CHAPTER 3

THERMAL RADIATION EFFECTS ON FLOW PAST A PARABOLIC STARTED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS FLUX

3.1 INTRODUCTION

In the processes involving high temperatures, the radiation heat transfer in combination with conduction, convection and also mass transfer plays very important role in the design of pertinent equipments in the areas such as nuclear power plants, gas turbines and the various propulsion devices for aircrafts, missiles, satellites and space vehicles. Moreover, radiation effect is significant in the dynamics of fluid in chemical, environmental, mechanical and solar power engineering. The radiation effect on convective flow and heat transfer problems has become more important industrially. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of reliable equipment, the design of fins, steel rolling, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, as well as numerous agricultural, health and military applications.

The effect of radiation on free convection has been drawn forth not only for its fundamental aspects but also for its significance in the contexts of space technology and processes involving high temperature. The radiative heat and mass transfer flow of an electrically conducting fluid has wide applications in geophysics, geothermal, food processing and cryogenic engineering, engineering and solar physics.

England & Emery (1969) have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along an isothermal vertical plate was studied by Hossain & Takhar (1996). Das et al (1996a) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh & Naveen Kumar (1984). Basanth Kumar et al (1991) have analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

Cess (1966) investigated thermal radiation effects on heated vertical plate using singular perturbation technique. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh & Singh (1983). Agrawal et al (1998) studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic field. Agrawal et al (1999) further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

The driving force for natural convection is buoyancy, a result of differences in fluid density when gravity or any type of acceleration is present in the system. Raptis & Perdikis (1999) have studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Radiation effects on the oscillatory flow past vertical in the presence of uniform temperature analyzed by Mansour (1990). The governing equations are solved by perturbation technique. Soundalgekar et al (1994) considered the effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux. Muthucumaraswamy (2006) studied thermal radiation effects on vertical oscillating plate in the presence of variable temperature and mass diffusion. The MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation were considered by Mazumdar & Deka (2007). If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass flux.

In this Chapter to study the thermal radiation effects on unsteady flow past an infinite vertical plate subjected to parabolic motion with variable temperature in the presence of uniform mass flux. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

3.2 MATHEMATICAL FORMULATION

Here the thermal radiation effects on unsteady flow of a viscous incompressible fluid past an infinite vertical plate with uniform mass flux has been considered. The $x-$axis is taken along the plate in the vertically upward direction and the $y-$axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time $t' > 0$, the plate is started with a velocity $u = u_0\, t'^2$ in its own plane against gravitational field and the temperature from the plate is raised linearly with time t and the concentration level near the plate are also raised at an uniform rate. The plate is infinite in length all the terms in the governing equations will be independent of x and are functions of y and t' only. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Under the usual Boussinesq's approximation the unsteady flow is governed by the governing equations as discussed in Chapter 2.

With the following initial and boundary conditions:

$$u = 0,\ \ T = T_\infty,\ \ C' = C'_\infty\ \ for\ all\ \ y,\ \ t' \leq 0$$

$$t' > 0 :\ u = u_0\, t'^2,\ \ T = T_\infty + (T_w - T_\infty)\, A\, t',\ \ \frac{\partial C'}{\partial y} = -\frac{j}{D}\ \ at\ \ y = 0 \tag{3.1}$$

$$u \to 0,\ \ T \to T_\infty,\ \ C' \to C'_\infty\ \ as\ \ y \to \infty$$

where $A = \left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}$.

By using non dimensional quantities as discussed in Chapter 2 and the corresponding governing equations in dimensionless form are as follows

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{3.2}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\,\theta \tag{3.3}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{3.4}$$

The initial and boundary conditions in non-dimensional quantities are

$$U = 0,\ \ \theta = 0,\ \ C = 0\ \ for\ all\ \ Y, t \leq 0$$

$$t > 0 :\ U = t^2,\ \ \theta = t,\ \ \frac{\partial C}{\partial Y} = -1\ \ at\ \ Y = 0 \tag{3.5}$$

$$U \to 0,\ \ \theta \to 0,\ \ C \to 0\ \ as\ \ Y \to \infty$$

3.3 METHOD OF SOLUTION

The dimensionless governing Equations (3.2) to (3.4) and the corresponding initial and boundary conditions Equation (3.5) are solved by Laplace transform technique. The closed form solutions for the velocity, the temperature and the concentration are as follows:

$$\begin{aligned}
U = {} & \frac{t^2}{3}\left[(3+12\eta^2+14\eta^4)\, erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10+4\eta^4)\exp(-\eta^2)\right] \\
& + 2\,c\, erfc(\eta) + 2\, b\, c\, t\left[(1+2\eta^2)\, erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] \\
& - c\exp(bt)\left[\exp(2\eta\sqrt{bt})\, erfc(\eta+\sqrt{bt}) + \exp(-2\eta\sqrt{bt})\, erfc(\eta-\sqrt{bt})\right] \\
& - d\,t\sqrt{t}\left[\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1+\eta^2 Sc)\exp(-\eta^2 Sc) - \right. \\
& \left. \eta(6+4\eta^2)\, erfc(\eta) + \eta\sqrt{Sc}(6+4\eta^2 Sc)\, erfc(\eta\sqrt{Sc})\right] \\
& - c(1+bt)\left[\exp\left(2\eta\sqrt{Rt}\right)\, erfc\left(\eta\sqrt{Pr}+\sqrt{at}\right) + \exp\left(-2\eta\sqrt{Rt}\right)\right. \\
& \left. erfc\left(\eta\sqrt{Pr}-\sqrt{at}\right)\right] + bc\,\frac{\eta\, Pr\,\sqrt{t}}{\sqrt{R}}\left[\exp\left(-2\eta\sqrt{R\,t}\right)\right. \\
& \left. erfc\left(\eta\sqrt{Pr}-\sqrt{at}\right) - \exp\left(2\eta\sqrt{R\,t}\right)\, erfc\left(\eta\sqrt{pr}+\sqrt{at}\right)\right] \\
& + c\exp(bt)\left[\exp\left(2\eta\sqrt{Pr(a+b)t}\right)\, erfc\left(\eta\sqrt{Pr}+\sqrt{(a+b)t}\right) + \right. \\
& \left. \exp\left(-2\eta\sqrt{Pr(a+b)t}\right)\, erfc\left(\eta\sqrt{Pr}-\sqrt{(a+b)t}\right)\right],
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
\theta = {} & \frac{t}{2}\left[\exp\left(2\eta\sqrt{R\,t}\right)\, erfc\left(\eta\sqrt{Pr}+\sqrt{at}\right) + \exp\left(-2\eta\sqrt{R\,t}\right)\right. \\
& \left. erfc\left(\eta\sqrt{Pr}-\sqrt{at}\right)\right] - \frac{\eta\, Pr\,\sqrt{t}}{2\sqrt{R}}\left[\exp\left(-2\eta\sqrt{R\,t}\right)\right. \\
& \left. erfc\left(\eta\sqrt{Pr}-\sqrt{at}\right) - \exp\left(2\eta\sqrt{R\,t}\right)\, erfc\left(\eta\sqrt{Pr}+\sqrt{at}\right)\right],
\end{aligned} \tag{3.7}$$

and

$$C = 2\sqrt{t}\left[\frac{\exp(-\eta^2 Sc)}{\sqrt{\pi}\sqrt{Sc}} - \eta\, erfc(\eta\sqrt{Sc})\right] \tag{3.8}$$

where

$$a = \frac{R}{Pr},\ \ b = \frac{R}{1-Pr},\ \ c = \frac{Gr}{2b^2(1-Pr)},\ \ d = \frac{Gc}{3(1-Sc)\sqrt{Sc}}$$

and

$$\eta = \frac{Y}{2\sqrt{t}}.$$

3.4 RESULTS AND DISCUSSION

The aim of the study is to numerically analyze the effects of the different physical parameters like thermal Grashof number, mass Grashof number, thermal radiation parameter, Schmidt number and time. The value of the Prandtl number Pr is taken as 0.71, which represent air. The numerical values of the Schmidt number Sc are chosen such that they represent a reality in case of air. The effect of Schmidt number play an important role in the concentration field.

3.4.1 Effect on temperature distribution

The effect of thermal radiation parameter is important in the temperature profiles. The temperature profiles for different values of the thermal radiation parameter R with $t = 0.2$ are plotted in Figure 3.1. It is observed that temperature profiles decreases rapidly from $\theta = 0.2$ in the axial direction from $\eta = 0$ to $\eta = 0.5$ and then it decreases slowly from $\eta = 0.5$ to $\eta = 1.5$ and it linearly towards $\eta = 3$. It is found that the temperature decreases continuously and considerably with the increase of the radiation parameter. This shows that the heat loss is more due to higher thermal radiation.

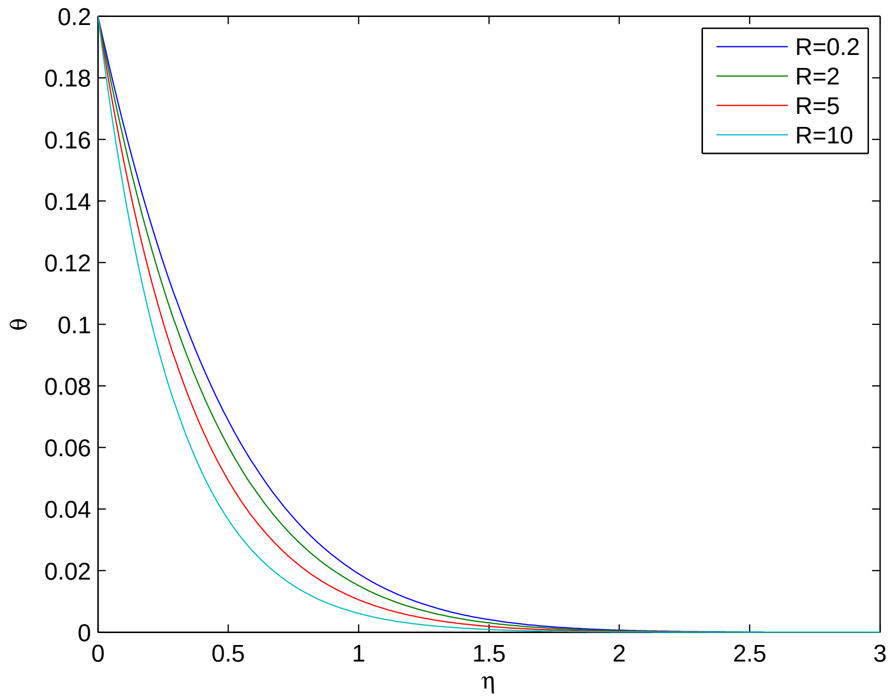

Figure 3.1 Temperature profile for different values of R

3.4.2 Effect on concentration field

The effect of concentration profiles for different values of Schmidt number Sc as shown in Figure 3.2 with time $t = 0.2$. The effect of Schmidt number is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is noted that the wall concentration decreases considerably with increasing values of the Schmidt number. Also, the wall concentration decreases slowly in the axial direction from $\eta = 0$ to $\eta = 3$.

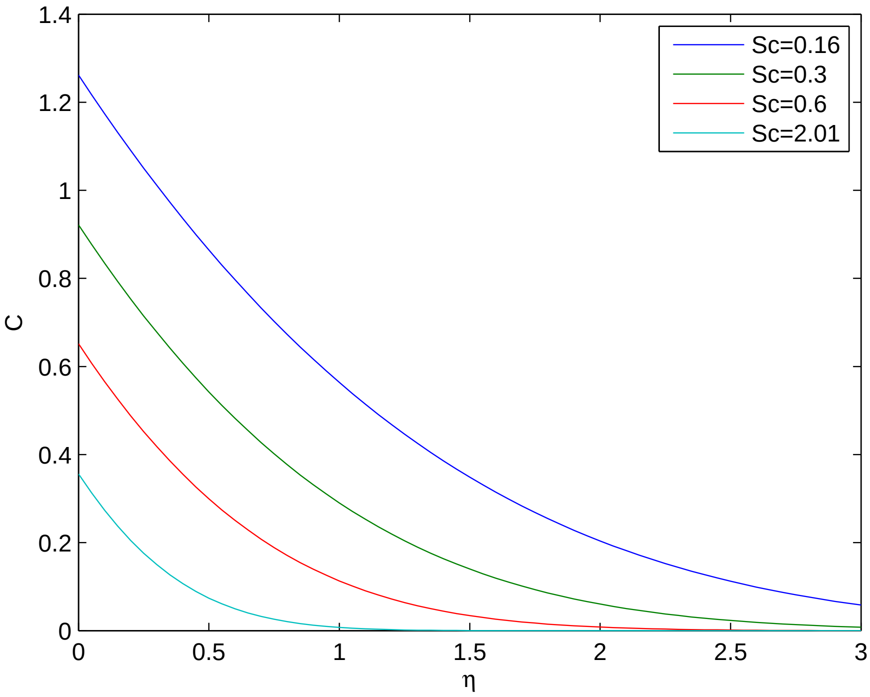

Figure 3.2 Concentration field for different values of Sc

3.4.3 Effect on velocity profiles

The velocity profiles are of particular interest, since they provide a detailed description of the flow field. Figure 3.3 exhibits the effect of velocity for different values of the radiation parameter $R = 0.2, 2, 5, 10$ with $Gr = 5 = Gc$, $Sc = 0.6$ and $t = 0.2$. One can notice that the velocity increases slightly as radiation parameter decreases. The trend shows that the velocity increases rapidly and attains maximum value around $\eta = 0.5$ and then it decreases gradually from $\eta = 0.5$ to $\eta = 2$. Subsequently, it decreases asymptotically from $\eta = 2$ to $\eta = 3$. Also, it is seen that the velocity decreases in the presence of high thermal radiation. This shows that the buoyancy effect on the temperature distribution is very significant in the air.

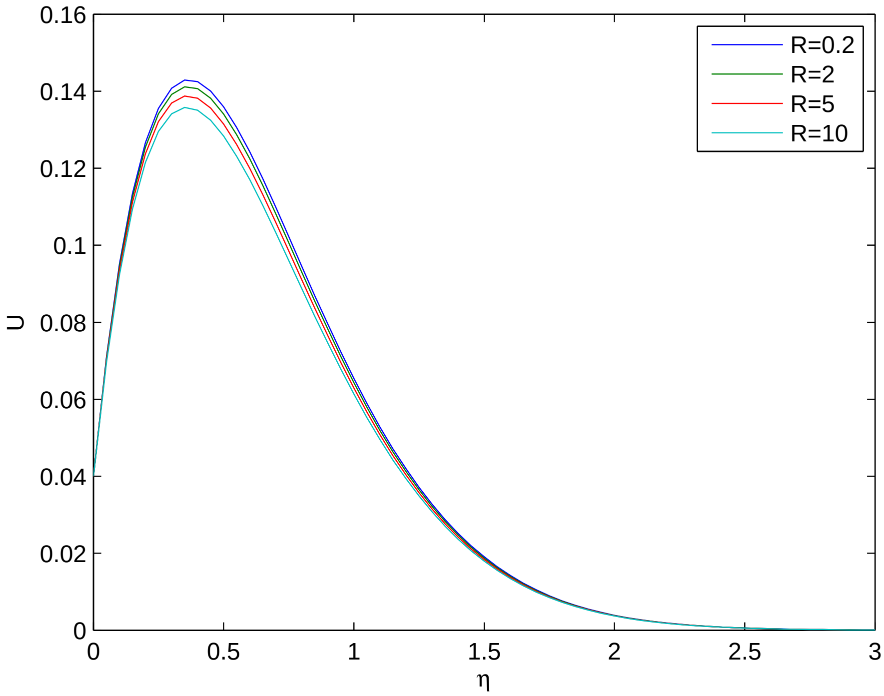

Figure 3.3 Velocity profile for different values of R

Figure 3.4 depicts the variation of velocity with axial distance for different values of Schmidt number Sc for $t = 0.2$. It is clear that the velocity decreases continuously and significantly in the axial direction with the increase of the Schmidt number Sc. It is found that velocity increases rapidly and attains maximum value around $\eta = 0.5$ and then it decreases gradually from $\eta = 0.5$ to $\eta = 3$. The velocity boundary layer seems to grow in the direction of motion of the plate.

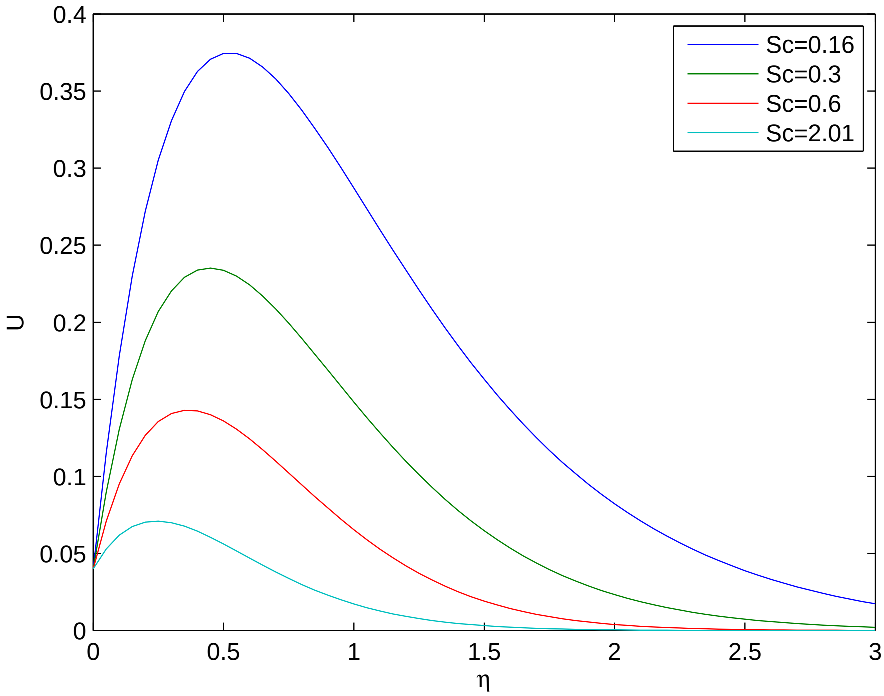

Figure 3.4 Velocity profile for different values of Sc

The effects of different thermal Grashof number $Gr = 2, 5$ and mass Grashof number $Gc = 5, 10$ on the velocity profile with $t = 0.2$ is sketched in Figure 3.5. It is observed that the velocity increases significantly with increasing values of the thermal Grashof number or mass Grashof number. It is seen that the peak values of the velocity increases rapidly near the plate as thermal Grashof number and mass Grashof number increases and then decays to the free stream velocity. One may observe that for $Gr = 5$ and $Gc = 10$, the velocity is significantly higher than that of other noted values of Gr and Gc.

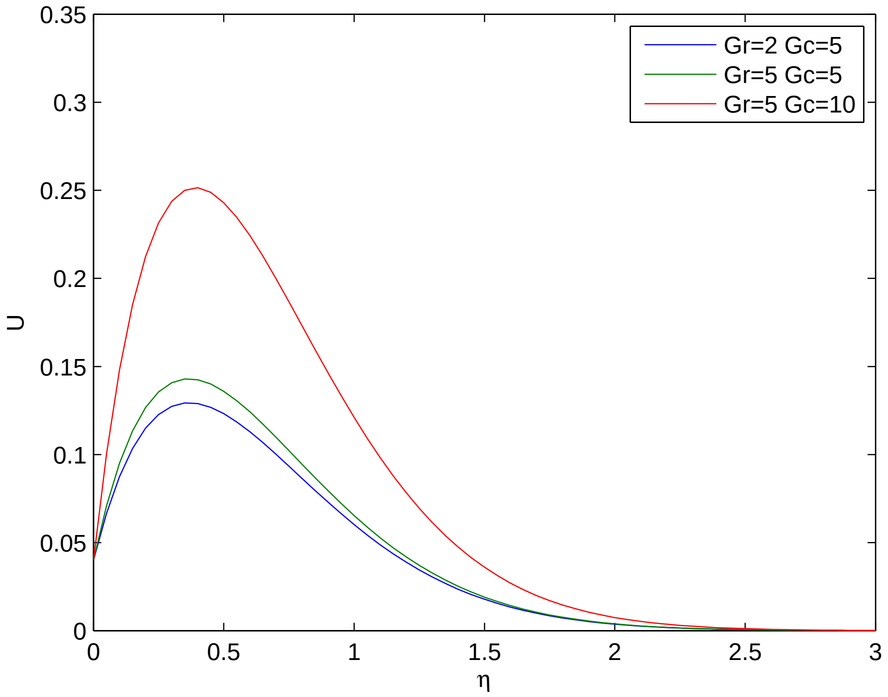

Figure 3.5 Velocity profile for different values of Gr and Gc

The effects of velocity profiles for different values of time are illustrated in Figure 3.6. It is found that the velocity increases significantly with respect to time t, as we expected. One can notice that the velocity increases rapidly from $\eta = 0$ to $\eta = 0.4$ and then it decreases gradually with axial direction.

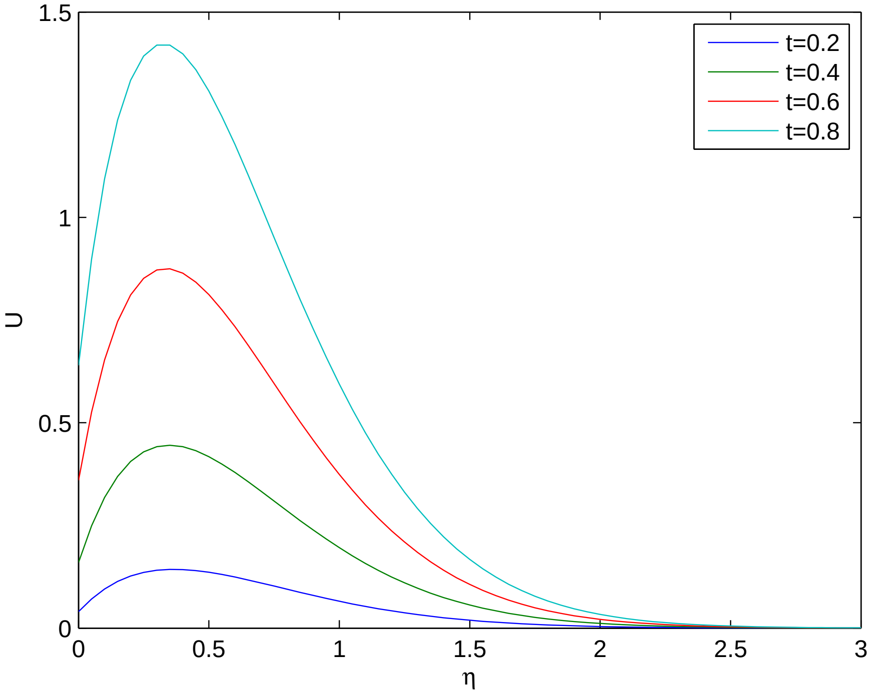

Figure 3.6 Velocity profile for different values of time t

3.5 CONCLUSION

The present study deals with the effects on flow past a parabolic starting motion of an infinite vertical plate in the presence of thermal radiation when mass is supplied to the plate at constant rate. The dimensionless governing equations are solved by the Laplace transform technique. Numerical evaluations of closed form solutions were performed, and some graphical results were obtained to illustrate the details of flow, heat and mass transfer characteristics and their dependence on some physical parameters like thermal radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. The conclusions of the study are as follows:

- The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the thermal radiation parameter or Schmidt number.
- The velocity increases with increased values of the time t.
- The temperature of the plate increases with decreasing values of the thermal radiation parameter. This shows that the heat loss is more in the presence of higher thermal radiation.
- The concentration near the plate increases with decreasing values of the Schmidt number.
- Comparison of flow past an infinite isothermal vertical plate subjected to parabolic motion with uniform mass flux with that of flow past an infinite vertical plate subjected to parabolic motion with variable temperature in the presence of uniform mass flux, it is concluded that there is a drastic change in the temperature profile in the case of variable temperature than in the case of isothermal vertical plate.

- In general, the study conclude that the heat loss is more in the presence of higher thermal radiation.