CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

In this thesis, a new framework for image denoising using a multiscale image representation scheme namely fast multiscale directional filter banks (FMDFB) is proposed. This chapter presents a general framework of multiscale transform domain denoising, comprehends the methodology used in this work namely wavelet, contourlet, and FMDFB, and discusses various image quality assessment metrics that are used in the proposed work.

3.2 GENERAL IMAGE denoising FRAMEWORK USING MULTISCALE TRANSFORMS

Image denoising is a process of mitigating the noise present in the images. The denoising process can be considered as an estimation problem that estimates the original image from its noisy observation. In recent years, multiscale transform domain denoising is an attractive tool for denoising, as it is computationally and qualitatively efficient, compared to the spatial and Fourier domain denoising techniques. The general multiscale image denoising framework is shown in Figure 3.1, and undergoes the following steps.

**Step 1:** Read the noisy image.

**Step 2:** Decompose the noisy image using the forward multiscale transform.

**Step 3:** Estimate the threshold value to eliminate noisy coefficients.
Step 4: Threshold the transform coefficients based on the threshold value, using hard or soft threshold method.

Step 5: Reconstruct the thresholded transform coefficients using inverse multiscale transform.

Step 6: Obtain the denoised image.

![Diagram of the denoising process](image)

Figure 3.1 General multiscale image denoising framework

3.3 WAVELET TRANSFORM

Fourier transform has been successfully used for analyzing stationary signals. Besides, Fourier transform loses the time information (Boggess & Narcowich 2009) that is more useful in many contexts, when it analyzes a non-stationary signal. Wavelet transform is a mathematical tool that is utilized for various applications in many fields, especially in engineering and technology (Akansu et al 2010). The literal meaning of the word ‘wavelet’ is ‘a small wave’ (i.e., wavelet is an oscillation that decays quickly). The necessary conditions to be satisfied by a wavelet function are as follows.

- Finite energy
- Zero mean or local
- Admissibility
These conditions are mathematically expressed in Equations (3.1), (3.2), and (3.3).

\[
\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty \quad (3.1)
\]

\[
\int_{-\infty}^{\infty} |\psi(t)| dt = 0 \quad (3.2)
\]

\[
\int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \quad (3.3)
\]

where \(\hat{\psi}(\omega)\) is the Fourier transform of \(\psi(t)\). Wavelets are family of functions that are derived from scaling and shifting of a single function called mother wavelet that is mathematically expressed as Equation (3.4).

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \quad a,b \in \mathbb{R}, a \neq 0 \quad (3.4)
\]

where

- a  – Scale parameter that measures the degree of compression
- b  – Shift parameter that determines the time location of the derived wavelet

If \(|a|<1\), then wavelet function offers smaller support to time domain; else if \(|a|>1\), then it offers a larger support in time domain. The former case corresponds to higher frequencies while the latter corresponds to lower frequencies. Thus, the wavelet functions have time domain support,
adapted to their frequencies. Wavelet functions can effectively isolate the fine
details as the basis function is flexible in terms of scaling and shifting.

The continuous wavelet transform (CWT) of a continuous, square
integrable function \( x(t) \), at a scaling parameter \( a > 0 \), and shift parameter \( b \in \mathbb{R} \)
is denoted as \( X_w(a,b) \) and expressed as in Equation (3.5).

\[
X_w(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) \psi^*(t) \left( \frac{t-b}{a} \right) dt
\]  

(3.5)

where

\( X_w(a,b) \) – CWT of \( x(t) \)
\( \psi^*(t) \) – Complex conjugate of mother wavelet function \( \psi(t) \).

The inverse continuous wavelet transform is expressed by Equation (3.6).

\[
x(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{a^2} X_w(a,b) \frac{1}{\sqrt{|a|}} \tilde{\psi} \left( \frac{t-b}{a} \right) db \, da
\]  

(3.6)

CWT is practically useful in detecting the hidden transients of the
signal, as it is very redundant and can represent fine details. The major
drawback of CWT is its high computational complexity. Typical examples of
CWT are Mexican hat wavelet, Morlet wavelet (Soman et al 2010), etc.

Discrete version of the wavelet function, called discrete wavelet
transform (DWT), is derived by sampling the wavelet function in the dyadic
grid. In general, the DWT can be constructed from two functions namely,
scaling and wavelet functions that correspond to lowpass and highpass filters,
respectively. The DWT decomposes the discrete signal \( x[n] \) into different
frequency bands by successive highpass and lowpass filtering that results in
coarse coefficients and detail coefficients, respectively.
The signal $x[n]$ is passed through a half-band lowpass filter $g[n]$ and a highpass filter $h[n]$ (Polikar 1996). This results in one-level wavelet decomposition and can mathematically be expressed as in Equations (3.7) and (3.8).

\[
y_{l}[k] = \sum_{n} x[n] \cdot g[2k-n]
\] (3.7)

\[
y_{h}[k] = \sum_{n} x[n] \cdot h[2k-n]
\] (3.8)

where

$y_{l}[k]$ – Output of the lowpass filter after sub-sampling by a factor 2

$y_{h}[k]$ – Output of the highpass filters after sub-sampling by a factor 2

This process is called subband coding that halves the time resolution and doubles the frequency resolution. This can be repeated for further higher levels of decomposition. The highpass and lowpass filters can be derived from one another using Equation (3.9).

\[
h[L-1-n] = (-1)^{n} \cdot g[n]
\] (3.9)

where

$g[n]$ – Coefficients of the lowpass filter

$h[n]$ – Coefficients of the highpass filter

$L$ – Length of the filter.

To reconstruct the signal from the wavelet coefficients, initially the signal is upsamples by the factor 2 at each level. Then, it is passed through the lowpass $g^*[n]$ and highpass $h^*[n]$ synthesis filters, and the resultant
signals are added. The analysis and synthesis filters are time reversal to one another (Polikar 1996). Hence, the reconstructed signal $x_0[n]$ is expressed as in Equation (3.10).

$$x_0[n] = \sum_{k=-\infty}^{\infty} (y_h[k] \cdot h[-n + 2k]) + (y_l[k] \cdot g[-n + 2k]) \quad (3.10)$$

Perfect reconstruction is not possible, if the $h[n]$ and $g[n]$ are not ideal half-band filters. A reasonable number of wavelet families have been proposed in wavelet history. This includes the very old Haar wavelet, the most famous Daubechies wavelets, Symlets, Coiflet (Soman et al 2010), etc.

### 3.3.1 Multiresolution Analysis (MRA) and Filter Structure

The concept of multiresolution analysis (MRA) is the backbone of wavelet transform. Multiresolution analysis is viewed as analyzing the signal at different resolutions. The fundamental fact of multiresolution analysis is that, each frequency component is resolved differently unlike Fourier representation (Daubechies 1990). The multiresolution analysis offers poor frequency resolution and good time resolution at high frequencies and offers poor time resolution and good frequency resolution at low frequencies (Polikar 1996). This is useful when the signal has low-frequency components for long duration and high frequency components for short duration. Generally, the real-time signals are rich in low-frequency components and they have less high-frequency components. Multiresolution analysis is helpful to analyze the noisy components in denoising applications, because the noisy components occur in real-world signals and are attributed to high frequency. The multiresolution versions of a signal of interest are derived using the downsampling operator in the analysis filter. In the synthesis filter structure,
the original resolution is achieved with the help of the upsampling operator. The analysis and synthesis filter structure of one-stage one-dimensional discrete wavelet transform (1D DWT) is shown in Figure 3.2, where

- \( x[n] \) – Input image
- \( g[n] \) – Coefficients of the analysis lowpass filter
- \( h[n] \) – Coefficients of the analysis highpass filter
- \( \downarrow 2, \uparrow 2 \) – Downsampling and upsampling by a factor 2
- \( y_l[n] \) – Lowpass subband coefficients
- \( y_h[n] \) – Highpass subband coefficients
- \( g^*[n] \) – Coefficients of the synthesis lowpass filter
- \( h^*[n] \) – Coefficients of the synthesis highpass filter
- \( x_0[n] \) – Reconstructed image

![Figure 3.2 One-stage 1D DWT](image)

The multiresolution analysis can be extended to signals of higher dimensions. For example, wavelet decomposition of an image of size \( N \times N \) would result in an approximation subband and three detail subbands, each of size \( N/2 \times N/2 \). And the subbands resulting from decomposition of approximation subband of size \( N/2 \times N/2 \) would result in an approximation subband and three detail subbands, each of size \( N/4 \times N/4 \). The process of 2D wavelet decomposition is further addressed in Chapter 4. The major
advantage of multiresolution analysis is that, the total number of wavelet coefficients is the same as the size of the original image for any levels of multiresolution decomposition. This ensures the critical sampling property of wavelets. The wavelet basis functions of fixed square shape efficiently represent the discontinuities in the images such as edges. But, they do not consider the discontinuity points situated over smooth contours such as edges and boundaries of objects in natural images, thereby capture only limited directional information (Do & Vetterli 2005). This impelled the evolution of multiscale transforms that are highly directional as discussed in the successive sections.

3.4 THE CONTOURLET TRANSFORM

Wavelet transform is good in detecting point discontinuities, but the natural images have such point discontinuities arranged along smooth contours (e.g., edges). Wavelet cannot link these point discontinuities to represent a meaningful shape; thus, it is not capable of representing edges that are high-frequency components effectively. In fact, wavelet transform decomposes the high-frequency components into only three directional subbands (horizontal, vertical, and 45° diagonal). The edges in these high-frequency subbands can be seen, yet the directional information of the edges cannot be seen from the high-frequency subbands. To overcome this issue (Do & Vetterli 2005), more number of wavelet coefficients at different scales are required to represent and to reconstruct the edges without loss (Masaebi & Moghaddam 2012). This led to the evolution of the efficient multiscale image representation schemes that are local, directional, and support multiresolution expansion.
According to Do & Martin Vetterli (2005), a multiscale image representation scheme must possess the following properties.

(i) **Multiresolution:** The representation should allow images to be successively approximated, from coarse to fine resolutions.

(ii) **Localization:** The basis elements in the representation should be localized in both the spatial and the frequency domains.

(iii) **Critical sampling:** For some applications (e.g., compression), the representation should form a basis, or a frame with small redundancy.

(iv) **Directionality:** The representation should contain basis elements oriented at a variety of directions, much more than the few directions that are offered by separable wavelets.

(v) **Anisotropy:** To capture smooth contours in images, the representation should contain basis elements using a variety of elongated shapes with different aspect ratios.

Separable wavelet representation is local, critically sampled, and supports multiresolution analysis, but has limited directionality and fixed anisotropy. This led to the development of an efficient multiscale image representation scheme namely, pyramidal directional filter banks or the contourlet transform (Do & Vetterli 2005). This is an iterated filter bank structure as shown in Figure 3.3 that uses a pyramidal transform namely, Laplacian pyramid (LP) for scale decomposition and a directional transform namely directional filter banks (DFB) for directional decomposition that links point discontinuities to a meaningful shape. The major advantage of contourlet transform is that, it allows the directional decomposition of high-frequency components at each scale. It offers perfect reconstruction if the
filters used in LP and DFB are perfect reconstruction filters. The contourlet transform allows a different number of directions at different scales. Hence, it offers a flexible multiscale and directional expansion of an image. However, it has a small redundancy ratio, which is less than 4/3. The basic building blocks of contourlet transform namely LP and DFB are explained in the following subsections.

![Figure 3.3 Iterated filter bank structure of the contourlet transform](image)

**3.4.1 The Laplacian Pyramid**

An efficient way of obtaining multiscale decomposition is to use a pyramidal structure (Burt & Adelson 1983) called Laplacian pyramid (LP). It is an overcomplete decomposition in which input image is represented by a set of bandpass images and a lowpass image. The theory behind the construction of LP is borrowed from Gaussian pyramid. Gaussian pyramid (GP) of an image is constructed by successively lowpass filtering with the Gaussian kernel and downsampling the lowpass filtered image. As a result, an image is represented with a set of lowpass images $G_0$, $G_1$, $G_2$, ..., $G_n$ at different resolutions as shown in Figure 3.4. The relationship between Gaussian and Laplacian pyramids is shown in Figure 3.5. As observed from
Figure 3.5, the GP shows high degree of correlation among the lowpass images $G_0$, $G_1$, $G_2$, …, $G_n$. This is not a desirable characteristic of a compact image representation.

The LP eliminates this problem by representing the image as a set of bandpass images and a lowpass image (Burt & Adelson 1983). The image ($G_0$) is first lowpass filtered to get the reduced (decimated/downsamled) version $G_1$ called coarse approximation as in the case of Gaussian pyramid. Then, $G_1$ is expanded (interpolated / upsampled) to get $E_1$, and the prediction $L_1$ is obtained by subtracting $E_1$ from $G_1$.

![Diagram of Gaussian pyramid and Laplacian pyramid](image)

**Figure 3.4** (a) Structure of Gaussian pyramid (b) Structure of Laplacian pyramid

This process can be iterated on the lowpass image upto the desired level of decomposition to generate other bandpass images in lower frequency range. At each level of decomposition, the Laplacian prediction is computed as $L_n = G_n - E_n$. These $L_n$ are the set of bandpass images that constitute the Laplacian pyramid. It is worth mentioning that at each level of decomposition, the image resolution is reduced by a factor 4.
The major drawback of LP is the implicit oversampling. A block diagram of single-stage LP decomposition is shown in Figure 3.6. An interesting feature of the LP is that decimation is performed only on the lowpass image. Thus, there is no frequency scrambling in LP unlike separable wavelets. The directional decomposition is performed on the bandpass images at each scale (Do & Vetterli 2005). Single-stage analysis and synthesis structure of the LP are shown in Figure 3.6,

where

\[ h_L[n] \] - Lowpass analysis filter
\[ g_L[n] \] - Lowpass synthesis filter
\[ \downarrow 2 \] - Downsampling a factor of 2
\[ \uparrow 2 \] - Upsampling by a factor of 2
Figure 3.6 Single-stage analysis and synthesis structure of the LP

The reconstruction of the LP is straightforward as shown in Figure 3.6. In order to avoid aliasing in the LP, the stopband edge of $h_L[n]$ should be less than $\pi/2$. In order to satisfy the equiripple criteria, $h_L[n]$ with passband edge 0.3\pi and stopband edge 0.5\pi are designed (Cheng et al 2007) (Do & Vetterli 2003). Furthermore, the LP is designed with orthogonal filters that is $h_L[n] = g_L[-n]$ to ensure reconstruction without loss.

3.4.2 Directional Filter Bank

Directional filter bank (DFB) was introduced by Bamberger and Smith (1992). It is a 2D directional filter bank that performs directional decomposition with perfect reconstruction and maximally decimated property. The DFB partitions a frequency plane into a set of wedge-shaped regions as illustrated in Figure 3.7.

Figure 3.7 Frequency plane partitioning in a three-level DFB
Figure 3.8 Two-channel DFB

It is implemented efficiently in an ‘l’-level tree structure that results in $2^l$ directional subbands. The tree structure is implemented with a two-channel filter bank in which a complementary diamond-shaped filter pair is followed by a quincunx downsampling as shown in Figure 3.8 (Do 2002, Cheng et al 2007a),

where

- $H_0, H_1$ – Lowpass filters
- $R_i$ – Resampling matrix
- $Q_k$ – Quincunx downsampling

A resampler is employed before the two-channel filter bank. Its function is to shear the desired frequency partitions into diamond shape so that the two-channel filter bank can give the desired frequency bands.

3.4.2.1 Resampling

The definition of resampling matrix is that it is a 2×2 matrix whose entries are all integers and whose determinant is non-zero so that its inverse matrix is also a resampling matrix. It is a unimodular matrix that can change the diamond-shaped passband into a parallelogram passband. A unimodular matrix is a matrix whose determinant is ±1. Its inverse is also unimodular. Many resampling matrices have been proposed in the literature (Bamberger &
Smith 1992, Do 2002). Any one of the following (Equation 3.11) can be used according to the required frequency band.

\[
\begin{align*}
R_1 &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} & R_2 &= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} & R_3 &= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} & R_4 &= \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}
\end{align*}
\]  

(3.11)

As shown in Figure 3.9, for \( R_1 \) and \( R_2 \), the input image is extended along the vertical direction; while for \( R_3 \) and \( R_4 \), the image is extended along the horizontal direction.

![Figure 3.9 Parallelogram-shaped passbands produced by \( R_1 \), \( R_2 \), \( R_3 \), and \( R_4 \), respectively](image)

**3.4.2.2 Diamond-shaped filters**

Diamond-shaped filter pair splits the frequency spectrum of the input signal into lowpass and highpass channels (Bamberger & Smith 1992). Figure 3.10 shows the frequency spectrum of diamond-shaped filter. One filter pair can be derived from the other by simply modulating the filters by \( \Pi \) in either the \( \omega_1 \) or \( \omega_2 \) frequency variable. Perfect reconstruction is achieved by applying the same modulation to both analysis and synthesis filters. Because of sampling, subbands would suffer from spatial distortion. This problem can be solved by adopting backsampling at the output of the DFB (Do 2002).
The overall sampling matrix is given by Equation (3.12) and (3.13).

\[
S = \begin{cases} 
D_0^{-2} & \text{in } R_0 \\
D_i^{-2} & \text{in } R_i 
\end{cases} 
\]  
(3.12)

\[
D_0 = \begin{pmatrix} 2 & 0 \\
0 & 1 
\end{pmatrix} 
D_1 = \begin{pmatrix} 1 & 0 \\
0 & 2 
\end{pmatrix} 
\]  
(3.13)

where

\( i \) – Index of output stage of DFB

\( R_j \) – \( j=0, 1 \) are the spectral regions shown in Figure 3.11

\( D_0, D_1 \) – Downsampling matrices defined by Equation (3.13)

The backsampling reorders the subbands so that the overall sampling is diagonal.
3.4.2.3 Quincunx downsampling

The quincunx downsampling matrix denoted as $Q_k$ in Figure 3.8 is a generalized downsampling matrix whose entries are $\pm 1$ with determinant 2. The resampling matrices are used to perform the shearing operation (Do 2002). The quincunx matrices that can be used in directional filter banks are derived as in Equations (3.14) and (3.15).

$$Q_1 = R_2 D_0 R_3 = R_3 D_1 R_2$$ (3.14)

$$Q_2 = R_1 D_0 R_4 = R_4 D_1 R_1$$ (3.15)

Of the above four quincunx sampling matrices, anyone can be used for downsampling. These sampling matrices generate the same sublattice but the downsampling operation rotates the input image by $-45^\circ$ and $45^\circ$. Quincunx down sampling results in downsampled and rotated representations as shown in Figure 3.12.

**Figure 3.12** (a) Example of quincunx downsampling lattice in $(n_1, n_2)$ space (b) An input image (c) and (d) Quincunx downsampled outputs by $Q_1$ and $Q_2$, respectively
As observed from Figure 3.3, the DFB is designed to handle the high-frequency components effectively, but it does not handle low-frequency components. This causes leaking of some low frequents into bandpass subbands. This is the reason why DFB is applied only on the bandpass image. The corresponding lowpass components are removed by a multiscale decomposition (Laplacian pyramid) before applying DFB (Do & Vetterli 2005).

### 3.5 MULTISCALE DIRECTIONAL FILTER BANKS (MDFB)

The MDFB (Cheng et al 2007b) alters the pyramidal directional filter bank (PDFB) or contourlet transform by introducing scale decomposition in its finest scale (Chan et al 2003). It contains two basic building blocks namely LP and DFB. Scale decomposition is carried out by performing low pass filtering with cut-off frequency $0.75\pi$ to an input image.

![Figure 3.13 Structure of MDFB](image-url)
First scale is generated by subtracting the lowpass image from the input image. Second scale is obtained as the first bandpass image in LP. Third scale is the second bandpass image in LP and so on.

Figure 3.13 shows the structure of MDFB,

where

\[ h_0[n], h_1[n] \quad – \quad \text{Analysis lowpass filters of LP} \]
\[ g_0[n], g_1[n] \quad – \quad \text{Synthesis lowpass filters of LP} \]
\[ h[n] \quad – \quad \text{Lowpass filter with cut off frequency } 0.75\pi \]

### 3.6 FAST MULTISCALE DIRECTIONAL FILTER BANKS (FMDFB)

The MDFB improves the radial frequency resolution of the contourlet transform. Higher frequency resolution is achieved at the expense of computational complexity (Cheng et al 2007b) that is incurred due to extra scale and directional decompositions. Cheng et al (2007a) proposed a fast structure for this MDFB named as fast multiscale directional filter banks (FMDFB) as shown in Figure 3.14,

where

\[ h_L, g_L \quad – \quad \text{Lowpass and highpass filters LP} \]
\[ \text{DFB} \quad – \quad \text{Directional filter bank} \]
\[ B_i \quad – \quad \text{Backsampling} \]
\[ B_i^{-1} \quad – \quad \text{Inverse backsampling} \]
\[ \downarrow 2 \quad – \quad \text{Downsampling a factor of 2} \]
\[ \uparrow 2 \quad – \quad \text{Upsampling by a factor of 2} \]
The basic building blocks of FMDFB are the same as that of MDFB namely, LP and DFB (Cheng et al 2007a). In the analysis filter bank structure, lowpass filtering and wavelet transform technique are applied for splitting the image into various scales. Usage of non-aliasing lowpass filters increases the effective bandwidth of the finer scale.

Figure 3.14 Structure for FMDFB (a) Analysis filter structure (b) synthesis filter structure
Directional decomposition with lower angular resolution is performed before scale decomposition. Hence, one set of operations for directional decomposition with lower angular resolution is saved by sharing between directional and scale decompositions.

As stated by Cheng et al (2007a), perfect reconstruction is always possible at all scales regardless of the lowpass filters that are used for decomposition. This FMDFB achieved 33.5%–37.5% of reduction in computational complexity when compared to original MDFB. Figure 3.15 shows the frequency resolution of contourlet, MDFB, and FMDFB where $\omega_1$ and $\omega_2$ are frequency variables. Moreover, the total number of directional subband coefficients is equal to the size of the original image, thereby maintaining the maximally decimated property.

![Figure 3.15 Frequency partitioning (a) contourlet (b) MDFB and (c) FMDFB](image)

**Figure 3.15** Frequency partitioning (a) contourlet (b) MDFB and (c) FMDFB

### 3.7 IMAGE QUALITY ASSESSMENT (IQA) METRICS

Image quality assessment (IQA) metrics are in practice to evaluate the quality of the processed image. On the other hand, the IQA metrics aid to quantify the effectiveness of the image processing algorithm which is
employed. The IQA is broadly classified as subjective and objective (Seshadrinathan et al 2010) image quality assessment methods.

The subjective IQA is based on human visual perception (He et al 2009). It requires one or more assessors (Moorthy et al 2013) to judge the image quality. Subjective IQA methods are accurate in determining how far the image distortion can be perceived. Conversely, the subjective IQA methods are expensive and time consuming (Sheikh et al 2006) and hence they are not suitable for real time image quality assessment. The objective IQA methods operate directly on the intensity of the image. They are expressed with the straightforward mathematical equations and they are easy to calculate. Moreover, objective IQA metrics such as structural similarity index (SSIM) and feature similarity index (FSIM) are consistent (Zhang et al 2012) with the subjective IQA scores.

This proposed work uses the objective image quality measurements such as mean square error (MSE), peak signal-to-noise ratio (PSNR), image enhancement factor (IEF) (Esakkirajan et al 2011), structural similarity index (SSIM) (Wang et al 2004), and feature similarity index (FSIM) (Zhang et al 2011) to evaluate the quality of the processed image.

3.7.1 Mean Square Error (MSE)

The most common and frequently used image quality measures show deviations between the original and processed images such as mean square error (MSE) and signal to noise ratio (SNR). The effectiveness of the algorithm stands in minimizing the mean square error. The MSE is calculated as expressed in Equation (3.16).

\[
MSE = \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} (F(x,y) - I(x,y))^2
\]  

(3.16)
where

\[
\begin{align*}
F(x, y) & \quad \text{Original clean image} \\
I(x, y) & \quad \text{Denoised image} \\
(x, y) & \quad \text{Spatial coordinates of the image} \\
M, N & \quad \text{Number of rows and columns in the image}
\end{align*}
\]

### 3.7.2 Peak Signal-to-Noise Ratio (PSNR)

Larger PSNR indicates a smaller difference between the original uncorrupted image and the denoised image. This is the most widely used objective image quality/distortion measure. The main advantage of this measure is ease of computation. The PSNR is calculated as expressed in Equation (3.17).

\[
PSNR = 20 \log_{10} \left( \frac{F_{\text{max}}}{\sqrt{MSE}} \right)
\tag{3.17}
\]

where

\[
F_{\text{max}} = 255 \text{ for an 8-bit image.}
\]

### 3.7.3 Image Enhancement Factor (IEF)

IEF given in Equation (3.18) is a statistical approach to measure the effectiveness of denoising (Esakkirajan et al. 2011). The higher the IEF, the better is the denoised image.

\[
IEF = \left[ \frac{1}{M \times N} \sum_{x=1}^{M} \sum_{y=1}^{N} (G(x, y) - F(x, y))^2 \right]^{\frac{1}{2}}
\tag{3.18}
\]

\[
IEF = \left[ \frac{1}{M \times N} \sum_{x=1}^{M} \sum_{y=1}^{N} (I(x, y) - F(x, y))^2 \right]^{\frac{1}{2}}
\]
where

\[ F(x,y) \quad \text{– Original image} \]
\[ G(x,y) \quad \text{– Noisy image} \]
\[ I(x,y) \quad \text{– Denoised image} \]

### 3.7.4 Structural Similarity Index (SSIM)

Structural similarity provides an alternative and complementary approach to image quality assessment. It is based on the assumption that the human visual system (HVS) is greatly adapted for extracting structural information from the image and a measure of structural similarity need to be a good estimate of perceived image quality. Multiscale structural similarity method (Wang et al 2003, Wang et al 2004) is an image synthesis-based approach to adjust the parameters that weight the relative significance among different scales. The original image is designated as scale one and the highest scale is M. For two images \( x_1 \) and \( x_2 \), the luminescence comparison \( I(x_1, x_2) \) is calculated only at scale M. At \( j^{th} \) scale, the contrast \( c(x_1, x_2) \) and structural \( s(x_1, x_2) \) comparisons are calculated using the Equations (3.19) – (3.21).

\[
I_M(x_1, x_2) = \frac{\mu x_1 \mu x_2 + C_1}{\mu x_1^2 + \mu x_2^2 + C_1} \quad (3.19)
\]

\[
c_j(x_1, x_2) = \frac{2 \sigma x_1 \sigma x_2 + C_2}{\sigma x_1^2 + \sigma x_2^2 + C_2} \quad (3.20)
\]

\[
s_j(x_1, x_2) = \frac{\sigma x_1 x_2 + C_3}{\sigma x_1 \sigma x_2 + C_3} \quad (3.21)
\]
where
\[ C_1 = (K_1 L)^2 \]
\[ C_2 = (K_2 L)^2 \]
\[ C_3 = \frac{C_2}{2}. \]

L – Dynamic range of the image

K_1, K_2 – Scalar constants equal to 0.01 and 0.03, respectively

\( \mu_{x_1}, \mu_{x_2} \) – Mean of the images \( x_1 \) and \( x_2 \)

\( \sigma_{x_1}, \sigma_{x_2} \) – Standard deviation of the images \( x_1 \) and \( x_2 \)

The multiscale SSIM is calculated as expressed in Equation (3.20)
where \( \alpha > 0, \beta > 0, \gamma > 0. \)

\[
SSIM(x, y) = \left[ I_M(x, y) \right]^{\alpha} \prod_{j=1}^{M} \left[ c_j(x, y) \right]^{\beta} \left[ s_j(x, y) \right]^{\gamma}
\]

Generally, \( \alpha = \beta = \gamma = 1 \) is used to balance the relative significance of the three components \( l, c, \) and \( s. \)

3.7.5 Feature Similarity Index (FSIM)

It is a full reference IQA proposed based on the fact that human visual system (HVS) understands an image mainly according to its low-level features. FSIM uses the phase congruency (PC) that is a measure of local structure as primary feature, and the image gradient magnitude (GM) as the secondary feature in FSIM. Again, PC is used as a weighting function to derive a single quality score (Zhang et al 2011) equated in Equations (3.21) and (3.22).
\[ FSIM = \frac{\sum_{x \in \Omega} S(x) PC_m(x)}{\sum_{x \in \Omega} PC_m(x)} \]  

(3.21)

\[ PC_m(x) = \max(PC_1(x), PC_2(x)) \]  

(3.22)

where

- \( PC_1(x) \) – Phase congruency of the image \( x_1 \)
- \( PC_2(x) \) – Phase congruency of the image \( x_2 \)
- \( PC_m(x) \) – Maximum phase congruency of the images \( x_1 \) and \( x_2 \)
- \( S(x) \) – Similarity between images \( x_1 \) and \( x_2 \) at a position \( x \).
- \( \Omega \) – Whole image in spatial domain

### 3.8 CONCLUSION

In this chapter, the general framework of multiscale transform domain denoising is presented. The methodology (computational tools) used in this thesis namely wavelet, contourlet, and FMDFB are discussed in detail. Further, various image quality assessment metrics used in this work such as MSE, PSNR, IEF, SSIM, and FSIM are also discussed.