CHAPTER 3

FREQUENCY DOMAIN MODELLING OF AC-DC-AC CONVERTER SYSTEMS

3.1 INTRODUCTION

The present day systems are powered by non-ideal sources whose output impedance is not negligible, besides most of the loads are non-linear in nature (Uffe Borup et al 2001). Also the insertion of switched mode power converters makes the systems variable structured. The analysis of harmonic components is an inevitable part of the study, due to the requirements of higher power quality. Numerical techniques offer a good representation of the non-characteristic waveform distortion generated by the converters. The most widely used method to calculate the harmonic components is a numerical time domain simulation method, in which the various components are analyzed by solving differential equations.

The time domain methods are easy to use and allow verification of system operation under any number of different operating states. However they do not provide an analytical insight required for optimal design; besides frequency dependence cannot be accurately modeled (Wood & Arrillaga 1995a, 1995b). An alternative method for calculating the harmonic currents of a power converter uses the Fourier series and the switching functions. With a frequency domain model, the closed loop frequency responses can be established, which will facilitate the analysis of system stability and design optimization. The frequency response test is cumbersome to perform, for
systems with large time constants, as the time required for the output to reach the steady state for each frequency of the test signal is exceedingly long. However frequency domain modeling is significant for power electronic circuits, which offer a faster response.

3.2 PROBLEM FORMULATION

The switched mode operation of the power electronic converter systems along with their complex structure make them variable structured system. For instance the inverter output waveforms are usually rectilinear in nature and as such contain harmonics. Besides in any inverter system, changes in load current and pattern may introduce some degree of non linearity. Another feature in such systems is its inability to continue to operate in the linear range due to the effects of pulse dropping. These issues cause the accurate modeling of such system challenging.

The harmonic currents cause detrimental effects, such as an abrupt termination of the load power or oscillations, which may impose higher stresses on all the components of the power path. The impact of the interaction between non-linear loads and the power sources need to be characterized, which necessitates the accurate model. It is envisaged to develop a frequency domain based model of a single phase uncontrolled rectifier and a SPFB inverter, in order to evaluate the performance of the AC-AC conversion system, suitable for variable frequency system through MATLAB simulation and FPGA based hardware implementation. The simulation results are to be validated by comparing with those obtained using TDA. Besides, the analytical FDA and experimental results are also to be compared.
This chapter presents a TSF based FDA for ASD system consisting of an uncontrolled inverter and PWM inverter. The developed FDA results are compared with TDA and experimental results.

### 3.3 FDA OF UNCONTROLLED RECTIFIERS

A typical Single Phase Diode Rectifier (SPDR) is shown in Figure 3.1. Generally the rectifier operates as a power converter, since its primary function is to convert the fundamental power frequency AC (50 or 60 Hz) to DC. The modulation is achieved by the alternate switching action of the diodes. The instantaneous output voltage, \( V_{dc} \) shown in Figure 3.2 (c), is expressed in terms of the rectifier switching function ‘\( S \)’ and AC source voltage, \( V_{ac} \) as in Equation (3.1). Figure 3.2 (b) shows the switching function of the SPDR, which represents the switching of the alternate diode pairs, to connect the supply voltage to the DC-bus. This switching function operates as a frequency transfer function in that it describes the way an AC side frequency signal is transferred to the DC side. The Fourier series of switching function is given in Equation (3.2).

\[
V_{dc} = V_{ac} \times S \quad \text{(3.1)}
\]

\[
S = a_0 + \sum_{n=1} \frac{4}{n\pi} \sin(n\omega t) \quad \text{(3.2)}
\]

As the switching function is symmetrical, the Fourier coefficients \( a_0 \) and \( a_n \) are zero and the switching function \( S \) is

\[
S = \sum_{n=1,3,5,...} \frac{4}{n\pi} \sin(n\omega t) \quad \text{(3.3)}
\]
Substituting Equation (3.3) in Equation (3.1) gives

\[ V_{dc} = \sum_{n=1,3, \ldots}^{n \pi} \frac{1}{n \pi} \sin(n \omega t) \cdot V_m \sin(\omega t) \]  

\[ = \frac{2V_m}{\pi} - \frac{4V_m}{\pi} \sum_{n=2,4, \ldots}^{n \pi} \frac{\cos(n \omega t)}{n^2 - 1} \]  

\[ \text{Figure 3.1 Uncontrolled rectifier} \]  

\[ \text{Figure 3.2 (a): Front end diode rectifier (b): Switching function and (c): Rectifier output} \]
The final rectifier output is given by Equation (3.5) and the rectifier load current is given by

\[ I_{dc} = \frac{2V_m}{\pi R} \cdot \frac{4V_m}{\pi} \sum_{n=1,2,3,...}^{\infty} \frac{\cos(2n\omega t)}{2n} \]

Where, \( Z_{2n} \) is the impedance offered to even order harmonic components. The rectifier source side current can also be obtained by using the same switching function (S) and the load side current, expressed as

\[ I_{ac} = I_{dc} \cdot S \]

By substituting Equation (3.6) and Equation (3.3) in Equation (3.7)

\[ I_{ac} = \frac{8V_m}{\pi^2 R} \sum_{n=1,3,5,...}^{\infty} \frac{\sin(n\omega t)}{n} + \frac{16V_m}{\pi^2} \sum_{n=1,2,3,...}^{\infty} \frac{\cos(2n\omega t) \cdot Z_{2n}}{2n} \cdot \sum_{n=1,3,5,...}^{\infty} \frac{\sin(n\omega t)}{n} \]

### 3.4 FREQUENCY DOMAIN ANALYSIS OF PWM INVERTERS

The power circuit of single-phase inverter is shown in Figure 3.3. The typical switching function ‘\( S \)’ of the inverter is shown in Figure 3.4 (b) which is derived from comparison of sine reference and triangular carrier. Figure 3.4 (a) and 3.4 (b) show the inverter input and output respectively. The switching angles are found using following expressions.

\[ p_i^{th} \text{ intersection, } \alpha_{im} + \frac{\pi}{2M} M_a \sin\alpha_{im} - \frac{2j}{2Mf} = 0 \]  

\[ p_{i+1}^{th} \text{ intersection, } \alpha_{im} - \frac{\pi}{2M} M_a \sin\alpha_{im} - \frac{2j}{2Mf} = 0 \]
The Fourier coefficients for a pair of pulse is given as

\[
B_n = \frac{4}{n\pi} \sin \left( \frac{n\delta_m}{4} \right) \left[ \sin n(\alpha_m + \frac{3\delta_m}{4}) - \sin n(\pi + \alpha_m + \frac{\delta_m}{4}) \right]
\]  \hspace{1cm} (3.11)

Where, \( \alpha_m \) is the starting point of the pulse and \( \delta_m \) is the width of each pulse.

The overall switching function is given as
\[ S_I = \sum_{n=1.3.5...}^{P} \frac{4}{n\pi} \sin \left( \frac{n\delta_m}{4} \right) \begin{bmatrix} \sin(n\alpha_m + \frac{3\delta_m}{4}) \\ \sin(n\alpha_m + \frac{3\delta_m}{4}) \\ \sin(n\omega t) \end{bmatrix} \]  

(3.12)

The output of the inverter is given by

\[ V_o (t) = V_{dc} \cdot S_I \]  

(3.13)

\[ V_o (t) = \sum_{n=1.3.5...}^{P} \frac{4V_{dc}}{n\pi} \sin \left( \frac{n\delta_m}{4} \right) \begin{bmatrix} \sin(n\alpha_m + \frac{3\delta_m}{4}) \\ \sin(n\omega t) \end{bmatrix} \]  

(3.14)

The inverter output current is the ratio between the output voltage and the load resistance, expressed as

\[ I_o (t) = \frac{V_{dc}}{nR} \sum_{n=1.3.5...}^{P} \frac{4V_{dc}}{n\pi} \sin \left( \frac{n\delta_m}{4} \right) \begin{bmatrix} \sin(n\alpha_m + \frac{3\delta_m}{4}) \\ \sin(n\omega t) \end{bmatrix} \]  

(3.15)

The inverter input current is obtained by using the inverter output current and the same switching function \((S_I)\), given by

\[ I_i (t) = I_o (t) \cdot S_I \]  

(3.16)

\[ I_i (t) = \frac{V_{dc}}{R} \sum_{n=1.3.5...}^{P} \frac{4V_{dc}}{n\pi} \sin \left( \frac{n\delta_m}{4} \right) \begin{bmatrix} \sin(n\alpha_m + \frac{3\delta_m}{4}) \\ \sin(n\omega t) \end{bmatrix} \]  

(3.17)
3.5 SIMULATION RESULTS

The simulation is performed on a SPWM inverter using MATLAB both in time and frequency domains for various values of $M_a$ and $M_f$. The results obtained by the analytical method are compared with those available in the time domain for $M_a=0.8$ and $M_f=10$. The results are obtained for a rectifier of load resistance of $10\Omega$ and inductance of $0.1\text{H}$. The FDA results are shifted in the Y-axis scale for clarity. Figure 3.5 (a) shows the DC voltage of the rectifier. It shows that the output voltages using FDA and TDA are almost the same, but the output DC current shown in Figure 3.5 (b) reveals that the TDA takes a longer time to reach the steady state value.

![Figure 3.5 (Continued)](image-url)
Figure 3.5 (a): Output voltage and (b): Output current waveform (FDA-calculated, TDA-simulated)

The source side current of the rectifier is seen in Figure 3.6. The output voltage of inverter and fundamental component of the output are depicted in Figure 3.7 (a) and (b) respectively. Figure 3.8 shows the simulated harmonic spectrum of the inverter output voltage. The dominant harmonic components of PWM controlled inverter are pushed to higher frequency as expected. It is seen that the inverter output current waveform is the same as that of the voltage for a resistive load.
Figure 3.6 Source current waveform of rectifier (FDA -Calculated, TDA-simulated)

Figure 3.7 (Continued)
Figure 3.7  (a): Output voltage and (b): Fundamental component of output voltage ($M_a = 0.8$, $M_f = 10$, $V_{dc} = 300V$)

Figure 3.8  Harmonic spectrum of output voltage – SPWM $M_a = 0.8$, $M_f = 15$ and $V_{dc} = 300V$
The SPWM strategy is implemented for the SPFB inverter using Spartan-6 FPGA (XC6SLX45). The design is compiled, simulated using ModelSim and finally downloaded to the device through Xilinx software. The PWM pulses are generated using Time Ratio Recursive (TRR) algorithm, in which the basic idea is to generate carrier waves of any frequency, acquired by fetching the triangular samples while the reference of any magnitude is obtained through a suitable multiplying factor ( Jeevananthan et al 2006). The concept of the TRR algorithm is carrier wave of any frequency can be achieved through fetching rate of the triangle pattern while reference of any magnitude is through the multiplying factor. For any $M_f$, with fixed reference frequency, the carrier wave is obtained by increasing the fetching rate $M_f$ times i.e. fetching the triangular pattern recursively in the rate of $M_f$ in one half cycle of the reference wave. Figure 3.9 and Figure 3.10 demonstrate the basic principle of the proposed TRR algorithm.

Figure 3.9 Obtaining carrier and reference in TRR
To find the magnitude of any kind of wave, a simple and practical method called the look-up table method is used, wherein the amplitude of the waveform is digitized at discrete points along the phase axis, and the digital values are stored in sequentially addressed locations. For continued generation of any periodic waveform, at most, one cycle needs to be stored. With the digitized values of sine and triangular waves stored in a memory, the magnitude of these can be read at any time for any desired phase angle, at any rate (decides frequency of the wave). To minimize the processing time and memory occupancy, half cycle of sine and triangular waves are stored. The
magnitude is digitized into 8-bit word with the maximum value set to 1 p.u of
the peak amplitude. The values of sine and triangular waves are digitized
from 0 to π at an interval of one degree for each wave.

The FPGA processor acquires the values of $M_a$, $M_f$, base reference
wave (at $M_a=1$) and base carrier wave (at $M_f=1$) as inputs. The first step is to
modify the base reference to the given $M_a$ and obtain the actual reference
wave. The base reference samples are multiplied by a transitory $M_a$ (ten times
the actual $M_a$) and later divided by ten. The values of the actual reference
wave are stored in an array of fresh adjacent locations. The second step is to
compare the reference and carrier waves. The subroutine determines the
actual carrier wave from the base carrier wave. The modified sine pointer and
formed pattern pointer (where the PWM pattern is to be stored) are initialized
and thereafter the carrier pointer is calculated recursively.

Figure 3.11 shows the typical output waveform resulted in
hardware testing. The corresponding harmonic spectrum is presented in
Figure 3.12.
Figure 3.11   Experimental output voltage waveform -SPWM

Figure 3.12 Frequency spectrum with SPWM ($M_a = 0.8$, $M_f = 15$)
A detailed comparison of analytical and experimental results is presented in Figure 3.13 in terms of THD as a function of $M_a$.

![Figure 3.13 Comparison of analytical (FDA) and experimental values](image)

**Figure 3.13 Comparison of analytical (FDA) and experimental values**

Tables 3.1 and 3.2 shows similar comparison for $M_a = 0.8$ and $M_f = 15$ highlighting the dominant harmonics. Figure 3.14 shows the FDA and TDA results of DC link current while Figure 3.15 gives similar results of representative dominant harmonics.

The FDA (developed model) is very much genuine in imitating the AC-DC-AC system than the TDA. The THD error caused by the TDA is 10% while the error in FDA is only 2%. Also the TDA results irrelevant values of harmonics in few cases.
Table 3.1 THD, fundamental and lower order harmonics

<table>
<thead>
<tr>
<th>Method</th>
<th>THD (%)</th>
<th>V_3 (%)</th>
<th>V_5 (%)</th>
<th>V_7 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDA</td>
<td>76.68</td>
<td>12.45</td>
<td>8.62</td>
<td>2.28</td>
</tr>
<tr>
<td>FDA</td>
<td>68.02</td>
<td>0.18</td>
<td>0.12</td>
<td>0.03</td>
</tr>
<tr>
<td>Experimental</td>
<td>69.68</td>
<td>1.45</td>
<td>0.62</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 3.2 Comparison of carrier frequency harmonics

<table>
<thead>
<tr>
<th>Method</th>
<th>2M_r-3 V_27 (%)</th>
<th>2M_r-1 V_29 (%)</th>
<th>2M_r+1 V_31 (%)</th>
<th>2M_r+3 V_33 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDA</td>
<td>27.20</td>
<td>51.48</td>
<td>43.41</td>
<td>9.63</td>
</tr>
<tr>
<td>FDA</td>
<td>17.55</td>
<td>38.92</td>
<td>38.86</td>
<td>17.55</td>
</tr>
<tr>
<td>Experimental</td>
<td>19.20</td>
<td>40.48</td>
<td>36.41</td>
<td>13.63</td>
</tr>
</tbody>
</table>

Figure 3.14 Input current of inverter (M_a= 0.8, M_f=10, V_{dc}=300V, R=10Ω)
Figure 3.15 (a): 19\textsuperscript{th} harmonic and (b): 21\textsuperscript{st} harmonic components of SPWM inverter (M\textsubscript{a} = 0.8, M\textsubscript{r} = 10, V\textsubscript{dc} = 300V, R = 10\Omega)
3.7 SUMMARY

The approach has served to develop accurate frequency domain models of the front end rectifier and PWM inverter. The importance of switching functions has been illustrated through the analysis. The scheme has created a new dimension in the harmonic analysis of power converters. The results show that TDA is very accurate and reflects the circuit behaviour right from the first cycle of its working. The frequency domain modelling has highlighted a technique by which the linear operating range of the PWM inverters can be identified. The close comparison of the simulated and implemented results reveals the superiority of the proposed method. This idea will go a long way in exploring newer variable speed techniques suitable for AC drives to meet state-of-the-art applications.