

CHAPTER 4

FIRST ORDER CHEMICAL REACTION ON FLOW PAST A PARABOLIC STARTED VERTICAL PLATE WITH CONSTANT HEAT FLUX AND MASS DIFFUSION

4.1 INTRODUCTION

Coupled heat and mass transfer problems in presence of chemical reaction are of importance in many processes and thus have received considerable amount of attention in recent times. In processes such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower, flow in a desert cooler, hydrometallurgical industries, power industry and chemical process industries like in food processing, manufacturing of ceramics or glassware and polymer production, heat and mass transfer occur simultaneously. Mass diffusion rates can be changed tremendously with chemical reactions. Analysis of transport processes and their interaction with chemical reaction has the greatest contributions to many areas of chemical science. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. Therefore, the study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists.

The chemical reaction can be codified as either homogeneous processes: taking place there where the reactants are present or heterogeneous processes: taking place only at a reaction interface between phases. In majority cases, a chemical reaction depends on the concentration of the species itself. A reaction is of order n , if the reaction rate is proportional to the n th power of concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly

proportional to the concentration itself. The radioactive disintegration of unstable nuclei are the best examples of first order reactions. In nature, the presence of pure air or water is not possible. Some foreign mass may be present naturally mixed with air or water. The presence of foreign mass in air or water causes some kind of chemical reaction. The effect of chemical reaction on different geometry of the problem has been investigated by many authors.

Apelblat (1980) investigated analytical solution for mass transfer with a chemical reaction of the first order. Acrivos (1960) considered the laminar forced convection mass transfer with homogeneous chemical reaction. Chambre & Young (1958) have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das et al (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al (1999). The dimensionless governing equations are solved by the Laplace transform technique. Agrawal et al (1999) have analyzed study of heat and mass transfer past a parabolic started infinite vertical plate.

The chemical reaction effect on heat and mass transfer flow along a semi infinite horizontal plate has been studied by Anjalidevi & Kandaswamy (1999). Mass transfer effects on the flow past an uniformly accelerated vertical plate was studied by Soundalgekar (1982). Also, Effects of mass transfer on the flow past an oscillatory infinite vertical plate with constant heat flux analyzed by Soundalgekar (1994). Basant Kumar et al (1991) have analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh & Singh (1983).

It is found that in many chemical engineering processes, chemical reaction takes place between foreign masses (present in the form of ingredients) and the fluid. This type of chemical reaction may change the temperature and the heat content of the fluid and may affect the free convection process. However, if the presence of such foreign mass is very low then we can assume the first order chemical reaction so that heat generation due to chemical reaction can be considered to be very negligible. Here only first order chemical reaction is considered. We are particularly interested in cases in which diffusion and chemical reaction occur at roughly the same speed. When diffusion is much faster than chemical reaction, then only chemical factors influence the chemical reaction rate, when diffusion is not much faster than reaction, the diffusion and kinetics interact to produce very different effects. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction.

Due to the fast Growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy suppliers to telephone switch boards and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen and helium) and medium molecular weight. Free convection effect on a flow past an impulsively started or oscillating infinite vertical plate was studied by Revankar (2000).

Muthucumaraswamy & Ganesan (2002) studied diffusion and first order chemical reaction effects on unsteady flow past on impulsively started infinite vertical plate with variable temperature. A numerical solution of the first-order homogeneous chemical reaction in an unsteady free convective flow past a semi-infinite vertical plate were presented by Palani & Kwang Yong Kim (2011). Effects of mass

transfer on flow past an impulsively started infinite vertical plate with Newtonian heating and chemical reaction was analyzed by Rajesh (2012). Muthucumaraswamy & Meenakshisundaram (2006) have studied theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature.

In this Chapter an attempt has been made to study the chemical reaction effects on unsteady flow past an infinite vertical plate subjected to parabolic motion with mass diffusion in the presence of constant heat flux. The dimensionless governing equations are solved by using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

4.2 MATHEMATICAL FORMULATION

Here the chemical reaction effects on unsteady free convection flow of a viscous incompressible fluid past an infinite vertical plate is studied. The x -axis is taken along the plate in the vertically upward direction and parallel to the free stream velocity and the y -axis is taken normal to the plate. Initially, at time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ . At time $t' > 0$, the plate starts with a velocity $u = u_0 t'^2$ in its own plane against gravitational field. At the same time, the plate concentration is raised to C'_w with heat supplied at a constant rate to the plate. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate K_l between the diffusing species and the fluid. Since the plate is infinite in length therefore all the terms in the governing equations will be independent of x and are functions of y and t' only. A chemically reactive species which transforms according to a simple reaction involving the concentration is emitted from the plate and diffuses into the fluid. The reaction is assumed to take place entirely in the stream. Under the Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = \mathbf{g} \beta (\mathbf{T} - \mathbf{T}_\infty) + \mathbf{g} \beta^* (C' - C'_\infty) + \nu \frac{\partial^2 \mathbf{u}}{\partial y^2} \quad (4.1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = \kappa \frac{\partial^2 T}{\partial y^2} \quad (4.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_l (C' - C'_\infty) \quad (4.3)$$

With the following initial and boundary conditions:

$$t' \leq 0 : u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y$$

$$t' > 0 : u = u_0 t'^2, \quad \frac{\partial T}{\partial y} = -\frac{q}{\kappa}, \quad C' = C'_w \quad \text{at } y = 0 \quad (4.4)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y \rightarrow \infty$$

The dimensional quantities are defined as

$$U = u \left(\frac{u_0}{\nu^2} \right)^{\frac{1}{3}}, \quad t = \left(\frac{u_0^2}{\nu} \right)^{\frac{1}{3}} t'$$

$$Y = y \left(\frac{u_0}{\nu^2} \right)^{\frac{1}{3}}, \quad \theta = \frac{T - T_\infty}{\left(\frac{q \nu^{\frac{2}{3}}}{\kappa u_0^{\frac{1}{3}}} \right)}$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gr = \frac{\mathbf{g} \beta \left(\frac{q \nu^{\frac{2}{3}}}{\kappa u_0^{\frac{1}{3}}} \right)}{(\nu u_0)^{\frac{1}{3}}}$$

$$Gc = \frac{\mathbf{g}\beta^* (\mathbf{C}'_w - \mathbf{C}'_\infty)}{(\nu u_0)^{\frac{1}{3}}}, \quad K = K_l \left(\frac{\nu}{u_0^2} \right)^{\frac{1}{3}},$$

$$Pr = \frac{\mu C_p}{k} \quad \text{and} \quad Sc = \frac{\nu}{D}.$$

The governing Equations (4.1) to (4.3) reduce to the following non-dimensional form:

$$\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} \quad (4.5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (4.6)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - K C \quad (4.7)$$

Non-dimensional quantities of initial and boundary conditions are

$$U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \leq 0$$

$$t > 0 : U = t^2, \quad \frac{\partial \theta}{\partial Y} = -1, \quad C = 1 \quad \text{at } Y = 0 \quad (4.8)$$

$$U \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty$$

4.3 METHOD OF SOLUTION

The dimensionless governing Equations (4.5) to (4.7) and the corresponding initial and boundary conditions Equation (4.8) are solved by Laplace transform

technique. The closed form solutions for the temperature, the concentration and the velocity are as follows:

$$\theta = 2\sqrt{t} \left[\frac{\exp(-\eta^2 Pr)}{\sqrt{\pi} \sqrt{Pr}} - \eta \operatorname{erfc}(\eta \sqrt{Pr}) \right], \quad (4.9)$$

$$C = \frac{1}{2} \left[\exp\left(2\eta\sqrt{KtSc}\right) \operatorname{erfc}\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{KtSc}\right) \operatorname{erfc}\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) \right] \quad (4.10)$$

and

$$\begin{aligned} U = & \frac{t^2}{3} \left[(3 + 12\eta^2 + 4\eta^4) \operatorname{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4\eta^2) \exp(-\eta^2) \right] \\ & + 2c \operatorname{erfc}(\eta) - c \left[\exp\left(2\eta\sqrt{KtSc}\right) \operatorname{erfc}\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) \right. \\ & \left. + \exp\left(-2\eta\sqrt{KtSc}\right) \operatorname{erfc}\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) \right] \\ & - c \exp(at) \left[\exp\left(2\eta\sqrt{at}\right) \operatorname{erfc}\left(\eta + \sqrt{at}\right) \right. \\ & \left. + \exp\left(-2\eta\sqrt{at}\right) \operatorname{erfc}\left(\eta - \sqrt{at}\right) \right] \\ & + c \exp(at) \left[\exp\left(2\eta\sqrt{Sc(K+a)t}\right) \operatorname{erfc}\left(\eta\sqrt{Sc} + \sqrt{(K+a)t}\right) \right. \\ & \left. + \exp\left(-2\eta\sqrt{Sc(K+a)t}\right) \operatorname{erfc}\left(\eta\sqrt{Sc} - \sqrt{(K+a)t}\right) \right] \\ & - dt\sqrt{t} \left[\frac{4}{\sqrt{\pi}}(1 + \eta^2) \exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1 + \eta^2 Pr) \exp(-\eta^2 Pr) \right. \\ & \left. - \eta(6 + 4\eta^2) \operatorname{erfc}(\eta) + \eta\sqrt{Pr}(6 + 4\eta^2 Pr) \operatorname{erfc}(\eta\sqrt{Pr}) \right] \end{aligned} \quad (4.11)$$

where

$$a = \frac{K Sc}{1 - Sc}, \quad c = \frac{Gc}{2a(1 - Sc)}, \quad d = \frac{Gr}{3(1 - Pr)\sqrt{Pr}}$$

and

$$\eta = \frac{Y}{2\sqrt{t}}.$$

4.4 NUMERICAL RESULTS AND DISCUSSION

The aim of the study is to numerically analyze the effects of the different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. The values of Prandtl number are taken as $Pr = 0.71$ and 7.0 , they represent air and water respectively.

4.4.1 Effect on temperature distribution

The temperature profiles for different values of the Prandtl number Pr and time t with axial distance η are plotted in Figure 4.1. It is observed that the temperature profiles increases with decreasing Prandtl number Pr . This trend shows that the magnitude of temperature is maximum at the plate and then decays to zero asymptotically. This is due to the fact that thermal conductivity of fluid decreases with increasing Prandtl number, resulting in a decrease in thermal boundary layer thickness. Further, the magnitude of the temperature for air is higher than that of water. This shows that the heat transfer is more in air than in water. Furthermore, it decreases steeply for $Pr = 7$ than that of $Pr = 0.71$. Also, it is observed that the temperature profiles increases with increasing values of time t .

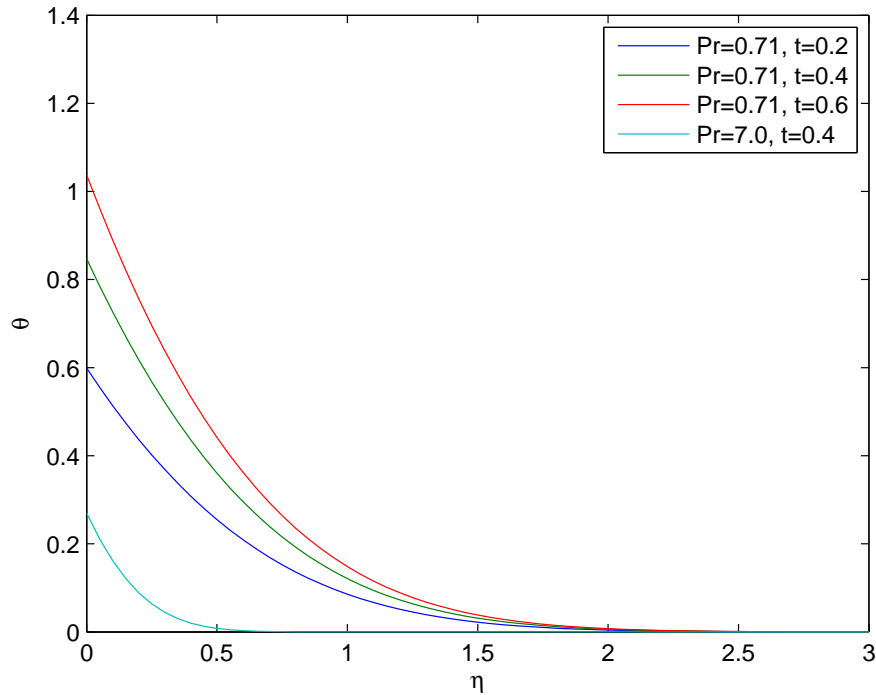


Figure 4.1 Temperature distribution profile for different values of Pr and t

4.4.2 Effect on concentration field

The effect of concentration profiles for different values of Schmidt number Sc and chemical reaction parameter K with time $t = 0.2$ are shown in Figures 4.2 and 4.3 respectively. The effect of Schmidt number is important in concentration field. It is noted that the concentration decreases considerably with increasing values of the Schmidt number. Further, an increase in the value of Schmidt number leads to a fall in the concentration since increase of Schmidt number, decrease of molecular diffusivity which results in decrease of concentration boundary layer. Hence, the concentration of species is higher for small values of Schmidt number. It is also seen that the concentration decreases marginally with an increase in chemical reaction parameter.

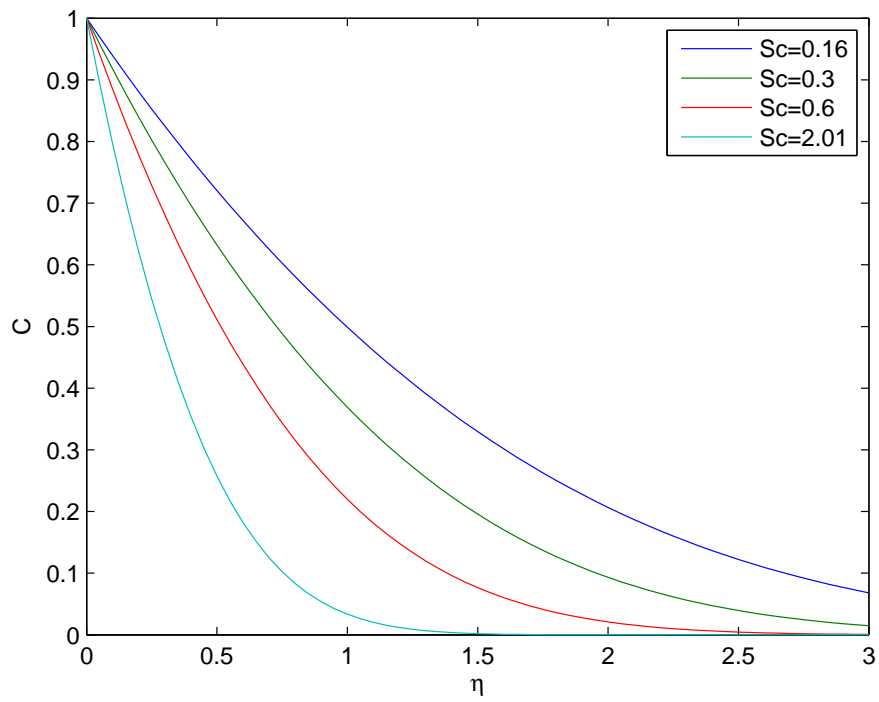


Figure 4.2 Concentration profile for different values of Sc

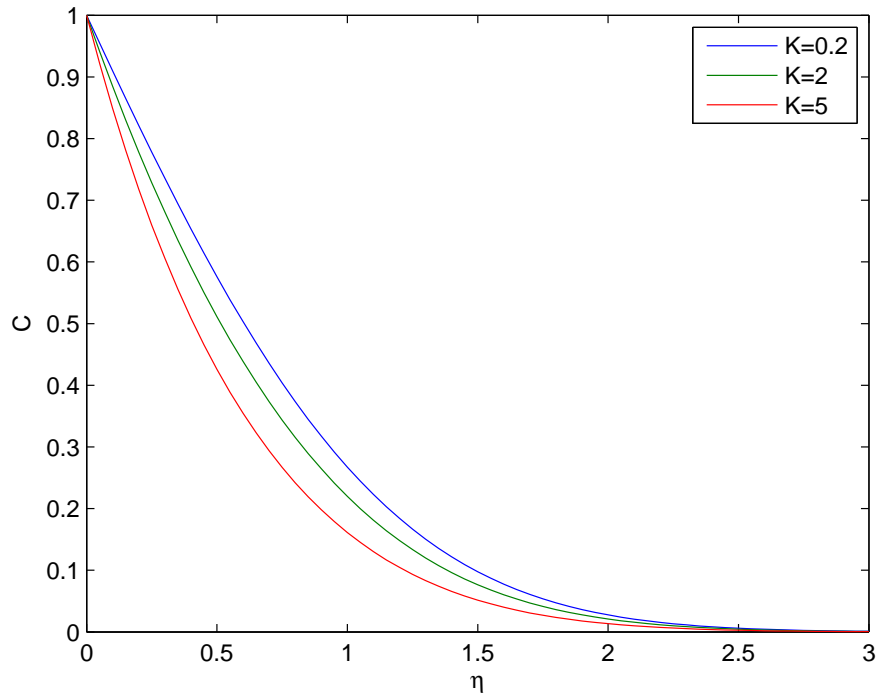


Figure 4.3 Concentration profile for different values of K

4.4.3 Effect on velocity profiles

The velocity profiles are of particular interest, since they provide a detailed description of the flow field. Figure 4.4 exhibits the effect on velocity for different values of the chemical reaction parameter $K = 0.2, 2.0, 5.0$ with $Gr = 5 = Gc$, $Sc = 0.6$, $Pr = 0.71$ and $t = 0.2$. It is found that the velocity increases marginally with an decrease in chemical reaction parameter. The trend shows that velocity increases rapidly and attains its maximum value in the vicinity of the plate ($\eta \leq 0.5$) and then it decreases gradually in the axial direction.

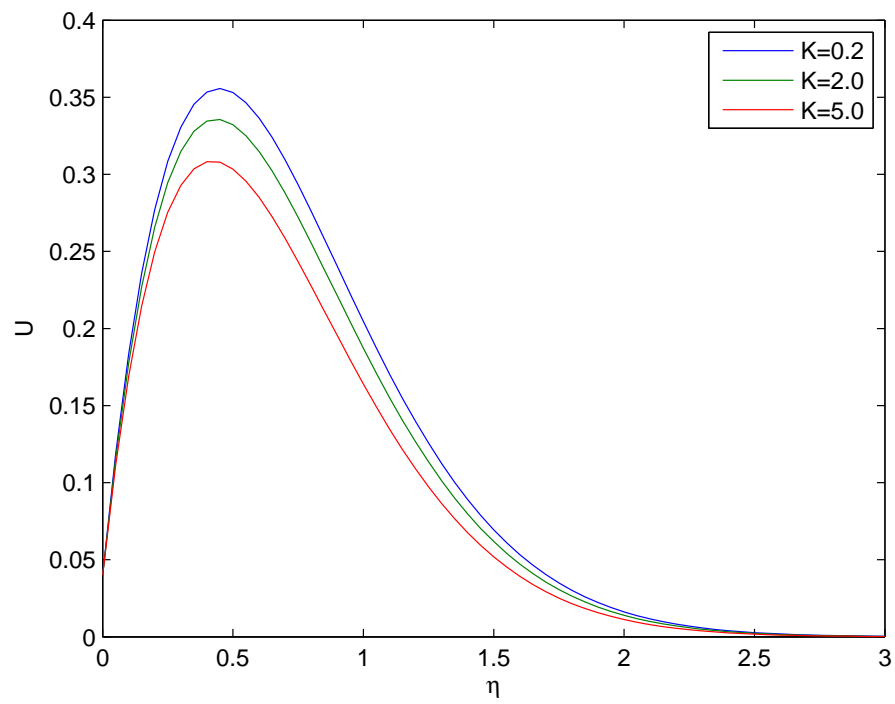


Figure 4.4 Velocity profile for different values of K

The effects of different thermal Grashof number $Gr = 2, 5$ and mass Grashof number $Gc = 5, 10$ on the velocity profile with $Pr = 0.71$, $Sc = 0.6$, $K = 0.2$ and $t = 0.2$ is sketched in Figure 4.5. It is observed that the velocity increases significantly with increasing values of the thermal Grashof number or mass Grashof number. It is seen that the peak values of the velocity increases rapidly near the plate as thermal Grashof number and mass Grashof number increases and then decays to the free stream velocity. One may observe that for $Gr = 5$ and $Gc = 10$, the velocity is significantly higher than that of other noted values of Gr and Gc .

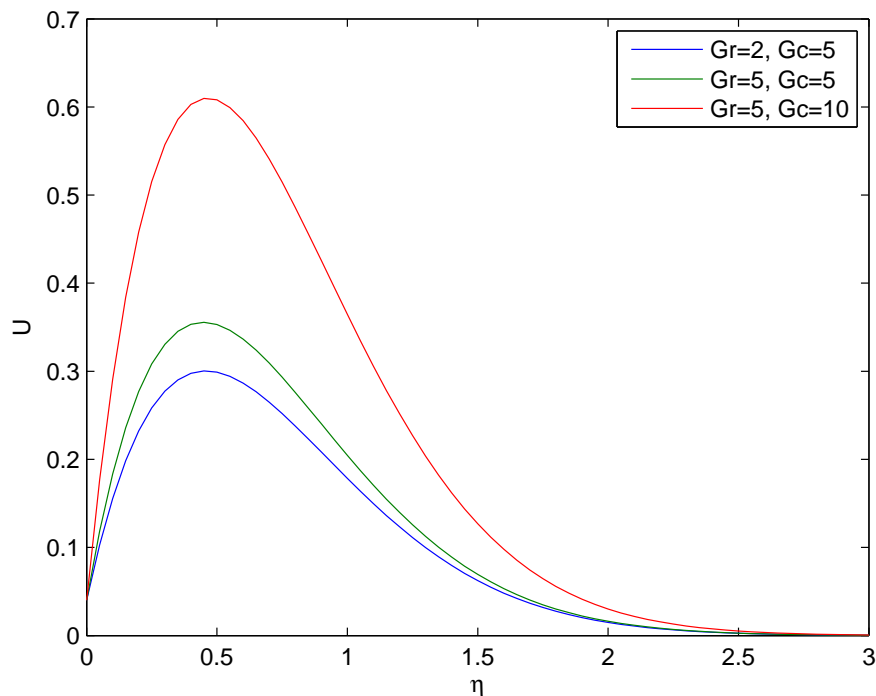


Figure 4.5 Velocity profile for different values of Gr and Gc

Figure 4.6 depicts the variation of velocity with axial distance for different values of Schmidt number $Sc = 0.16, 0.3, 0.6, 0.78$ for $Pr = 0.71, Gr = Gc = 5, K = 0.2$ and $t = 0.4$. It is clear that the velocity decreases continuously with an increase in Schmidt number Sc . It is noticed that the maximum velocity attains near the plate and then decreases and it vanishes far away from the plate.

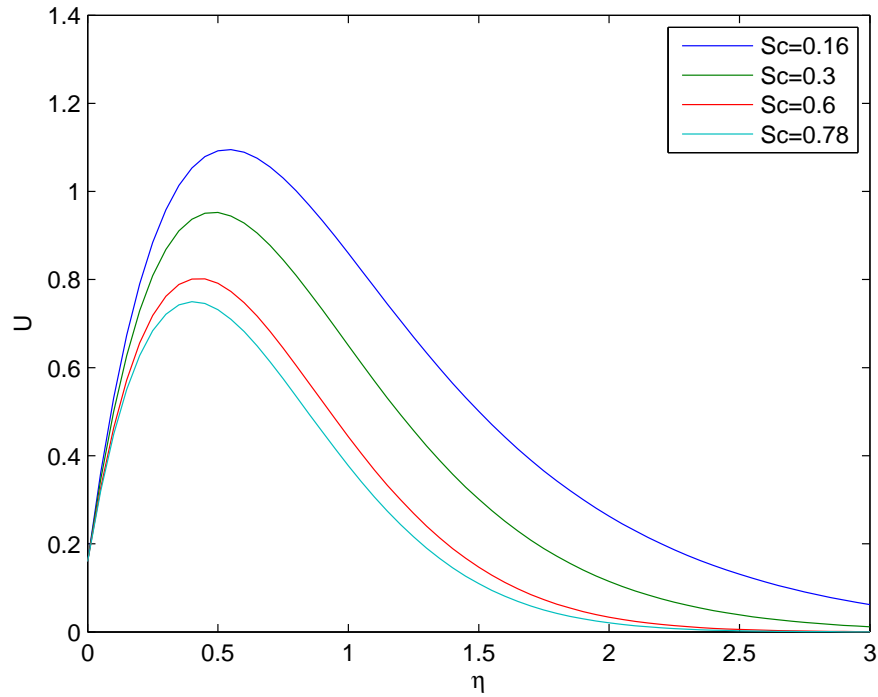


Figure 4.6 Velocity profile for different values of Sc

The effects on velocity profiles for different values of time t with $Pr = 0.71$, $Gr = Gc = 5$, $Sc = 0.6$ and $K = 0.2$ are illustrated in Figure 4.7. It is obtained that the velocity increases significantly with increased values of time t at each point in the flow field for water vapor. Moreover, the effect of time on the velocity is more dominant than other parameters like K , Sc and Gc or Gr .

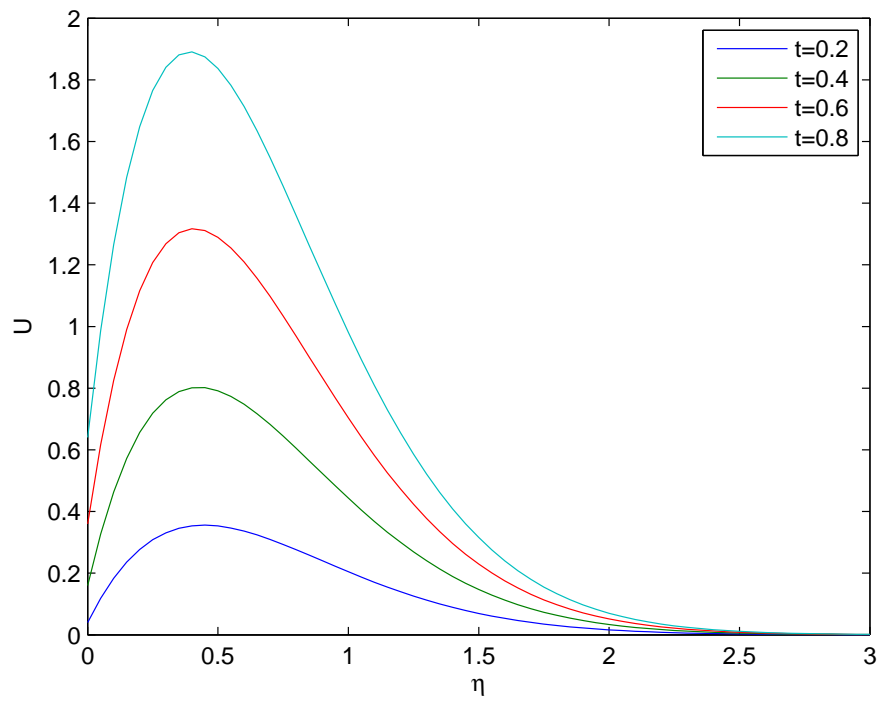


Figure 4.7 Velocity profile for different values of t

4.5 CONCLUSION

The theoretical solution of unsteady flow past a parabolic starting motion of an infinite vertical plate in the presence of chemical reaction when heat is supplied to the plate at constant rate has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like chemical reaction parameter, Prandtl number, Schmidt number, thermal Grashof number, mass Grashof number and time are studied graphically. The results of the flow problem indicates the following:

- The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the chemical reaction parameter or Schmidt number.
- The velocity increases with increased values of the time t .
- The temperature increases with decreasing values of the Prandtl number whereas it increases with increasing time.
- The concentration profile decreases with an increase in chemical reaction parameter and Schmidt number.