

CHAPTER 5

UNSTEADY FLOW PAST A PARABOLIC STARTED VERTICAL PLATE WITH HEAT FLUX AND VARIABLE MASS DIFFUSION IN THE PRESENCE OF CHEMICAL REACTION

5.1 INTRODUCTION

Heat and mass transfer involve buoyancy driven flows induced by a combination of temperature and concentration gradients. Many transport processes occur in nature and industrial applications in which combined heat and mass transfer takes place simultaneously due to combined effects of thermal diffusion and diffusion of chemical species. The phenomenon of heat and mass transfer is encountered in chemical process industries such as polymer production and food processing as well as in other fields such as oceanography, geology, biology, astrophysics.

The study of heat and mass transfer combined along with chemical reaction plays an important role in chemical industries like heat exchangers that are used for the cooling of electronic circuits, packed bed chemical reactor and also in radioactive waste georepositories. In addition, mass transfer with chemical reaction has special significance in chemical and hydrometallurgical industries. The formation of smog represents a first order homogeneous chemical reaction. For instance, one can take into account the emission of NO_2 from automobiles and other smoke-stacks. Thus, NO_2 reacts chemically in the atmosphere with unburned hydrocarbons and produces peroxyacetylnitrate, which forms a layer of photochemical smog.

The chemical reaction effects depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single

phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In majority cases, a chemical reaction depends on the concentration of the species itself. A reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself (Cussler 1998). Some examples of first order chemical reactions are decomposition of N_2O_5 in CCl_4 and radioactive disintegration of unstable nuclei.

Coupled heat and mass transfer problems in presence of chemical reaction are of importance in many processes and thus have received considerable amount of attention in recent times. In processes such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower and flow in a desert cooler, heat and mass transfer occur simultaneously. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. Therefore, the study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists. Andersson et al (1994) have studied the diffusion of a chemically reactive species from a stretching sheet. Anjalidevi & Kandasamy (1999) investigated the effects of chemical reaction and heat and mass transfer on a laminar flow along a semi-infinite horizontal plate. Flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species have been investigated by Takhar et al (2000).

Radiation and chemical reaction effects on unsteady double diffusive convective flow past an oscillating surface with constant heat flux analyzed by Arpita Jain (2013). Agrawal et al (1999) have discussed study of heat and mass transfer past a parabolic started infinite vertical plate. Apelblat (1980) studied mass transfer with a chemical reaction of the first order: Analytical solutions. Asogwa et al (2012) have examined flow past an exponentially accelerated infinite vertical plate and

temperature with variable mass diffusion. Chambre & Young (1958) have observed on the diffusion of a chemically reactive species in a laminar boundary layer flow.

Chandrakala & Bhaskar (2012) have presented radiation effects on oscillating vertical plate with uniform heat flux and mass diffusion. Das et al (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al (1999). The dimensionless governing equations were solved by the usual Laplace Transform technique.

Muthucumaraswamy & Ganesan (2002) have considered diffusion and first order chemical reaction effects on unsteady flow past on impulsively started infinite vertical plate with variable temperature. Muthucumaraswamy & Kulandaivel (2003) have investigated chemical reaction effects on moving infinite vertical plate with uniform heat flux and variable mass diffusion. Manivannan et al (2009) have developed radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh (1983).

It is proposed to study the effects of chemical reaction on unsteady flow past an infinite vertical plate subjected to parabolic motion with variable mass diffusion in the presence of uniform heat flux. The dimensionless governing equations are solved by using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

5.2 MATHEMATICAL FORMULATION

Here the chemical reaction effects on unsteady flow of a viscous incompressible fluid past an infinite vertical plate with uniform heat flux and variable mass diffusion is studied. The x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ . At time $t' > 0$, the plate is started with a velocity $u = u_0 t'^2$ in its own plane against gravitational field and the temperature from the plate is raised uniformly at a constant rate and also the level of concentration is raised to t . The plate is infinite in length all the terms in the governing equations will be independent of x and are functions of y and t' only. Under the Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = \mathbf{g} \beta (T - T_\infty) + \mathbf{g} \beta^* (C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (5.1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = \kappa \frac{\partial^2 T}{\partial y^2} \quad (5.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_l (C' - C'_\infty) \quad (5.3)$$

With the following initial and boundary conditions

$$t' \leq 0 : u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y$$

$$t' > 0 : u = u_0 t'^2, \quad \frac{\partial T}{\partial y} = -\frac{q}{\kappa}, \quad C' = C'_\infty + (C'_w - C'_\infty) A t' \quad \text{at } y = 0 \quad (5.4)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y \rightarrow \infty$$

where $A = \left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}$.

Let us introduce the following dimensionless variables

$$U = u \left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, \quad t = \left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}} t',$$

$$Y = y \left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, \quad \theta = \frac{T - T_\infty}{\left(\frac{q\nu^{\frac{2}{3}}}{\kappa u_0^{\frac{1}{3}}}\right)},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gr = \frac{\mathfrak{g}\beta \left(\frac{q\nu^{\frac{2}{3}}}{\kappa u_0^{\frac{1}{3}}}\right)}{\left(\nu u_0\right)^{\frac{1}{3}}},$$

$$Gc = \frac{\mathfrak{g}\beta^* \left(C'_w - C'_\infty\right)}{\left(\nu u_0\right)^{\frac{1}{3}}}, \quad K = K_l \left(\frac{\nu}{u_0^2}\right)^{\frac{1}{3}},$$

$$Pr = \frac{\mu C_p}{k} \quad \text{and} \quad Sc = \frac{\nu}{D}.$$

Using in the Equations from (5.1) to (5.4), reduce to the following dimensionless form

$$\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} \quad (5.5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (5.6)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - K C \quad (5.7)$$

Non-dimensional quantities of initial and boundary conditions are

$$U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \leq 0$$

$$t > 0 : U = t^2, \quad \frac{\partial \theta}{\partial Y} = -1, \quad C = t \quad \text{at } Y = 0 \quad (5.8)$$

$$U \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty$$

5.3 METHOD OF SOLUTION

The dimensionless governing Equations (5.5) to (5.7) and the corresponding initial and boundary conditions Equation (5.8) are solved by Laplace transform technique. The closed form solutions for the temperature, the concentration and the velocity are as follows

$$\theta = 2\sqrt{t} \left[\frac{\exp(-\eta^2 Pr)}{\sqrt{\pi} \sqrt{Pr}} - \eta \operatorname{erfc}(\eta \sqrt{Pr}) \right], \quad (5.9)$$

$$\begin{aligned}
C = & \frac{t}{2} \left[\exp \left(2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{Kt} \right) \right. \\
& + \left. \exp \left(-2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{Kt} \right) \right] \\
& - \frac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{k}} \left[\exp \left(-2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{Kt} \right) \right. \\
& \left. - \exp \left(2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{Kt} \right) \right]
\end{aligned} \tag{5.10}$$

and

$$\begin{aligned}
U = & \frac{t^2}{3} \left[(3 + 12\eta^2 + 4\eta^4) \operatorname{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4\eta^2) \exp(-\eta^2) \right] \\
& + 2c \operatorname{erfc}(\eta) + 2ac t \left[(1 + 2\eta^2) \operatorname{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] \\
& - c \left(1 + at \right) \left[\exp \left(2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{Kt} \right) \right. \\
& + \left. \exp \left(-2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{Kt} \right) \right] \\
& + 2ac \left(\frac{\eta\sqrt{Sc}\sqrt{t}}{\sqrt{K}} \right) \left[\exp \left(-2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{Kt} \right) \right. \\
& \left. - \exp \left(2\eta\sqrt{KtSc} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{Kt} \right) \right] \\
& - c \exp(at) \left[\exp \left(2\eta\sqrt{at} \right) \operatorname{erfc} \left(\eta + \sqrt{at} \right) + \exp \left(-2\eta\sqrt{at} \right) \operatorname{erfc} \left(\eta - \sqrt{at} \right) \right] \\
& + c \exp(at) \left[\exp \left(2\eta\sqrt{Sc(K+a)t} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{(K+a)t} \right) \right. \\
& + \left. \exp \left(-2\eta\sqrt{Sc(K+a)t} \right) \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{(K+a)t} \right) \right] \\
& - dt\sqrt{t} \left[\frac{4}{\sqrt{\pi}} (1 + \eta^2) \exp(-\eta^2) - \frac{4}{\sqrt{\pi}} (1 + \eta^2 Pr) \exp(-\eta^2 Pr) \right. \\
& \left. - \eta(6 + 4\eta^2) \operatorname{erfc}(\eta) + \eta\sqrt{Pr}(6 + 4\eta^2 Pr) \operatorname{erfc}(\eta\sqrt{Pr}) \right]
\end{aligned} \tag{5.11}$$

where

$$a = \frac{K Sc}{1 - Sc}, \quad c = \frac{Gc}{2a^2(1 - Sc)}, \quad d = \frac{Gr}{3(1 - Pr)\sqrt{Pr}}$$

and

$$\eta = \frac{Y}{2\sqrt{t}}.$$

5.4 RESULTS AND DISCUSSION

In order to get a physical view of the problem, numerical calculations are carried out for different values of chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. The values of Prandtl number are taken as $Pr = 0.71$ and 7.0, they represent air and water respectively. The values of the Schmidt number are chosen as hydrogen (0.16), helium (0.3), water vapor (0.6), ammonia (0.78) and ethyl benzene (2.01).

5.4.1 Effect on temperature distribution

The temperature profiles for different values of the Prandtl number Pr and time t with axial distance η are plotted in Figure 5.1. It is observed that the temperature profiles decrease with increasing Prandtl number Pr . This trend shows that the magnitude of temperature is maximum at the plate and then decays to zero asymptotically. This is due to the fact that thermal conductivity of fluid decreases with increasing Prandtl number, resulting in a decrease in thermal boundary layer thickness. Further, the magnitude of the temperature for air is higher than that of water and it decreases steeply for $Pr = 7$ than that of $Pr = 0.71$. Also, it is observed that the temperature profiles increase with increasing values of time t .

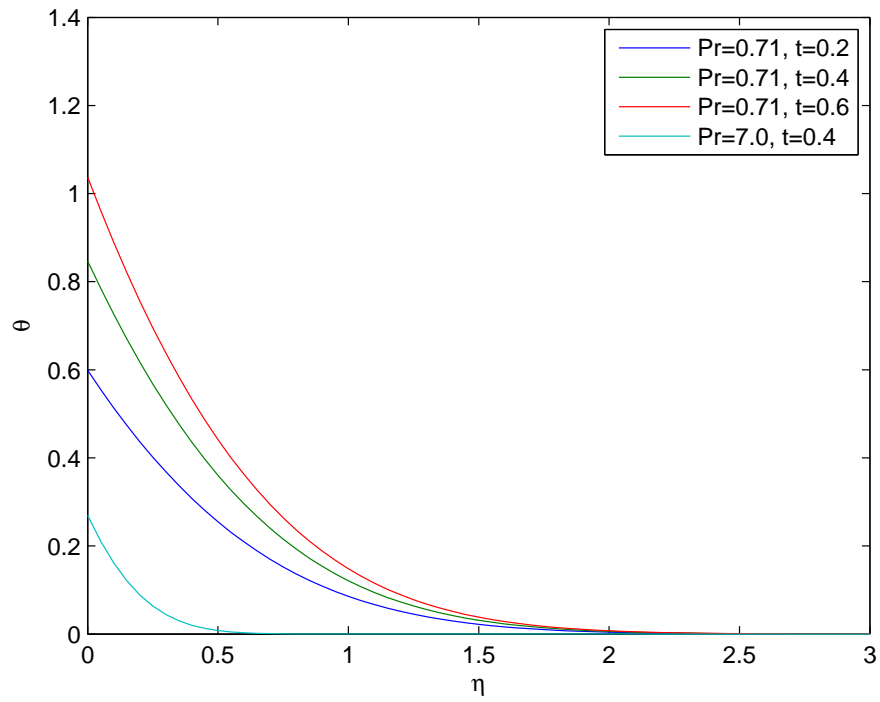


Figure 5.1 Temperature distribution profile for different values of Pr and t

5.4.2 Effect on concentration field

The effect of concentration profiles for different values of Schmidt number Sc and chemical reaction parameter K with time $t = 0.2$ are shown in Figures 5.2 and 5.3 respectively. The effect of Schmidt number is important in concentration field. It is noted that the concentration decreases considerably with increasing values of the Schmidt number. Further, an increase in the value of Schmidt number leads to a fall in the concentration since increase of Schmidt number, decrease of molecular diffusivity which results in decrease of concentration boundary layer. Hence, the concentration of species is higher for small values of Schmidt number. It is also seen that the concentration decreases marginally with an increase in chemical reaction parameter.

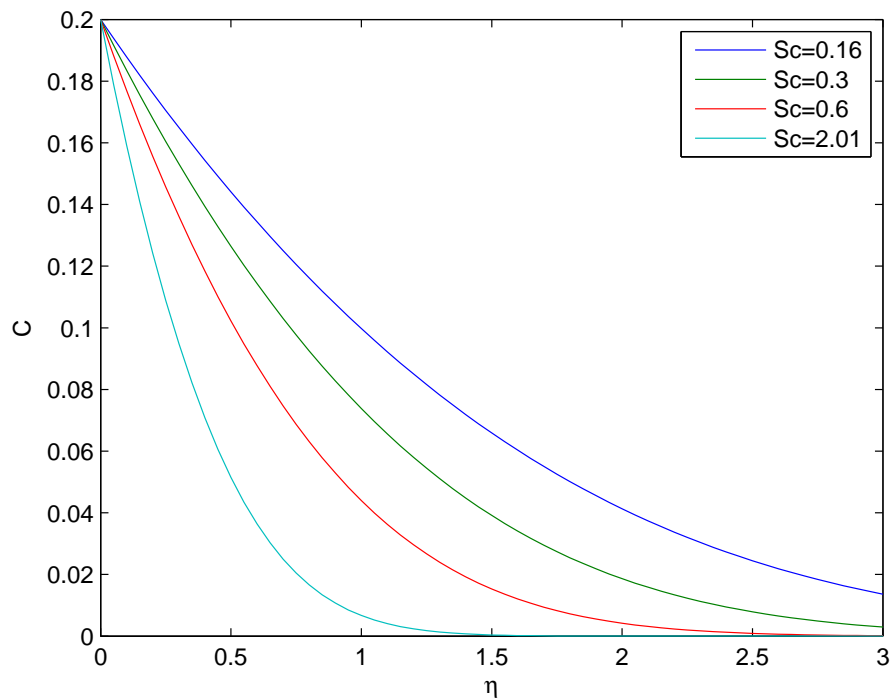


Figure 5.2 Concentration profile for different values of Sc

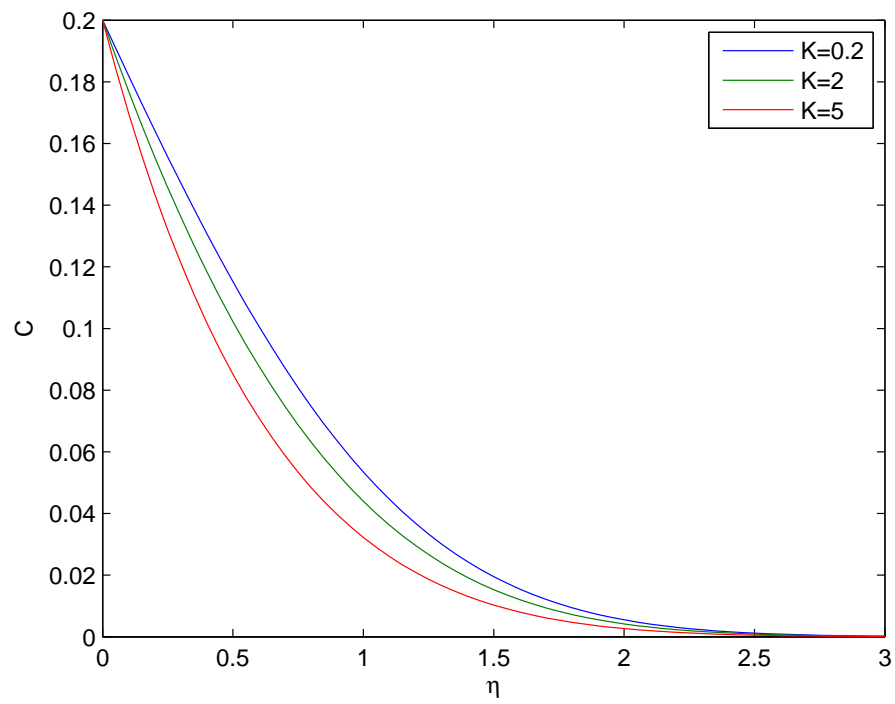


Figure 5.3 Concentration profile for different values of K

5.4.3 Effect on velocity profiles

The velocity profiles are of particular interest, since they provide a detailed description of the flow field. Figure 5.4 exhibits the effect on velocity for different values of the chemical reaction parameter $K = 0.2, 2.0, 5.0$ with $Gr = 5 = Gc$, $Sc = 0.6$, $Pr = 0.71$ and $t = 0.2$. It is found that the velocity increases marginally with an decrease in chemical reaction parameter. The trend shows that velocity increases rapidly and attains its maximum value in the vicinity of the plate ($\eta \leq 0.5$) and then it decreases gradually in the axial direction.

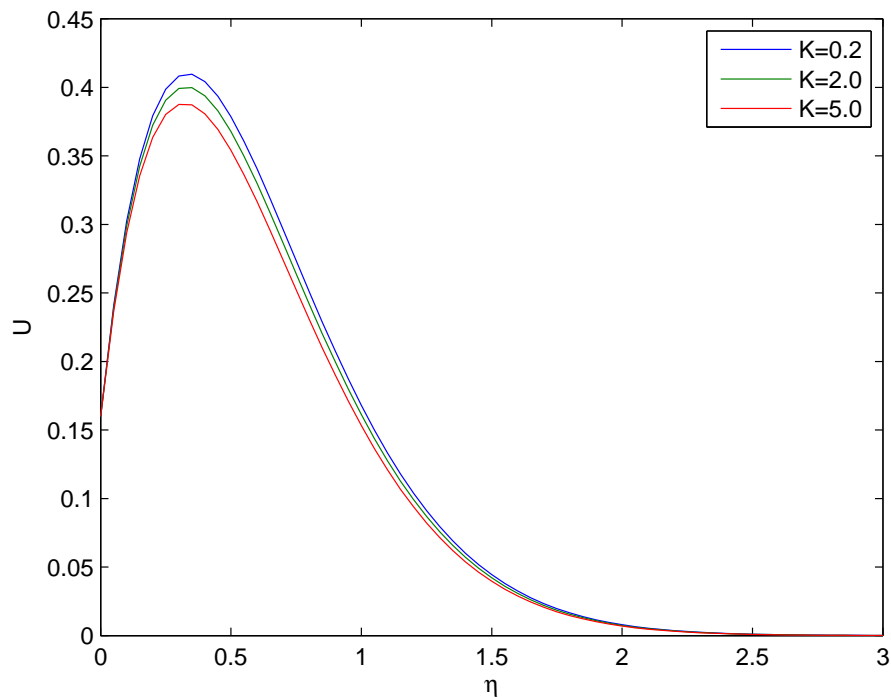


Figure 5.4 Velocity profile for different values of K

The effects of different thermal Grashof number $Gr = 2, 5$ and mass Grashof number $Gc = 5, 10$ on the velocity profile with $Pr = 0.71$, $Sc = 0.6$, $K = 0.2$ and $t = 0.2$ is sketched in Figure 5.5. It is observed that the velocity increases significantly with increasing values of the thermal Grashof number or mass Grashof number. It is seen that the peak values of the velocity increases rapidly near the plate as thermal Grashof number and mass Grashof number increases and then decays to the free stream velocity. One may observe that for $Gr = 5$ and $Gc = 10$, the velocity is considerably higher than that of other noted values of Gr and Gc .

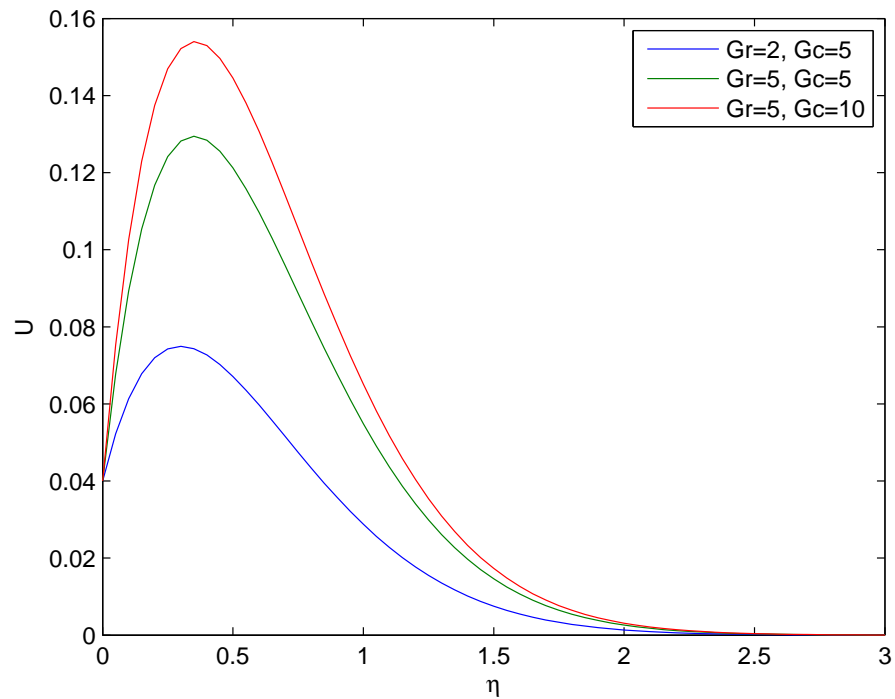


Figure 5.5 Velocity profile for different values of Gr and Gc

Figure 5.6 depicts the variation of velocity with axial distance for different values of Schmidt number $Sc = 0.16, 0.3, 0.6, 0.78$ for $Pr = 0.71, Gr = Gc = 5, K = 0.2$ and $t = 0.4$. It is clear that the velocity decreases continuously with an increase in Schmidt number Sc . It is noticed that the maximum velocity attains near the plate and then decreases and it vanishes far away from the plate.

The effects on velocity profiles for different values of time t with $Pr = 0.71, Gr = Gc = 5, Sc = 0.6$ and $K = 0.2$ are illustrated in Figure 5.7. It is obtained that the velocity increases significantly with increased values of time t . Moreover, the effect of time t on the velocity is dominant than other physical parameters like K, Sc and Gc or Gr .

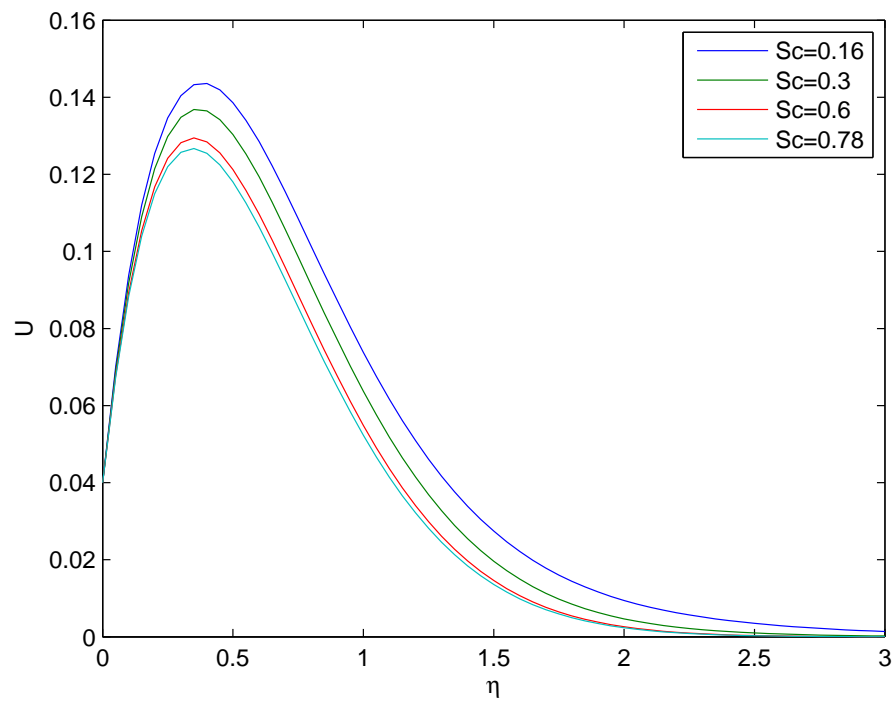


Figure 5.6 Velocity profile for different values of Sc

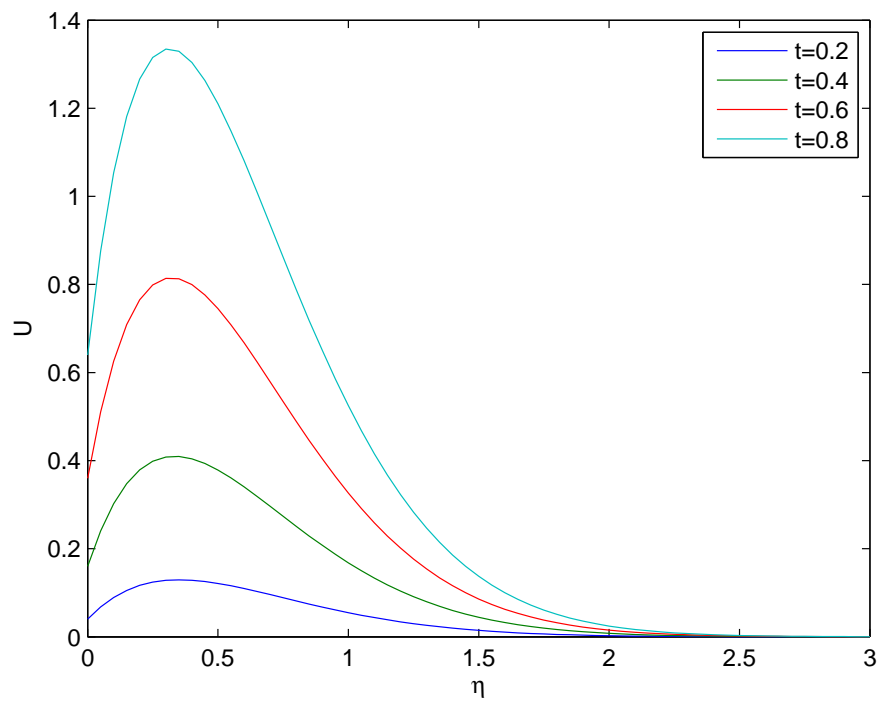


Figure 5.7 Velocity profile for different values of t

5.5 CONCLUSION

The theoretical solution of unsteady flow past a parabolic starting motion of an infinite vertical plate in the presence of chemical reaction has been investigated. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like chemical reaction parameter, Prandtl number, Schmidt number, thermal Grashof number, mass Grashof number and time are studied graphically. The conclusions of the study are as follows:

- The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the chemical reaction parameter or Schmidt number.
- The velocity increases with increased values of the time t .
- The temperature of the plate increases with decreasing values of the Prandtl number whereas it increases with increasing time.
- The concentration near the plate increases with decreasing values of the Schmidt number and chemical reaction parameter.
- Comparison of flow past an infinite vertical plate subjected to parabolic motion with mass diffusion in the presence of constant heat flux with that of flow past an infinite vertical plate subjected to parabolic motion with variable mass diffusion in the presence of uniform heat flux, it is concluded that there is a drastic change in the concentration profile in the case of variable wall concentration than in the case of wall concentration.