘one’, and is a unit only in this relation to the ‘measure’). To measure means to isolate units and to determine their quantity. Students make a clear distinction between the quantity being measured, the ‘measure’ and the number being used to designate the relationship between them. Davydov and his colleagues designed a series of activities oriented to give young children opportunities to cultivate, in a natural way, simple forms of reasoning that involve the relations of equality and order.

Below ‘design experiments’ along Davydov’s curriculum framework are reviewed.

2.2.2: Studies in Davydov’s curriculum framework

Yoshida (2004) studied 40 third graders on how they handled the everyday concepts of fraction in mathematics. A series of seven fraction classes were observed for five days in Hiroshima, Japan in 2001. The learners did not have concepts of ‘unit whole’, or ‘one –whole’. The teacher provided situations based on three elements.

- multiple meanings of fractions ‘partition fractions’ and ‘quantity fractions’
- multiple objects for fractions –rectangle and square papers
- multiple ways of dividing equally –into three and into four

He found that conflict between everyday and mathematical concepts led to the development of concepts for fractions in children. The children’s everyday concepts were exposed in class and these contradicted the system of school mathematical concepts, and thus, the everyday and school mathematical concepts were preserved as unified concepts having both system and concrete contexts.

Schmittau (2003) presents a study based on Davydov’s curriculum which is grounded in Vygotskian cultural-historical theory/psychology. The curriculum is based
on measurement rather than counting as genesis of number and multiplication. The first 
grade course starts with comparison of two quantities (length, area, volume or weight) 
which differ to an extent that it permits visual determination of their equality or 
inequality. Then learners are given quantities that do not differ much such that they 
require alignment to effect a determination as to which is greater or smaller (Davydov, 
Gorbov, Mikulina, and Saveleva, 1999, cited in Schmittau, 2003). After these tasks, 
learners are given tasks that require them to compare quantities that cannot be aligned 
(for example they could be asked to compare volume of liquid in two containers having 
very different shapes – the learners have to find a third container into which to pour the 
original liquids to determine which of them has greater or lesser volume). Then later as 
learners gain confidence they are confronted with tasks of comparing two long line 
segments with an intermediary unit such as a short strip of paper to use for this purpose. 
The learners now have to measure each one of the strips of paper. The measure is 
expressed as a ratio of the length of the original segment to the length of the unit. The 
measure may be a whole number or a fraction or even an irrational number.

It has been reported that the genesis of number from measure gives greater 
coherence to the category of real number and spares the learner successive conceptual 
upheavals. The learners master one solution method after another, each time they are 
given/or are confronted with a problem for which the previous method no longer is 
adequate. In the process the learners develop the power to analyze situations such 
models afford them. Appropriately constructed models give learners the ability to grasp 
conceptual structure at its most abstract level, thereby enabling them to ascend from the 
abstract to the concrete (p.234). Also these models allow conceptual connections 
between mathematical actions previously viewed as separate operation and also provide 
learners with tools of analysis required for problem solving.

The development of models that form such a curriculum, however, require an 
epistemological analysis of the concepts that encompasses both historical and conceptual 
analyses. According to Davydov (1990) this entails a lengthy and arduous process, but a 
necessary one, since symbolic forms of thought “absorb the genesis of a concept, making 
it “necessary to trace all of the historically available methods of solving the same 
problems in order to see the initial forms behind the abbreviated curtailed thought
processes (represented symbolically) to find the laws and rules for this curtailment and then to detail the complete structure of the thought processes being analyzed” (p.322, cited in Schmittau, 2003, p. 232)

Schmittau (2003) also reports on a cross-cultural study of the conceptual structure of multiplication between 40 secondary and university students in the United States and 24 elementary and secondary students in Russia. The study was about finding out how, “Students who experience a curriculum designed to foster the development of a generative metonymic structure for the categories of real number and multiplication differs from that of students instructed in Davydov’s curriculum which develops the concepts of number and multiplication very differently.” (p. 235)

The subjects were assigned the task of rating instances of multiplication on a scale of 1 to 7 for degree of membership in the category. The instances to be rated included integers, fractions, irrational, monomial and binomial products, and a product of length and width yielding rectangular area. A flexible clinical interview format was employed in probing subject’s responses to questions of “what is multiplication?”, “In what sense do you consider this to be multiplication?” (referring to inter, irrational, binomial etc).

The results on the rating task indicated that the American student’s multiplication possessed a prototypical structure; they assigned 4 x 3 a rating of 1 while the other instances were rated considerably less representative of multiplication. Further the U.S. university students and two-thirds of the secondary students indicated that they did not see area of a rectangle as a multiplication. The Russian counterparts, however, did not give evidence of prototypical on the ratings task. Further the Russian students were able to obtain the product of two binomials and explained in what sense it represented multiplication.” (p. 237). She concludes that “the U.S. subjects’ solutions were as a result of attempting at formation of a scientific concept through the cognitively dysfunctional means of empirical abstraction. The pedagogical experiences of the Russian students on the other hand were the result of an extensive historical, conceptual and psychological analysis on the part of Davydov and his colleagues.” (p. 242). They generated real numbers through action of measuring.
This study apart from providing a prototype of pedagogy informed by Vygotskian psychology, has much to contribute to consideration of epistemological and psychological foundations for curriculum and instruction.

Schmittau (2004) conducted another study on uses of concept mapping in teacher education in mathematics which was also located in the social-historical theory. She studied a case of two pre-service teachers who presented their concept maps on their understanding of multiplication. The study was meant to illustrate or show how a concept map can alert a teacher educator to whether or not students are understanding mathematics as a conceptual system (in which procedures are fully integrated), or grasping it at the level of more formalism (Schmittau, 2003).

The results showed that the maps reveal much about whether future teachers grasp the nature of mathematics as a conceptual system, understand the conceptual content of mathematical procedures, and possess the requisite pedagogical content knowledge to mediate such understanding to future learners. The map of participant A (Katie) revealed her understanding of what multiplication is “a change in units in order to take an indirect measure” (cf. Davydov, 1998). The other participant B (Shawn), however, saw multiplication as an “operation which is composed of an “operator” and “operands”.

Katie’s map reflected the centrality of the concept of area to her understanding of multiplication. Her map according to Schmittau (2003) also reflected the participant’s knowledge on her part of the cultural historical development of the solution e.g. quadratic equation by factoring. She concludes that Katie understanding of the multiplication reflects internalization of the relevant content and pedagogical content knowledge necessary to teach this concept meaningfully.

These studies highlight the need for Vygotskian theory to unfold the historically developed conceptual content from its encapsulation in symbolic expression in order to pedagogically mediate the full restructuring of the concepts (Davydov, 1990).

Cobb P and Hodge L (2002) present a research report based on Davydov’s curriculum principles, on learning, identity and statistical data analysis. This study is located in the socio-cultural perspective focusing on students’ development of a sense of who they are in relation to statistics as an integral aspect of their learning. The research targeted 12 year olds who participated in a design experiment. The participants engaged in, saw
value in, and viewed themselves as competent at developing data-based arguments. The study also focused on how the design experiment supported the students' development of the positive orientations towards statistics.

The experiment was conducted over a 14 week period with a group of 11 American eighth-grade students. The composition of the participants were 7 African Americans, 1 Asian American, and 3 Caucasian Americans. The first part of the instructional activity involved the teacher and students talking through how they could generate data that would enable them to address a particular problem or issue (Davydov's curriculum framework). The second part of the instructional activities, students analyzed the data individually or in small groups using a computer-based analysis tool that was developed for the experiment.

They found that within the design experiment students learnt significant statistical ideas; they developed relatively deep understanding of the process of generating data. They were said to able to anticipate that the legitimacy of the conclusions that they would be able to draw from data depended crucially on the soundness of the process by which those data were generated (Cobb & Tzou, 2001). Thus, they concluded that 'design experiment classroom' would constitute an appropriate case in which to investigate issues relating to students development of identities as doers of statistics.

Other studies and observations have shown that pure procedural knowledge, whether learned in mathematics or in any other subject domain, tends to remain meaningless and non-transferable (Bruer, 1993; Davydov, 1990; Hiebert and Wearne, 1985; Karpov, 2003; Talyzina, 1981). As Leontiev (1983) indicated, "In order for a child to develop the highest generalization (that is, a concept), it is necessary to develop in him/her the system of psychological operations that are relevant to this highest generalization." (p.347, cited in Karpov, 2003, p.68). A good example cited is the learning of the concept of 'perpendicular lines', he outlines that procedures that underlie the concept should be the methods of identifying within the given pair of lines those attributes that are necessary and sufficient for associating (or not associating) this pair of lines. A combination of conceptual and procedural knowledge has a much higher quality than pure verbal scientific knowledge or meaningless non transferable procedures (Karpov, 2003).
Recent years has seen a trend in research attempting to establish links between the abstract and the concrete aspects of human activity. Below studies which particularly focused on this aspect are reviewed.

2.2.3: Studies linking everyday and scientific concepts

Saljo R and Wyndhamn J (1996) in their study of solving of everyday problems in the formal setting; an empirical study of the school as context for thought, they set a task for students to work on everyday problem of establishing what it would cost to send a letter by using the official table of postage rates of the Swedish post office. (The task given to students was focused/formulated as follows; what would it cost to send a letter that weighs 120 grams within Sweden?) They indicate that the task was fairly familiar to students though it was also viewed as an abstract problem. (The students were not performing the authentic action of sending a letter, but rather were attending to a hypothetical instance of an everyday action).

The research targeted 214 students (aged 15 – 16 in the 8th and 9th grades) in Swedish compulsory schools. Results were analyzed for 211 students. The participants solved the problem individually as a paper and pencil task, and the analyses were based on the written solutions and explanations presented by students. They found that there were a higher proportion of answers in the reading off category among the grade nine students. However, the task was given to the students in two contexts; in a mathematics class and in a social studies class. The results showed that the overall context in which the participants found themselves tended to determine their interpretation of the task. Dealing with postage rates as part of a mathematics lesson lead students to perceiving the task as mathematical in nature and conversely, when the task was embedded in the context of social studies class, there was a tendency to read the table and to abstain from calculating.

They found that participants’ interpretation of how to solve a problem related closely to their implicit and explicit assumptions about what is a natural mode of proceeding in a certain situation and given in certain type of task. The actions of participants provide evidence that they hold strong implicit pre definitions of what it means to take part in mathematics lessons and to be part of the community. They conclude that studies of
human cognition should be based on an epistemology recognizing that the world is inherently complex, multifaceted, and open to interpretation.

Steinbring and Seeger (1994) studied how children while doing measurements in different situations conceptualized the relation between the quantity to be measured and the unit of measurement. The exploration of the relationship was embedded in different concrete activities of measurement and comparison. Estimation of measures played a more important role than exact calculations. Addition with 'concrete numbers' can be integrated into the open situation very naturally and produces nearly no problems of understanding. But as soon as the theoretical nature of mathematical concepts increases, the problem of integrating the mathematical formal knowledge (formula) becomes problematic. “The ‘funnel pattern’ interaction (that is question-answer interplay between teacher and learners) produces not only an algorithmization of mathematical knowledge, it also deprives it of its theoretical character in the course of the interaction reduction process. (Steinbring, 1988, 1989, 1994). It gets reduced to procedural or an algorithmic matter, where it can be dissolved into small steps.

The clash of the ‘natural’, everyday and the ‘formal’, mathematical context is problematic. Learners move within the open context of everyday problems safely and with considerable creativity, but in the ‘formal’ mathematics the story changes. “The emergence of theoretical thinking and reasoning is dependent on mythical thinking and at the same time one has to overcome the simple forms of mythical thinking” (Steinbring and Seeger, 1994, p. 178). They write that traditional teaching processes tend to introduce (implicitly) a universal epistemological basis for school mathematical knowledge with learners taking mathematics as objects arranged according to strange rules, and objects to operate with according to mysterious regulations. A major methodical goal of mathematics teaching is the integration of new, theoretical knowledge into the natural context of communication. “The conflict between the naturalness of everyday knowledge and the systemic and self-referential character of theoretical mathematical knowledge constitutes a basic problem for teaching-learning process. On the one hand, the social context of teaching requires a certain amount of naturalness to make communication and understanding in the classroom possible. On the other hand,
the theoretical nature of new knowledge is in conflict with immediate naturalness of theoretical relations.” (p.181)

Thus, learners and the teachers fail to communicate, although they may be using identical expressions/terms. They operate in incompatible mathematical worlds; they play according to different rules. According to Sfard (1994) rather than try to make his/her meta-mathematical principles explicit, the teacher should aim at providing a student with an appropriate experience. The student is more likely to be converted to a new mathematical practice by living in a new mathematical world than just by talking about it. “The master-apprentice mode of learning, based on participation in doing rather than on explicit dialogue between the teachers and the students, seems to be a promising pedagogical option.” (p.285).

Learning mathematics in school has also been interpreted as a special case of culture (Cobb and Yackel, 1998; Nickson, 1994) that is participating in the process of mathematics classroom is participating in a culture of Mathematizing (Steinbring, 2005). The meaning of the culture concept for scientific mathematics and for school mathematics has been emphasised by different authors (Wilder, 1981; Bishop, 1988). According to Steinbring every mathematical knowledge requires ‘certain sign or symbol systems’ in order to gather and code the knowledge. The signs and codes themselves do not have meaning. But for these to have meaning they require appropriate ‘reference contexts’ and they are actively constructed by the epistemic agents.

She showed the epistemological role of mathematical signs/symbols in the interaction construction process of mathematical knowledge through the example of number. The mathematical sign ‘4’ stands for the conceptual number ‘4’, and this is an abstract conceptual idea, but in order to let learners access this abstract idea, a multitude of materials to which ‘4’ relates are utilized, for example 4 little balls/or coloured chips are presented or given to depict the number ‘4’.

But mathematical signs/symbols ultimately always relate to universal mathematical conceptual ideas not to concrete mathematical ‘objects’. Thus, the mediation between signs and structured reference contexts requires conceptual mediation (Steinbring, 2005, p.22). A fundamental conceptual idea is necessary in order to regulate the mediation between a sign and reference context. Thus, this contrasts diversely to
understanding of numbers as representing concrete objects. “Mathematical knowledge does not relate directly to concrete or real objects. Mathematical concepts and mathematical knowledge, coded in signs and symbols, represent abstract relations, structures and patterns (Steinbring, 2005). She goes on to say that, at the same time this mathematical knowledge is not pre-given, finished product, but interpreted according to the epistemological conditions of its dynamic, interactive development.

Mathematical concepts are constructed in interaction processes as symbolic relational structures (the symbolic relations have to be actively constructed and controlled by the subject in interactions.) According to Steinbring meaning-making (construction of meaning) for unfamiliar new mathematical signs require building up reasonable relations between signs and possible contexts of reference and of interpretation.

Sfard (2001b) presents a study which conceives learning mathematics as developing a discourse. She presents classroom data as she explores the learning of concepts not apparent in the everyday experiences of the learners. The context is two classrooms where learning or new mathematical topics was going on. One lesson is on ‘negative numbers’ and the other is on ‘triangles’. The analysis is focused on how children acquire the concepts of negative number or triangles or how they construct the concepts for themselves. The learners were twelve-year olds, seven graders, and the language of instruction was Hebrew (translated into English by Sfard).

Learning was considered as taking part in a discourse or communicating about the concepts, that is, the process of changing one’s discursive ways in a certain well defined manner. Thus, as learners learnt about triangles or negative numbers, they altered and extended their discursive skills so as to become more able to communicate on these topics with members of the mathematical community. In the framework of learning mathematics as a development of a mathematical discourse, she investigated learning as the ways in which children modified their discursive actions in:

- their vocabulary;
- the visual means in which communication mediated and
- the meta-discursive rules that navigated the flow of communication and tacitly told the participants what kind of discursive moves would count as suitable for this or that discourse and which were inappropriate (Sfard, 2001b)
Based on the results she found, she challenged some of the perspectives on the beliefs on learning mathematics such as; that 'learning with understanding stresses the importance and primacy of conceptual understanding over formalization and skill.' She states that one can make sense of mathematical discourse only through persistent participation and not prior to it. That is, whenever one wishes to become fully fluent in mathematical community one has to persist in practicing mathematical discourse; she also cautions/challenges beliefs, that effective learning requires that mathematics activities be kept embedded in real-life contexts, that is, keeping school mathematical discourse as part of everyday discourse. This, she asserts would be a contradiction to the aim of 'schooling', which require learners to be initiated to a special type of mathematical discourse (school mathematics). Further, she highlights the importance of significant other in the presence of the learners, as learners on their own right may not develop the mathematical discourse.

Ben-Yehuda, et al (2005) in their study “Doing wrong with words: What bars students access to Arithmetic discourses”, they investigated mechanisms of failure in mathematics. They adopted the communicative approach to cognition, that is, learning mathematics as an initiation to a certain type of discourse. As they searched for factors that impede student’s participation in arithmetic communication, they examined the arithmetical discourses of two 18 year old girls with long histories of learning difficulties. In order to analyze the learning difficulties, they decomposed the arithmetic operation into further elementary operations. They also analyzed the two participants’ self-referential remarks and narratives. From the analysis they make two claims;

1) “Considering the multiplicity of ways in which any arithmetic task can be performed, almost any person may become a skillful participant in arithmetic discourse, provided, first, that a discursive mode is found which makes the best of this person’s special strengths and bypasses her particular weaknesses and second, that in the process of teaching, the general social and cultural context of learning are taken into account as having a central role in enabling or barring one’s access to literate discourses.

2) If this potential for successful participation remains often unrealized it is not only because of the current, insufficiently developed, ways of assessing individual
students' strengths and weaknesses, but also because of such school practices as privileging mathematics over other subjects, invalidating colloquial arithmetic discourse, and using literate mathematical discourses as a general success - predictor”

Brown and Renshaw (n.d.) examined the learning from concrete (spontaneous) to abstract (scientific) knowledge in the classroom situation. They examined Vygotsky's distinction between everyday and scientific concepts. The discourse formats used in their analysis were obtained from published research on classroom talk, but the data was obtained from Brown's (2001) Ph.D thesis. The data comprised video/audio recordings of teacher-student interactions gathered over the course of one school year in a year 7 classroom. In particular the analysis focused on the final group's ideas and representations to the whole class (class presentations). In their analysis they identified two types of discourses in the interactions, which they modeled as 'replacement type' and 'interweaving type'.

The replacement pattern of discourse emphasised adoption of precise vocabulary and acting within the ground rules of particular discourses genres, thereby replacing the more concrete and everyday way of representing knowledge. In this frame of discourse, the pedagogical process required children to work within the system of signs and symbols with its own logic and set of meanings. Also they found that it was the teacher who focused on the mathematical practices to be represented and compared. The interweaving model referred to a discourse where children populated the classroom talk with their own purposes, that is, their inventive ideas were interwoven with the conventions of mathematics or where the children's individual approaches to doing mathematics were interwoven with the more flexible representations systems used by mathematicians.

They state that the 'replacement' and the interweaving' patterns of discourses were alternatives to the Initiation-Response-Evaluation format (Mehan, 1979) which reject outright the initial heuristics conversations from learners. Although the 'replacement' pattern script is similar to the Initiation-Response-Evaluation type, they differ in that the 'replacement' format, "delays the teacher's evaluation of students' responses in favour of recontextualising what the students 'think' within the discourses
practices of a mathematical community". Thus, there is a phase where the learners' concrete and experiential knowledge is rephrased, re-represented and replaced by the more abstract and general concepts of mathematics.

The interweaving format offers a hybrid form of understanding, where the everyday and scientific concepts grow together. They conclude that either format, replacement or interweaving extends the students participation in mathematics, linking their inventions to the conventions of mathematics.

In light of these different epistemological perspectives in the two areas, how do the children/learners behave in the two contexts? Below, section 2.3 reviews literature dealing with common mathematical knowledge out-of-school and in-school.

2.3: Learners mathematical practices in and out-of-school

A number of studies have been done in last fifteen years describing how children use mathematics in and out of school situations. The results indicated that the knowledge/practices are different in a number of ways in the two areas. The studies show that the local strategies developed in practice are more effective than the arithmetic algorithms taught in schools (Bonotto, 2001; Lave, 1988; Lave & Wenger, 1991; Carraher et al, 1985; Nunes, 1993).

Carraher, Carraher & Schliemann (1985) and Saxe (1991) investigated mathematics amongst street children. The street sellers and other workers in Recife, Brazil, with restricted school experience showed that in addition, subtraction, multiplication and division the participants performed fluidly and virtually error free, and without using pencil and paper or traditional algorithms. Their understanding and use of the properties of the decimal system when they dealt with addition and subtraction problems in the context of commercial activities were almost flawless compared to their counterparts in school. This however was located within a sense-making and relevant activity, such as commercial trading of their products. In contrast, when participants were given the same mathematical problems to solve using school algorithms, the students could not demonstrate the same degree of proficiency. The results indicated that the children were overwhelmingly more capable at solving problems in the informal setting.

Nunes et al (1993) report of studies done in Brazil under a research programme of the Universidade Federal de Pernambuco, Recife. The studies compared the strategies
used by the children first while selling their wares in street markets and later using written mathematics. It was observed that people who depended on oral arithmetic made very few errors and adopted efficient ways to prevent deviating far from the correct answer.

Scribner (1983) carried out a study amongst the dairy warehouse workers. She contrasted these with ninth grade students and clerks (who were new to the task, just like students). The task was what dairy warehouse workers perform in their everyday work loading dairy products of differing sizes on trucks in accordance with specific customer orders. The two groups were observed closely in problem-solving situations to reveal important differences. She found that the students tended to be single algorithm problem solvers drawing upon a range of school-based mathematical practices, while the workers used arithmetic short-cuts in combination with the visual display.

Davidenko, Masingila and Wisniowska (1996) present research from several of their studies that illustrate some of the differences between in-school and out-of-school mathematics practice, in the hope of finding a framework for connecting these experiences. They also look at the work of Saxe (1991) who gave a framework as a method for studying the interplay between socio-cultural and cognitive development processes. In their own work on differences between the two settings they found that the differences involved the goals of the activity, the conceptual understanding of persons in each context, and flexibility in dealing with constraints. Using Saxe’s intent for conducting research to better understand the interplay among various cognitive forms through practice, that is, using his ideas and terminology, they came up with ways of connecting in-school with out-of-school experiences.

First, that the goal structures of activities must be similar for in and out-of-school activities from which students may construct similar mathematical knowledge. Second, social interaction should be an essential part of classroom mathematics practice. Third, in-school activities should make use of cultural artifacts and conventions that students can use to interpret problems and make sense of them. Fourth, teacher can build on students’ prior understandings through encouraging them to bring it to bear as they create their own problem situations, solve problems in more than one way and share their
solution methods with each other (Lester, 1989) and to let them focus on semantics rather than syntax.

Bonotto (2001) explored through a study for which special cultural artifacts were used, that is, supermarket receipts, to try to construct with 9-year old pupils new mathematical knowledge. The task was establishing the algorithm for multiplication of decimal numbers. They introduced estimation and approximation processes too. The children who already knew decimal number and the units of measurements for length, also had knowledge of kilogram, hectogram, and gram was from their out-of-school (everyday) experiences. Their study found that the link with out-of-school experience facilitated by the use of the artifact (supermarket receipt) contributed in giving meaning to new mathematical knowledge and to reinforcing previous knowledge. The use of the procedure of estimation and approximation allowed the children to participate more freely focusing on the mathematical concepts rather than on pure and simple computations.

A link between mathematical knowledge of children prior to school and practice of school mathematics appeared to be a way forward as Tsai W-H (in press) carried out a study which tested a model he referred to as 'Cultural Conceptual Learning Teaching (CCLT) model. The model looked at the ways in which children's real experiences and cultural practices could connect to mathematical classroom lessons. The study was done in Taiwan at Hsin-Chu among the second-grade classes in mathematics. The study found that school mathematics for those classes using the model based on cultural activities improved more than for classes in the control group.

The studies outlined above look at the strategies in out-of-school and in-school (Nune et al 1993), type of cognition in the two contexts (Resnick, 1987); comparisons between these in terms of effectiveness, procedures (Lave, 1977; Masingila et al 1996); differences in terms of type of activities, goals, values (Carher N.T., Carraher D.W, Schliemann, 1985, Saxe, 1991), type of learning in each of the contexts, out-of-school and in-school (Lave & Wenger, 1991), nature of mathematical knowledge in cultural settings (Panda, 2004) and development of number concepts (Renick, 1987). These studies have something in common about the mathematics done in and out of school, according to Masingila et al;
"- people look efficacious as they deal with complex tasks,
- the mathematics practice is structured in relation to ongoing activity and setting,
- people have more than sufficient mathematical knowledge to deal with problem,
- mathematics practice is nearly always correct,
- problems can be changed, transformed, abandoned and/or solved since the problem has been generated by the problem-solver and
- procedures are inverted on the spot as needed" (Masingila et al 1996).
This contrasted with school mathematics which emphasized individual cognition, abstract thought and general principles (Resnick, 1987).

While these studies supported the assertion that use of children’s prior mathematical knowledge was beneficial to them for purposes of transfer of learning to new situations and also provided ample knowledge about differences in strategies in the two settings, there were still gaps in knowledge about how school based concepts/ideas develop from everyday concepts and further, how epistemic practices which are different in the two areas are reconciled while dealing within the common mathematical knowledge. Section 2.4 reviews some studies which have dealt with epistemological issues in classroom situations.

2.4: Other studies focusing on epistemology in the classroom

Jablonka (2002) carried out a study on the role of context in school mathematics. In her study she analyzed about a hundred examples from mathematics textbooks and teaching materials for primary and secondary students. She analyzed the data using categories from epistemology and linguistics to determine how particular out-of-school practices of using mathematics were re-contextualized in written texts containing tasks. She found that there were epistemological claims in the examples or contextual problems/tasks that did not go hand in hand with the school mathematics frame. The epistemological claims were that mathematical concepts and structures were assumed to meet some essential features of empirically given phenomena and thus mathematics was seen as appropriate method for the production of new knowledge. She highlighted a number of these claims in her study and she concluded by cautioning that “When developing or using written texts for students; explicit attention should be given to the values that inform the selection of the context and descriptions of the problems. The
confusion of different practices of using mathematics … has to be considered problematic because it causes wrong perceptions of the usefulness of mathematical knowledge”.

Evans (1998) reports about a study he did illustrating the problems of transfer of classroom mathematical knowledge to practical situations. He carried out the study with 25 college students of about 21 years of age. The focus of the study was student’s thinking and affect as well as their earlier experiences with mathematics and numbers in relation to each positioning themselves in specific practices. From the study he concludes that “... the differences in goals and values, social relations and regulation and especially language/signification and emotional association – between different discursive practices which make transfer, in the sense of the application of concepts or ‘skills’ from school math to everyday practices, highly problematical” (p.284). The study suggests that transfer because of vagaries of signification and emotional changes are difficult to predict or control.

This study looked at possibilities of transfer from school knowledge to everyday practices and further the research target was adults in various previous experiences.

Also Steinbring (1998) carried out a study examining the ‘disparity’ between teacher’s and the students’ background knowledge. He found that the difficulties arose due to student’s background understanding of the nature of mathematical concepts (school mathematics). He investigated mathematical understanding in classroom interaction and treated mathematical understanding as the deciphering of social and epistemological signs and symbols which has a reciprocal process of understanding between teachers and students. In the first part of the study, he analyzed the understanding of written symbols with the help of procedures describing the correct mathematical operations, as he looked at addition and multiplication of decimal fractions and a diagram that dealt with division of fractions. In the second part of the study, he looked at classroom interaction and understanding. The first episode dealt with the topic “What is relative frequency”, while the second episode dealt with “The area formula for the trapezoid”.

He observed for the first part of the study that, “… the more the accepted answers are restricted to the social/conventional side, the more the students are deprived of some epistemological means of justification. They were only able to make proposals to be
approved or rejected by the teacher” (p. 358). He goes on to say that the restriction of the interaction on the social/conventional level turns the understanding of a piece of mathematical knowledge into a process of negotiating the right words, names, and rules, and in most cases its validity can be delivered only by the teacher’s authority (Steinbring 1991, ibid p. 359)

In the second part of the study he concludes that, “A re-establishment of the balanced interplay between understanding new mathematical knowledge and organizing it in a socially conventionalized discursive frame of understanding is only possible in everyday mathematics interaction when the teacher becomes aware of the need to understand something in this process deeply for himself or herself, and that not everything is already understood beforehand.” (p.366). That’s the process of understanding shows the interaction of social, conventional, and epistemological constraints that has to be developed and related in order to produce real understanding.

Struve (1990) carried out a study on the nature of mathematics peculiar to school and compared it with the characteristics of modern mathematics (university mathematics). He found and concluded that a pure mathematics discover the truths about mathematics reality (and understood the meanings of the objects) because of the formal interrelatedness among the objects in context of a mathematical theory. In order to understand the meaning of a mathematical object, the mathematician can content himself/herself with its formal definition and its theoretical use. In contrast in school (especially elementary school) the meaning of a mathematical object was related to empirical aspects. The number concept is related to materials, geometric concepts are related to physical space and so forth. Thus he concluded that the epistemological bases of school mathematics differ from those of university mathematics.

The study by Jablonka above draws attention to the dangers of superimposing or taking the context, that is, information or activities with a particular epistemological perspective as universals. The study by Evans looked at the transfer of knowledge from one context to another and showed how difficult the exercise is. Steinbring study set the conditions for understanding to be achieved; these are social and epistemological signs and symbols as well as an understanding between teachers and learners. Struve study
contrasts the epistemological perspectives of school mathematics and university mathematics’ practices.

2.5: Summary

The review of the related literature above highlights the following aspects in the field: mathematics is a human activity and that all cultures endeavour in it as they solve problems in their everyday lives, further that mathematical practices differ from one cultural group to another. There is some commonality nonetheless between the community’s mathematical knowledge and the school mathematical knowledge. The practitioners of everyday mathematics are generally unaware of the ‘mathematics’ they practice. The mathematical knowledge is embedded in their on-going practices. The mathematical knowledge is local, segmentally organized, context specific and dependent, and it is directed towards achievement of specific, immediate and relevant goals of the comments/individuals.

Thus, according to Bernstein (1999) it has a particular epistemological basis which he describes as of horizontal discourse. In terms of learning, the everyday knowledge (mathematical knowledge included) utilized a number of strategies, many of them unconscious efforts such as keen observation and collaboration in groups, listening and contributing as and when ready. Within the zone of proximal development, caregivers and other companions offer access and often provide support and pointers in the context of shared community activities (Vygotsky, 1978). This, however, involves complex group relationships involving learners who learn to take responsibility for contributing to their own learning and to the community’s goals (Rogoff, 2003). Thus, according to Bernstein the pedagogy was in the context of the activity and was repeated till the competence is acquired. Everyday mathematical knowledge is contrasted with school mathematical knowledge. From the review Vygotsky makes a clear distinction between everyday concepts (knowledge) and what he terms scientific concepts (school related concepts). The scientific concepts begin in the domain of conscious awareness and volition and form a logical system, they are generalizable and they are removed from material experience and exist within a hierarchical network of related concepts. Nonetheless Vygotsky saw the two types of concepts as interdependent. It is noted that a
special process is required to bring together a child's everyday representations with the more abstract scientific representations.

The studies also reveal that appropriation of spontaneous and scientific concepts is the result of fundamentally different types of learning. Everyday involves empirical learning, while scientific involves theoretical learning. The studies also indicate that there is conflict between the naturalness of everyday knowledge and the systemic and self-referential character of theoretical mathematical knowledge, which constitutes a basic problem for teaching/learning process. Also studies following Davydov's curriculum suggest that the mathematical concepts require an epistemological analysis that encompasses both historical and conceptual analyses in order to pedagogically mediate the full restructuring of the concepts (e.g., teaching the genesis of number from the historical concept of 'measure').

However, other studies highlight the need to explore different epistemological perspectives between the teacher and the students. Some studies reveal that different epistemological perspectives are legitimate or justified within given contexts, and that learner's epistemological perspectives are to a large extent influenced by the context and these tend to be implicit and context-specific rather than explicit. In school, however, the epistemological basis is explicit and it manifests itself in the signs and symbols apart from the social interactions between teachers and learners.

These studies also re-assert that children's prior mathematical knowledge was beneficial to them for purposes of transfer of learning to new situations. While these studies shed light on how school based concepts develop from everyday concepts, of concern is how everyday and the scientific concepts interact and what the resulting epistemic practices are. The epistemological basis of the background knowledge of the learners once explored could provide opportunities for understanding the breakdowns in communication between teachers and learners. These studies have dealt with epistemological perspectives (basis) for everyday concepts and school mathematics concepts, however, the discourses that result in a classroom (utilizing vertical as well as horizontal discourses) have not been theorised. Thus, this study focuses on the epistemology of these practices in the classroom.

The next chapter presents the methodology used to carry out the study.