CHAPTER 1

INTRODUCTION

Recent studies investigating achievement scores in mathematics among primary school children in Zambia (that is, children in the first seven grades of formal education) have indicated that achievement levels have remained low including low motivation for the subject among both teachers and learners (Ministry of Education (MOE), 2004; Kelly, 1991, 1994). One study concluded that almost three quarters of grade six (6) pupils could not comprehend texts satisfactorily. The National Assessment Report of 2001 put the numeracy scores at 28.7 percent (MoE, 2001). The unsatisfactory state of development of numeracy problem solving skills was attributed to, “rote learning, unclear progression, following mathematical rules without understanding and calculations without meaning” (sic, MoE, 2004, p.5). This was despite pursuing fundamental goals of acquisition of basic numeracy and problem-solving skills as part of the curriculum for lower and middle basic classes (MOE, 1996).

The low levels of numeracy in schools have also been attributed to traditional teaching and learning approaches adopted at this level. The difficulties of many pupils in mathematics and science are believed to be caused by the way they were introduced to these basic concepts and ideas in primary school (MoE, 1996). Some of the difficulties were attributed to teachers not considering children’s experiences worthy of including in the classroom discourse. As rightly pointed out by Resnick, “the accumulated body of research on the development of children’s mathematical knowledge indicates that certain fundamental concepts are normally constructed by children and are, therefore, available as the basis for further mathematical development. Current school practice, however, seems not to build on this informal knowledge, and in some cases, it even suppresses it deliberately.” (1989, p.236).

The studies done in countries other than Zambia have also found that mathematics teaching and learning is traditionally viewed by the teachers as a body of facts and procedures characterized by conceptions of mathematical cognition as being independent of context in which it is acquired (e.g D'Ambrosio, 1985; Gerdes, 1997; Resnick, 1987; Lave, 1988; Rogoff, 2003 etc). Teaching and learning at school assume that the children
came to school without any kind of mathematical knowledge, so they have to be introduced to the subject for the first time in the school (Resnick, 1987). This leads to maintenance of separation of children everyday knowledge from what they learn in school.

The isolation of individuals from the social context in which their knowledge developed assumes is problematic as Doise and Mugny (1984) stated that, “It seems that the attempt to remove all cultural bias from a test of cognitive functioning proceeds from a deep misunderstanding about the nature of cognition. Cognition is not independent of culture; one cannot conceive of it as a set of principles of functioning independent of particular circumstances and of the intentions of the subject. In reality, we should attempt to make our measures of cognitive functioning more sensitive to cultural variations.” (Doise & Mugny, 1984, p.13). According to Dieckmann, Gutierrez, Irving (2007), to become mathematically literate required explicit attention to the development of an expansive linguistic and artifact rich repertoire, including the socio-cultural knowledge of what it meant to do mathematics.

Many empirical works on the role of culture in cognitive development suggest use of everyday tasks that are already part of people’s repertoires of practice (Gutierrez & Rogoff, 2003) as starting points (Cole, Gay, Glick and Sharp, 1971). The cultural groups have been found to vary in their use of mathematics in everyday lives, that is, how they count, measure, relate, classify and infer or broadly speaking, how they mathematise (e.g. D’Ambrosio, 1984; Gerdes, 1997; Bishop, 1988 etc). These social groups such as farmers, engineers, and professional groups of people as well as children enact a culture with their own specific jargon, patterns of problem solving, use of symbols and codes and their own ways of ‘mathematising’ (D’Ambrosio, 1984, pp.42-43 cited in Gutierrez, 2007).

Before children come to school, they acquire community knowledge and practices. One, therefore assumes that there may be multiple contact points between home and school mathematics. In other words, one assumes that there could be common mathematical knowledge which cuts across the different contexts; out-of-school and in-school. Children enter school possessing much mathematical ideas and knowledge, however, little was known about the qualitative nature of their experience prior to school
Also of significance were the social practices and activities that embody these mathematical ideas. These embedded knowledge need to be made explicit so that those can be used optimally in classrooms. In school however, the treatment of this mathematical knowledge was based on theoretical arguments related to 'mathematical objects', which was characteristics of school knowledge (Vygotsky, 1978). Thus, it is important for learners to extend or broaden their tools by encompassing this mathematical rationality. But, in order to make sense of an object or event, subjects (learners) use their background knowledge to form meaningful context for interpreting the object and thus, ambiguities and negotiation of meanings are essential features of mathematics in school which require epistemological consideration (Voigt, 1998). Moreover, it is necessary to examine the epistemic practices of the community and that of the school and thus, identify the kinds of practices (epistemic) that arise in the interface between out-of-school and in-school mathematics practices.

The present study focuses on the epistemic practices of this common mathematical knowledge and attempts to understand through the emerging paradigm of learning. Before presenting the study, it is important to discuss the theories that inform this study.

1.1: THEORETICAL FRAMEWORK

The study is located in three broad theoretical perspectives; the idea is to look beyond single frameworks, and to remain open to the different ways these theories from outside the discipline of mathematics education that help us raise some pressing and enduring questions in the field. Moreover, as cognition involves observation, thinking, reasoning, problem-solving, experimentation and so on and thus, it is a multidisciplinary phenomenon (Sacha, et al 1997) and should therefore be studied taking multi-disciplinary perspective.

1.1.1: Cultural-Historical Theory

Cultural-historical theory of cognitive development is the main theory within which the study is located. The theory posits that knowledge acquisition is essentially and inescapably a socio-historical cultural process, where culture is the product of social life and human social activity. In this theory, children’s cognitive development is
conceptualized as socialization into using the appropriate cognitive and communicative tools that have been passed down from generation to generation. It is through such socialization that children learn the accumulated knowledge by doing it in culturally appropriate and relevant ways (Vygotsky, 1978). The core premise of the cultural-historical theory is the intimate connection between the special environment that human beings inhabit and the fundamental distinguishing qualities of human minds. "The special quality of the human environment is that it is suffused with the behaviour adaptations of prior generations in external form." (Cole, 1996, p.59)

According to Vygotsky, human action is mediated by tools which are critical. These tools include among others language, various systems of counting, mnemonic techniques, algebraic symbol systems, works of art, writing, schemes, diagrams, maps and mechanical drawings, all sorts of conventional signs and so on (Vygotsky, 1981, quoted in Tulviste and Wertsch, 1996). Cognitive development and learning, according to Vygotsky, especially depends on the child's mastery of symbolic mediators, their appropriation and internalization in the form of inner psychological tools (Kozulin, 1998, 2003). From this perspective children's cognitive development is conceptualized as socialisation into learning and using the appropriate cognitive and communicative tools that are passed down from generation to generation. It is through such socialisation that children learn the accumulated ways of thinking and doing that are relevant in their culture(s) (Vygotsky 1978). Further, and related to this theory is the situated cognition theory, an offshoot of Vygotskian social-historical theory.

1.1.2: Situated Cognition

Situated cognition theories (offshoots of Vygotskian cultural-historical theory) posit the process of learning and understanding as socially and culturally constituted and that what is learned is integrally implicated in the forms in which it is appropriated. In this perspective learning is seen as social, collective rather than individualistic phenomena, and human beings are relational matter, generated in social living historically, in social formations whose participants engage with each other as a condition and pre-condition for their existence (Lave, 1996).

This is a move away from conceptualisations of context-embedded learning or logical and intuitive understanding with the school being the centre of learning. Thus,
when people engage for substantial periods of time, day by day, doing things in which their on-going activities are interdependent, learning is part of their changing participation in changing practices (Lave, 1996). Lave and other researchers in situated cognition showed that knowledge developed in everyday settings, and that cognitive performance is closely associated to the everyday meaning of the tasks. Thus, to a large extent cognition was not independent of culture. It is a set of principles functioning in particular circumstances and of the intentions of the subject(s).

Learning mathematics in this context is conceptualised as entering into a community of practice of learners of mathematics, where the ultimate goal of learning is to think and act like a member of that community. From this point of view, mathematical practice entailed making conjectures and devising mathematical arguments to be supported in a discursive mode. As Cummins (2001) put it, writing from a Vygotskian perspective, "Interactions with more expert or knowledgeable members of their social groups enable children to internalise cultural tools such as language, literacy and mathematics which have the power to transform higher psychological processes." (P. 60). A theory that explains how knowledge is held in the community is distributive cognition and section 1.1.3 highlights this.

1.1.3.: Distributive Cognition

Distributive cognition theory posits that cognition is socially constructed through collaborative efforts towards shared objectives in cultural surroundings (Salomon, 1993) and in this framework information is processed between individuals and tools and artifacts provided by the culture (Salomon, 1993, p.3, cited in Daniels, 2001, p.70).
Hutchins (1995) a leading proponent of distributive cognition highlights it by asserting that “All human societies face cognitive tasks that are beyond the capabilities of any individual member. Even the simplest culture contains more information than could be learned by any individual in a life-time, so that tasks of learning, remembering and transmitting cultural knowledge are inevitably distributed. The performance of cognitive tasks that exceed individual abilities is always shaped by a social organisation of distributed cognition. Doing without a social organisation of distributed cognition is not an option.” (Hutchins, 1995, p. 262, quoted in Daniels, 2001, p.70).

Thus distributed cognition can be seen through cultures and communities. Learning certain habits or following certain tradition can be seen as cognition distributed over a group of people. As Atran et al (2005) stated, “People’s mental representations interacted with other people’s mental representations to the extent that those representations could be physically transmitted in a public medium (language, dance, signs, artifacts etc). These public representations, in turn, are sequenced and channeled by ecological features of the external environment (including the social environment) that constrain psychophysical interactions between individuals.” (Atran et al, 2005, p. 751).

The structure of cognition is widely distributed across the environment, both social and physical (e.g. Atran et al, 2005; Hutchins, 1995, etc). As Brown et al (1989) explained, knowledge index the situation in which it arise and is used. “The embedding circumstances efficiently provided essential parts of its structure and meaning. So knowledge which came coded by and connected to the activity and environment in which it develop is spread across its components parts, some of which are in the mind and some in the world much as the final picture on a jigsaw is spread across its component pieces.” (Brown et al, 1989, p.40).

These theories within the situated cognition framework are (Vygotskian and post-Vygotskian theories) unlike Piagetian theories that take into account immediate context of an individual excluding societal and cultural factors are suited for the study as it involves epistemology of common mathematical knowledge which derives its essence from the community and the school. These theories share the assumption that learning has to be understood as actions and activities integrated in a complexity of social,
institutional, cultural and historical practices. They involve coordination between individual(s) and artifacts. For example, in the distributive cognition theory, the unit of analysis is widened from viewing the individual as a 'loner' to including the learner's practice in relation to activities in his/her communities of practice.

Thus human knowledge and cognition are distributed by placing memories, facts or knowledge on the objects, individuals and tools in the environment.

**Mathematical Cognition**

Mathematical cognition or learning is conceptualized as not purely intellectual activities, isolated from social, cultural and contextual factors, but as learners learn mathematics while interacting with each other to solve problems in culturally approved specific ways. It is a process of participation in a community of practice, participation that, at first, is legitimately peripheral but gradually increases in engagement and complexity. Mathematics learning/cognition is the construction of present versions of past experiences for several persons acting together (Lave, 1996). Mathematics knowledge and learning is viewed in this context as being 'distributed' throughout the complex structure of persons acting in a setting. Thus, mathematical cognition remains rooted to a large extent to the social, historical and economic dimensions and the concrete life of individuals (Radford, 1998).

If mathematical meaning derives from connections, then, the means by which learners constructively generate mathematical ideas is closely connected to a setting and articulated in terms of the medium of instruction within it (Noss, et al 1997, p.226). Thus, discourse is central, through conversation and dialogue, the children's mathematical leaning is encouraged and supported. In this case the role of capable others is very significant as Vygotsky puts it, those competent others facilitate the child's development in mathematics in the zone of proximal development. Everything depends on the ways in which connections are built, the kinds of discourse, which surround the activity and the structures that are constructed around it.

Focusing on mathematics learning, Tomasello outlined how language facilitates mathematical meaning making processes. Mathematics, like any other semiotic system, functions inside a cultural network of significations and, thus, he talks about causal understanding as the cognitive glue that gives coherence to human cognition in all types
of specialised content domains. He says that number and mathematics underline many important activities, from money to architecture to business to science. He, thus, outlined the process as follows:

"...cultural network of significations .... children encounter these in a cultural and linguistic matrix in which
• they are given specific pieces of knowledge and models of thinking and explaining via language directly.
• they operate with the structures of language including both causal structures and classification, and relational structures.
• they engage in discourse with others about the physical world and its working in ways that induce the kinds of perspective-taking on which some of these concepts depend." (Tomasello 1999, p. 189)

Thus, instead of viewing mathematics as a hierarchical formal and abstract skills/subject, it is seen or best learned through practice and collaborative inquiry and argument. Mcleod (1994) explains it as follows, "This model conceives of mathematics like other domains of knowledge, as social and cultural constructions, a view that makes it acceptable and encourages the inclusion of children’s intuitive strategies for computing and solving numerical problems. It also legitimizes using children’s cultural and linguistic backgrounds as a vehicle for learning." (Mcleod, 1994, p. 232).

Further, apart from these theories of learning/cognition the study is guided by the following concepts as outlined in section 1.2.

1.2: CONCEPTUAL FRAMEWORK

1.2.1: Mathematical Practices

In order to understand and identify mathematical practices in the community the researcher is guided by conceptualisation of mathematics from a wide spectrum of perspectives; though largely it refers to those aspects of the world pertaining to magnitude and number and the relations between them (Brown, 1996), that is, concerning arithmetic operations, structures and problem-solving activity with symbolic tools. Also considered are such aspects as isolation of dimensions and the idea of units especially of physical objects as observed in the discourse as opposed to such formerly favoured units such as concepts, mental schemes, or student’s knowledge” (Sfad, et al 2001). Thus in
the discourse, the researcher looks for exploration of relationships between objects or presentations, that are regulated socially and/or of logical rules.

In situations where the objects that are being counted do not matter, it is the relationship between quantities that matter. These can form model(s) or models are assumed in the discourse, but it is also the process or procedures of drawing conclusions or knowledge/facts from such occurrences that constitute mathematical knowledge (Nunes, et al, 1993).

1.2.2: Epistemic Practices (epistemology)

In order to establish an epistemological basis of the practices of the community and of the school, the focus was on the behaviour processes, methods and practices in terms of their contributions to the production of knowledge. Epistemology (theory of knowledge) is concerned with genesis, structure and justification of knowledge. In classical epistemology, the focus is on prepositional knowledge, thus for one to know a proposition, s/he must believe it, the proposition must be true, and the belief in it must be justified or rationally warranted.

However, the philosophers like Wittgenstein, Ryan, Polanyi and Kuhn argue that not all knowledge can be made explicit. Kuhn (1962/1970) and others view knowledge as shared among members of a culture or community and that it can no longer be regarded as exclusively sited in an individual’s mind. Thus, he describes practices of scientific research communities as being influenced by ‘social factors’, as he included tacit component in his account of scientific knowledge. Foucault (1977, 1980) adds yet another dimension to the argument by bringing in a political view; he argues that the so-called knowing-seeking, especially in modern world, really serve the aims of power and social domination. Barnes and Bloor (1982) put it this way, “... there are no context-free or super-cultural norms of rationality” (p.27).

Thus, Shera (1970) came up with the term ‘social epistemology’ to describe the kind of epistemology that is rooted in social fabric. “Social epistemology is the study of knowledge in society ... the focus of this discipline should be upon the production, flow, integration and consumption of all forms of communicated thought throughout the entire social fabric” (p. 86). Thus, in the social epistemological perspective, the focus is on social processes whereby knowledge is created, justified and learned. In addressi
social dimensions of knowledge, knowledge is seen as what is ‘believed’ or what beliefs are ‘institutionalised’ in this or that community, culture or context. It seeks to identify the social forces and influences responsible for knowledge production so conceived (Stanford Encyclopedia of Philosophy, 2006). The knowledge is justified through a person’s successful participation in some social activity or form of life or conformity to public norms. In case of practical know-how knowledge is justified through public performance and or through demonstration (Ernest, 1998).

Thus, the social dimensions of knowledge or information, that is, what is believed or held in the community, (institutionalized in the community) as a way of knowing or doing something (particularly mathematical) constitutes the epistemic practice(s) of their knowledge and/or mathematical knowledge. Justification of knowledge (validation) is looked at through people’s successful participation in social activity or form of life or through verification of information, but for practical know-how, public performance and demonstration were followed through. Its propagation or how it is passed on from generation to generation within the community and school is the basis for the research.

1.3: CONCEPTUAL FRAMEWORK FOR SELECTED METHODS

The various facets of interaction require a multi-disciplinary approach to understand the nature of knowledge and thus the methods need multi-theory studies.

1.3.1: Activity Theory

Activity as a unit of study allows reformulation of the relation between the individual and the socio-cultural environment, where each was inherently involved in the other’s definition (Rogoff, 1995). In order to collect and analyse data that will help understand the study we drew on the activity theory model as put forward by by Engestrom(1987) in his attempt to extend Vygotsky’s theory. According to Vygotsky interaction between individual and context is the appropriate unit of analysis. A child’s development can be seen as a conversion of social relations into mental functions (psychological functions). That is, every function in the child’s cultural development appears twice, first on the social level between people and, later, on the individual level, i.e. inside the child’s mind (Vygosky 1978).

Human activity can be explained as a mediated process triggered by artifacts (technical tools) or signs (psychological tools) in a social environment that contributes to
Engestrom's model views a human activity system as a collective activity system with complex interrelations between the individual subject and his/her community. The basic unit of activity analysis is an activity system. There are six components in the model which can be explained as follows; the *subjects* of the activity refer to individuals or groups of participants engaged in the activity, the *object* represents the purpose and intention of human activity that target certain objectives (it can be a material or mental product), *tools* refer to any mediating artifacts, material or conceptual, that shaped the activity, *rules* refer to norms, regulations and conventions that constrain the activity being carried out, *division of labour* refers to the allocations of responsibilities and variations in job roles of the subject as they carry out the activity, and *community* refers to the social and cultural context of the environment in which the subjects operate.

Another theory that informed the methods adopted is discourse analysis.
1.3.2: Discourse Analysis

The basic assumption of discourse analysis is that 'internal states' are constituted in social activity – especially 'discourse'. It is concerned with the ways in which language constructed objects, subjects and experiences, including subjectivity and sense of self. It focuses on the discursive resources that people draw on, that is, interpretive repertoires or discourses (Carla, 1999). Thus by studying how people talk and how they use language in practice, we can study many things which psychologists have previously thought occur 'within the heads' of participants. Discourse analysis involves a set of assumptions concerning the constructive effects of language and also reflexivity on the part of the researcher. It tries to explore how socially produced ideas and objects that populate the world are created, how they are maintained and held in place, thus discourse analysis tries to uncover the way in which it is produced (Hardy C and Phillips N, Personal communication).

Foucault (1969) explained how the process of analysis was done. He stated it as follows:

"The analysis of thought is always allegorical in relation to the discourse that it employs. Its question is unfailingly: What was being said in what was said? The analysis of the discursive field is oriented in a quite different way; we must grasp the statement in the exact specificity of its occurrences; determine its conditions of existence, fix at least its limits, establish its correlations with other statements that may be connected with it, and show what other forms of statements it excludes. We do not seek below what is manifest the half silent murmur of another discourse; we must show why it could not be other than it was" (Foucault, 1969, p.40 English translation, 1989, p.31)

Thus, discourse analysis looked at language as an activity embedded in social interaction. It was thus, interactive activity that mediated linguistics and socio-cultural knowledge (Thwaites et al, 1994).

This theory guided text/content analysis of the discourse practices and the symbol systems in school. The analysis focused on understanding of how the teacher and learners constructed the activity; identifying discursive meanings attached to topic or concepts; identifying discourses which informed the accounts of the topics or concepts.
and also identifying ‘a system of statements’ which constructed an object how these were utilized (Carla, 1999).

1.3.3: Summary

The theories mentioned above demonstrate how learning or production of knowledge is a cultural construct dependent as well as resulting from cultural elements that are passed on and changed along human history. The theories; Vygotsky’s cultural-historical theory and its offshoots, situated cognition and distributive cognition theories explain and locate learners in cultural context, thus making it possible to explore linkages between learners’ experientially constructed mathematical knowledge and their experiences with school mathematics.

These theories are elaborated by the Third Generation Activity Theoretical framework of Engestrom (1987) which takes care of the web of interactions as learners, teachers and other participants engage in everyday and school knowledge which is deeply embedded in the cultural context. The activity theory presents the learning activity as a collective endeavour. It offers a chance to look at the various dimensions of interactions in the learning process through tools, rules, object, division of labour, community and subject.

Further, since mathematical concepts are represented through relations or signs or symbols or graphics, but ultimately these artifacts are grounded in experience and created via neural mechanisms (Lakoff and Nunez, 2000) discourse was seen as semiotically mediated action thus, discourse analysis looked at language as an activity embedded in social interaction (Thwaites et al, 1994).

Thus, schematically, the proposed conceptual framework for the study utilizing these theories is as diagram (figure 1.3.2) depicts.
Cultural-history of the community
- indigenous knowledges
- artifacts
- rules
- norms
- economic activities
- social activities
- division of labour

Common math knowledge

Every day maths

School maths

History of mathematics
- mathematical community
- institutions
- rules
- norms
- educational policies
- teacher preparation

School context
- social, economic, political, cultural, historical practices, artifacts, values, physical environment

Fig. 1.3.2: Schematic diagram of the conceptual framework