CHAPTER 3

SIGNAL PROCESSING TECHNIQUES

3.1 FAST FOURIER TRANSFORM (FFT) ANALYSIS

3.1.1 Concept of Fourier Transform

Fast Fourier Transform (FFT) is used extensively in signal processing applications such as power systems, power electronics, communications, broadcasting, entertainment and many other areas. FFT provides frequency domain representation of any periodic or non-periodic signal. Before getting into FFT technique, it is important to understand the fundamentals and basic concepts of Fourier Transform and Discrete Fourier Transform.

The Fourier Transform (FT) is a generalization of the Fourier series. Instead of sines and cosines, as in a Fourier series, the Fourier transform uses exponentials and complex numbers. For a signal or function f(t), the Fourier Transform is defined as

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t} dt \qquad (3.1)$$

and the inverse Fourier Transform is defined as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\; e^{-i\omega t} d\omega \qquad (3.2)$$

where 'i' is the imaginary unity number, defined as the square root of -1 and ω is the range of angular frequencies associated with the signal, i.e. the frequency content of the signal. When working with time and the signal f(t), it is said to be working in the time domain, and the variables are real. When working with angular frequency and the Fourier transform F(ω), it is said to be working in the frequency domain, and F(ω) is complex. The FT can be thought of as an analog tool, since it is used for analyzing the frequency content of continuous signals.

3.1.2 Concept of Discrete Fourier Transform

For a digital (or) discrete signal, computation of Fourier transform is called as the Discrete Fourier transform (DFT). The DFT can be thought of as a digital tool, since it is used for analyzing the frequency content of discrete signals. The discrete Fourier transform is defined as

$$F(k\Delta f) = \sum_{n=0}^{N-1} f(n\Delta t) e^{-i(2\pi k\Delta f)(n\Delta t)} \tag{3.3}$$

for k = 0, 1, 2, ..., N-1. In the case of DFT, summation has been used in place of the integral since here discrete data rather than continuous data are being examined. The variables in the time domain are

- N = number of discrete samples.
- T = sampling time.
- Δt = time increment between samples.
- f_s = the sampling frequency.
- integers n and k have values from 0 to N-1.

In order to analyze the signals using DFT in practice, it is necessary to obtain a discrete signal by sampling technique and take a segment of N samples by windowing method. Next, assuming that the segment is periodic, i.e. the segment is extended on either side, compute the coefficients of one period of the discrete Fourier Transform. Then, move onto the next segment in the sequence of samples and repeat the computation of DFT coefficients. The representation through samples of the Fourier transform is in effect a representation of the finite-duration sequence by a periodic sequence.

3.1.3 Concept of Fast Fourier Transform

The Fast Fourier transform is simply a DFT that is faster to calculate on a computer. All the rules and details about DFTs described above apply to FFTs as well. For most FFTs, the computer algorithm restricts N to a power of 2, such as 64, 128, 256, and so on. However, some of the newer FFT algorithms do not have such a restriction. (For example, the FFT used in LabVIEW does not have this power of 2 restrictions).

By making use of periodicities in the sines that are multiplied to do the transforms, the FFT greatly reduces the amount of calculation required. Functionally, the FFT decomposes the set of data to be transformed into a series of smaller data sets to be transformed. Then, it decomposes those smaller sets into even smaller sets. At each stage of processing, the results of the previous stage are combined in special way. Finally, it calculates the DFT of each small data set. For example, an FFT of size 32 is broken into 2 FFTs of size 16, which are broken into 4 FFTs of size 8, which are broken into 8 FFTs of size 4, which are broken into 16 FFTs of size 2. Calculating a DFT of size 2 is trivial. It turns out that it is possible to take the DFT of the first N/2 points and combine them in a special way with the DFT of the second N/2 points to produce a single N-point DFT. Each of these N/2-point DFTs can be calculated using smaller DFTs in the same way. One (radix-2) FFT begins, therefore, by calculating N/2 2-point DFTs. These are combined to form N/4

4-point DFTs. The next stage produces N/8 8-point DFTs, and so on, until a single N-point DFT is produced.

The output, $F(k\Delta f)$, of an FFT subroutine is a series of complex numbers, one for each discretely sampled data point, representing each discrete frequency, only half of which are useful because of the Nyquist criterion. It is the magnitude or amplitude of the complex number generated by the FFT that is actually used to compare the relative importance of the various frequencies. Recall that for some complex number $z = x + iy$, where x is the real part and y is the imaginary part, the magnitude of z is given by $|z| = \sqrt{x^2 + y^2}$. The magnitude of a complex number is called the modulus.

A plot of the magnitude of the FFT output, $|f|$, as a function of frequency, f, is called the frequency spectrum. Over a given frequency range, the amplitude defined in this way indicates the relative importance of that frequency range to the signal. A frequency spectrum plot formed from an FFT is analogous to the harmonic amplitude plot formed from a Fourier series.

In order to understand FFTs, consider a simple 10 Hz sine wave with amplitude = 1. As a starting point, f_s is chosen to be 200 samples per second, and N is chosen to be 256. In other words, the 10 Hz sine wave is sampled at 200 Hz for 1.28 seconds ($T = N/f_s = 256/200 = 1.28$). Both the time trace (the signal in the time domain) and the frequency spectrum (the magnitude of the FFT output in the frequency domain) are shown below.

Figure 3.1 shows the time trace, showing the discretely sampled data of the sine wave signal

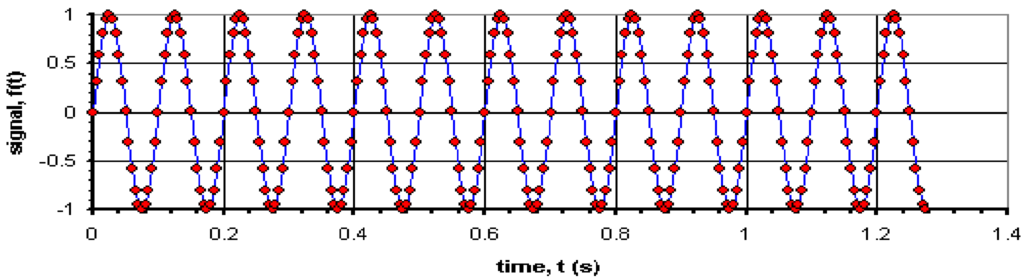

Figure 3.1 Discretely sampled data of sine wave signal

Figure 3.2 shows the frequency spectrum for N = 256 and $f_s = 200$. The frequency resolution for the spectrum is $\Delta f = 1/T = 1/1.28 = 0.781$ (to three significant digits).

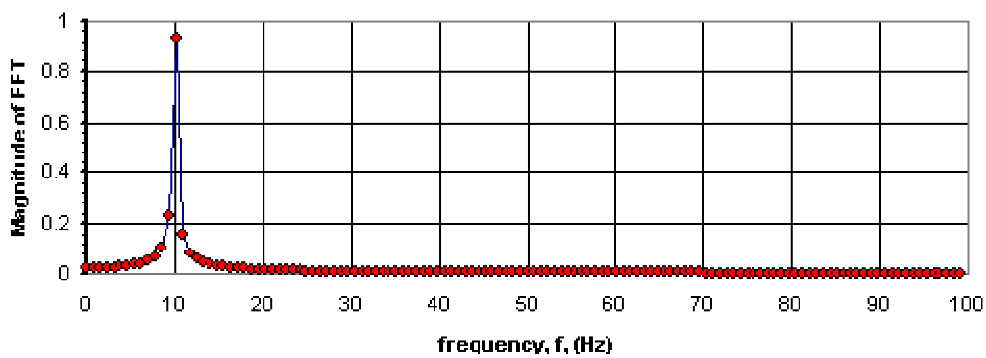

Figure 3.2 Frequency spectrum of sine wave signal

- Even though the frequency spectrum correctly shows a spike at 10 Hz, the spike is not infinitesimally narrow. In fact, it appears from the frequency spectrum that there is a significant component of the signal at frequencies near 10 Hz (specifically within about 5 Hz to either side of 10 Hz). This unphysical error in the FFT is called leakage, which appears when the discrete data acquisition does not stop at exactly the same phase in the sine wave as it started. In principle, if an infinite number of discrete samples are taken, leakage would not be a problem. However, any real data acquisition system performing FFTs uses a finite (rather than infinite) number of discrete samples, and there will always be some leakage.

- The maximum amplitude of the FFT is not exactly 1 (in fact it is less than 1), even though the amplitude of the original signal was exactly 1. This is another consequence of leakage - namely some of the energy at 10 Hz is erroneously distributed among frequencies near 10 Hz, thereby reducing the calculated amplitude at 10 Hz.
- The frequency range is from 0 to 100 Hz, half of the sampling frequency, as discussed above. This is a consequence of the Nyquist criterion.

However, the frequency spectrum obtained from the FFT analysis, clearly shows the energy content of the signal at various frequency ranges. It is also possible to understand the various dominant frequencies present in the discrete sampled signal from the FFT analysis.

3.1.4 Applications of the FFT

- The fast Fourier transform (FFT) is extremely useful in analyzing unsteady measurements, because the frequency spectrum from an FFT provides information about the frequency content of the signal.
- For example, in the case of power quality measurements, it is necessary to understand any frequency variations due to voltage sag, swell, notches, etc. In the case of condition monitoring techniques and image processing applications using spectral analysis, FFT is mostly used.

- In the area of energy auditing, most of the harmonic analyzer equipment use the concept of FFT to find the harmonic distortion values of voltage and current measurements.
- Examples also come from nearly every area of engineering, such as vibrations, where one needs to know the frequency content of the vibration; fluid flow, where one needs to know the frequency content of the turbulent fluctuations; and acoustics, where one needs to know the frequency content of a sound signal, to mention just a few.

3.2 DISCRETE WAVELET TRANSFORM ANALYSIS

3.2.1 Concept of Wavelet Transforms

Signal processing using wavelet theory has emerged as a powerful tool in the recent times to overcome some of the shortcomings of the Fourier transforms. The wavelet is a flexible tool for the analysis of transients and non-stationary signals. It allows simultaneous time and frequency analysis of signals. Two of the most common manifestations of the wavelet transform are the Continuous Wavelet Transform (CWT), which is commonly used in time-scale spectral estimation and the Discrete Wavelet Transform (DWT), which is mainly implemented for data compression and for pattern recognition/ image processing applications. Wavelets are extensively used for the purposes of filtration and preprocessing the data, analysis and prediction of stock markets situations, as well as for processing and synthesizing various signals, like speech or medical signals, training neural networks and so on.

3.2.2 Continuous Wavelet Transform Method

Wavelet analysis involves the breaking up of a signal into shifted and scaled versions of a single prototype function called the original or mother wavelet (Rioul et al 1991). A mathematical definition of continuous wavelet analysis is as follows.

Let x(t) be a signal defined in $L^2(R)$ space, which denotes a vector space for finite energy signals. R is a real continuous number system. The signal must satisfy the condition that;

$$\int_{-\infty}^{\infty} x^2(t)\,dt < \infty \tag{3.4}$$

The CWT of x(t) is then defined as

$$CWT_x^{\psi}(\tau,s) = \psi_x^{\psi}(\tau,s) = \frac{1}{\sqrt{|s|}}\int x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{3.5}$$

where ψ(t) is called a basic wavelet (or mother wavelet). The asterisk denotes a complex conjugate, and τ, s ϵ R, s ≠ 0 are the translation and dilation or scale parameters respectively. The functions that are used as mother wavelets have net area equal to zero. The time analysis is performed with a contracted i.e. low scale, high frequency version of the mother wavelet, while frequency analysis is performed with a dilated i.e. high scale, low frequency version of the same wavelet. For a given mother wavelet function and sampling period, mapping of a scale into a particular frequency can be achieved from the ratio of centre frequency of the wavelet to the product of scale value and the sampling period.

3.2.3 Discrete Wavelet Transform Method

Extraction of salient features of the output voltage data, which in turn actively drives diagnostic knowledge out of the raw data, plays a major role in the novel fault diagnostic technologies. In order to develop an efficient fault diagnostic system, it is necessary to perform both time and frequency domain analysis of output voltage signals. Discrete wavelet transform technique has been found to be efficient to extract features from the output voltage data. Multi resolution signal decomposition analysis of the Discrete Wavelet Transform aims at ultimately producing a time-scale representation of the given discretized signal x(n) at various decomposition levels.

Discrete wavelets are not continuously scalable and translatable, but can only be scaled and translated in discrete steps. In this process, the mother wavelet be dilated and translated discretely by selecting $s=a^m$ and $\tau=nba^m$ where a and b are fixed constants with a>1 , b>0, m , n ε Z and Z is the set of positive integers. Then DWT is given by,

$$DWT_x^{\psi}(m,n)=\frac{1}{\sqrt{a^m}}\int x(t)\psi^*\left(\frac{t-nba^m}{a^m}\right)dt \tag{3.6}$$

This method maps a continuous variable into a sequence of coefficients. In the above equation x(t) is discretized signal function and ψ(t) is the wavelet function.

The Multi-Resolution Signal Decomposition (MRSD) analysis is used to develop representations of a signal at various levels of resolution. In the implementation of MRSD, the input function is a sequence of numbers (digitized signal). Let $c_0(n)$ be a discrete time signal recorded using a certain measuring device. The signal has to be decomposed into a detailed and smoothened representation of the original signal (Vetterli et al 1992). The wavelet functions and the scaling functions are used as building blocks to decompose and construct the signal at different resolution levels. The wavelet function will generate the detail version of the decomposed signal and the scaling function will generate the approximated version of the decomposed signal. From the MRSD technique, the decomposed signals at scale 1 are $c_1(n)$

and $d_1(n)$, where $c_1(n)$ is the low pass version (approximated version) of the original signal and $d_1(n)$ is the high pass representation (detailed version) of the original signal $c_0(n)$ in the form of wavelet transform coefficients.

They are defined as,

$$c_1(n) = \sum_k h(k-2n)c_0(k) \tag{3.7}$$

$$d_1(n) = \sum_k g(k-2n)c_0(k) \tag{3.8}$$

where h(n) and g(n) are the associated filter coefficients that decompose $c_0(n)$ into $c_1(n)$ and $d_1(n)$ respectively. The next higher level decomposition is based on $c_1(n)$. The decomposition signal at scale 2 is given by

$$c_2(n) = \sum_k h(k-2n)c_1(k) \tag{3.9}$$

$$d_2(n) = \sum_k g(k-2n)c_1(k) \tag{3.10}$$

Higher scale decompositions are performed in the same way as described above. The implementation of the MRSD technique is evident in Figure 3.3.

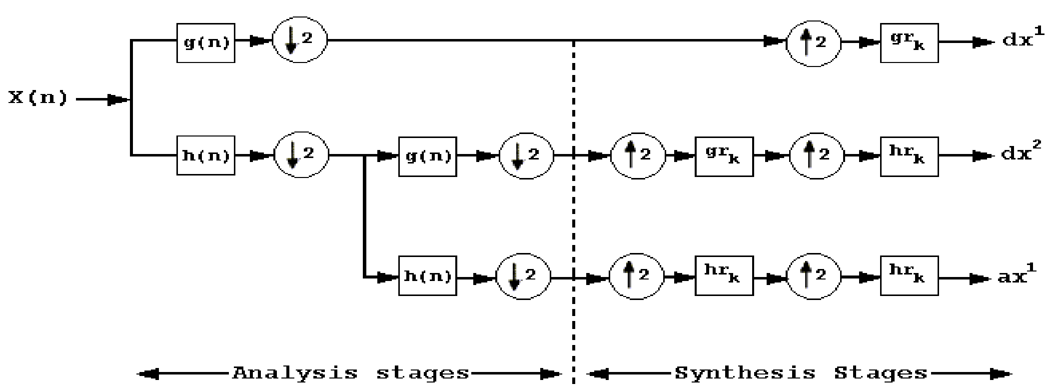

Figure 3.3 Schematic implementation of MRSD

In order to understand the working of multi resolution decomposition of DWT in a practical way, consider the Figure 3.4 overview of signal decomposition using DWT. The discrete sampled signal is passed to wavelet filters of both low pass filter and high pass filter. Then the wavelet coefficients obtained from the low pass filter are called as approximation coefficients and the wavelet coefficients obtained from the high pass filter are called as detailed coefficients.

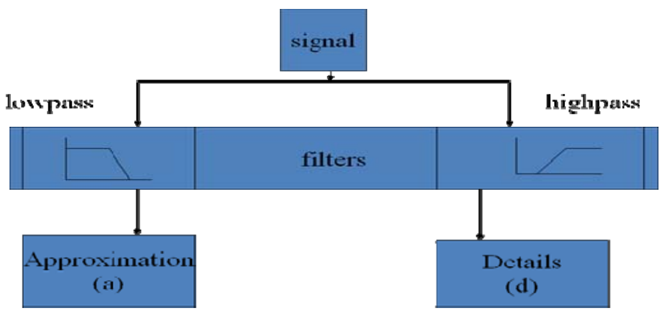

Figure 3.4 Overview of signal decomposition using DWT

It is possible to reconstruct the original signal at any stage of signal decomposition. Figure 3.5 shows the successive stages of multiresolution decomposition of signal and corresponding coefficients at each stage as A_1, A_2, D_1, D_2, etc. The reconstruction of the signal at stage 1 will be $S=A_1+D_1$, whereas at stage 2 will be $S=A_2+D_2+D_1$. During decomposition of signal downsampling is used and during reconstruction stages, upsampling concept is used.

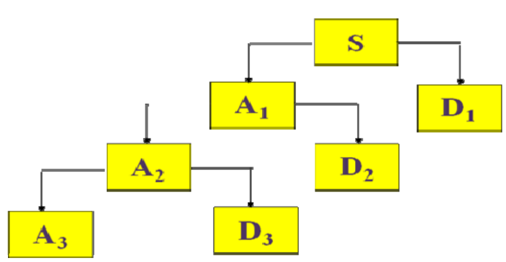

Figure 3.5 Block diagram representation of wavelet coefficients

MRSD is used to archive two important properties. The first is the localization property in time to identify any characteristic change. This will appear as large coefficients at the time of disturbance. The second is the partitioning of the signal energy into different frequency bands. This gives an idea of the frequency content in the distorted original signal. From this technique, a distorted signal can be decomposed into different resolution levels and any changes in the smoothness of the signal can be detected and localized at the finer resolution levels.

The choice of mother wavelet plays an important role in detecting and identifying the characteristic change in the signal. If the wavelet function and scaling function form an orthonormal basis, then according to Parseval's theorem, the energy of the signal can be related to the energy of the wavelet coefficients at different resolution levels. And also, a perfect reconstruction of the original signal is possible when the applied wavelet satisfies the orthonormal basis property. In the present work, the most famous orthonormal basis wavelet known as Daubechies 4 wavelet has been chosen for the analysis. Figure 3.6 shows the typical Daubechies mother wavelet function of order 4.

The standard deviation can be considered as a measure of the energy present in the signal with zero mean (Gaouda et al 1999). Therefore the standard deviation at different resolution levels of the decomposed signal has been considered as an important feature in the present work.

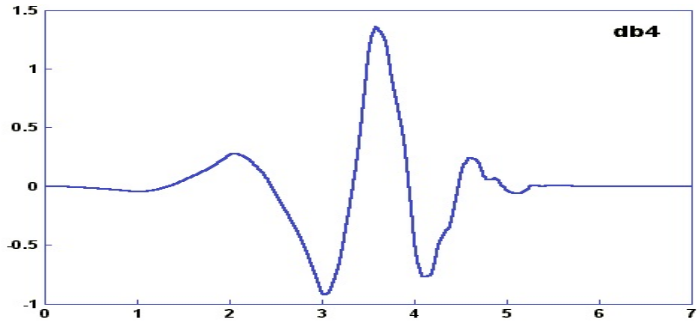

Figure 3.6 Daubechies 4 wavelet function

The purpose of carrying out the standard deviation is to identify the transient energy present at any point of time. Therefore, the Standard Deviation Multi Resolution Analysis (STD_MRA) curve, obtained by plotting the standard deviation values at different resolution levels of the decomposed signal, is used in the present work as an important feature.

In the present work, output voltage signals were decomposed up to 9 levels. The standard deviation values are calculated for detailed components to identify the transient energy present in the signal at different level of decomposition (D1 to D9). Standard deviation of the n^{th} level of detailed signal is calculated using the formula,

$$std = \sqrt{\frac{1}{N_n - 1}\sum_{j=1}^{N_n}[d_n(j) - \mu_n]^2} \quad (3.11)$$

where μ_n is the mean of the vector d_n and N_n is the length of the vector d_n. In the present work, energy content of the signal obtained from DWT MRA technique is considered as an important feature for the fault diagnosis of multilevel inverter system.