Chapter 3

Flowshop Scheduling with Loading and Unloading Time for Transport Agent between Two Machines

3.1 Introduction

The concept of transportation time was introduced by Maggu and Das [44], to the problem of two machines in tandem. They studied a system in which an infinite number of transport agents were available and no transport agent was required to return to stage 1 from stage 2. Later on, Khan et al. [40] further introduced the concept of a single transport agent. Aziz and Singh [6] further worked on the problem of machine tandems and item blocks with a single transport agent in between. They also considered the cases of high inventory costs between the two machines and further restrictions on input. The case was also generalised to the problem of m-machines and n-jobs.

Here, we consider the case of two machines in tandem where only a single transport agent is available who, after delivering the items to machine 2 has to come back to machine 1 for transporting the next item. We assume that machine 1 starts processing the next item immediately after finishing the preceding one. We have also considered the loading and unloading times for transport agent along with transportation times. The concept of loading and unloading times is sometimes very important visually in automobile sector where logistics done at a different location are to be brought back for assembling. We have considered four types of times – loading time, transportation time, unloading time and
returning time. An algorithm has been developed for finding the optimal schedule.

3.2 Problem Description

Let us consider \( n \) items \((I_1, I_2, \ldots, I_n)\) being processed through two machines \((A\) and \(B)\) in the order \(AB\) with a single transport agent available, who transports an item processed at machine \(A\) to the machine \(B\) and returns back empty to \(A\) to transport the second item to \(B\) and so on until all the items are taken to machine \(B\). Let an item \(i\) to be transported by agent takes loading and unloading times \(l_i\) and \(u_i\) respectively. Let \(t_i\) be the transportation time for item \(i\) to be carried from machine \(A\) to \(B\); \(A_i\) & \(B_i\) are the service times of \(i^{th}\) item on machines \(A\) and \(B\) respectively, and \(r_i\) is the returning time of transport agent from machine \(B\) to \(A\) after delivering item \(i\) at \(B\). The time by which the transport agent finishes with item \(i-1\), the job of \(i^{th}\) item on machine \(A\) may or may not get finished and the machine \(A\) after processing item \(i-1\) immediately takes up item \(i\) for processing so the item \(i\) will wait for transport agent if it is not returned back by that time. The problem is to find an optimal schedule of items so as to minimize the total production time for completing all the items.

3.3 Development of Procedure

We provide the procedure for an optimal schedule with the help of following theorem:
Theorem An optimal sequence is obtained by sequencing the item \( i - 1, i, i + 1 \) such that

\[
\min \{A_i + l_i + t_i + u_i + R_{i-1}, B_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i\} < \min \{A_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i, B_i + l_i + t_i + u_i + R_{i-1}\}
\]

where \( R_{i-1} = \begin{cases} l_{i-1} + t_{i-1} + u_{i-1} + r_{i-1} - A_i, & \text{if it is positive;} \\ 0, & \text{otherwise.} \end{cases} \)

Proof: Let \( S \) and \( S' \) denote the sequences of items given by :

\[
S = (I_1, I_2, \ldots, I_{i-1}, I_i, I_{i+1}, I_{i+2}, \ldots, I_n) \\
S' = (I'_1, I'_2, \ldots, I'_{i-1}, I'_i, I'_{i+1}, I'_{i+2}, \ldots, I'_n)
\]

Let \( (X_p, X'_p) \) and \( (C X_p, C' X'_p) \) be respectively the processing time and completion time of any item \( p \) on machine \( X (= A \text{ or } B) \) for the sequences \( (S, S') \). Let \( (l_p, l'_p) \) and \( (u_p, u'_p) \) respectively denote the loading and unloading times for an item to be transported by transport agent from machine \( A \) to machine \( B \) for the sequences \( (S, S') \). Let \( (t_p, t'_p) \) denotes the transportation times of item \( p \) from machine \( A \) to machine \( B \) for the sequences \( (S, S') \). \( (r_p, r'_p) \) is the returning time of the transport agent from machine \( B \) to machine \( A \) after delivering the \( p^{th} \) item at \( B \) for sequences \( (S, S') \).

Note that we have defined \( R_{p-1} = l_{p-1} + t_{p-1} + u_{p-1} + r_{p-1} - A_p \geq 0 \). The completion time of \( p^{th} \) item on machine \( B \) is given by

\[
CB_p = \max \left( CA_p + l_p + t_p + u_p + R_{p-1}, CB_{p-1} \right) + B_p \quad (3.1)
\]

We will choose the sequence \( S \) if

\[
CB_n < C'B_n \quad (3.2)
\]
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i.e., if

\[
\max \left( CA_n + l_n + t_n + u_n + R_{n-1}, CB_{n-1} \right) + B_n < \max \left( C'A_n + l'_n + t'_n + u'_n + R'_{n-1}, C'B_{n-1} \right) + B'_n
\]

As \( CA_n + l_n + t_n + u_n + R_{n-1} = \sum_{i=1}^{n} A_i + l_i + t_i + u_i + R_i = C'A_n + l'_n + t'_n + u'_n + R'_{n-1} \),

and \( B_n = B'_n \), result Eq. (3.2) will be true if:

\[
CB_{n-1} < C'B_{n-1}
\] (3.3)

Proceeding in this way we get that inequality Eq. (3.2) is true if:

\[
CB_{p} < C'B_{p} (p = i+1, i+2, \ldots, n \text{ and } i = 1, 2, \ldots n-1)
\] (3.4)

We now calculate the values of \( CB_{i+1} \) and \( C'B_{i+1} \)

\[
CB_{i+1} = \max \left\{ CA_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i, CB_i \right\} + B_{i+1}
\]

\[
= \max \left\{ CA_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i, \max \left( CA_i + l_i + t_i + u_i + R_{i-1}, CB_{i-1} \right) + B_i \right\} + B_{i+1}
\]

\[
= \max \left\{ CA_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i + B_{i+1}, CA_i + l_i + t_i + u_i + R_{i-1} + B_i + B_{i+1}, CB_{i-1} + B_i + B_{i+1} \right\}
\]

Now

\[
CB_{i+1} = \max \left\{ CA_{i-1} + A_i + A_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i + B_{i+1}, CA_{i-1} + A_i + l_i + t_i + u_i + R_{i-1} + B_{i+1}, CB_{i-1} + B_i + B_{i+1} \right\}
\] (3.5)

Similarly

\[
C'B_{i+1} = \max \left\{ C'A_{i-1} + A'_i + A'_{i+1} + l'_{i+1} + t'_{i+1} + u'_{i+1} + R'_i + B'_{i+1}, C'A_{i-1} + A'_i + l'_i + t'_i + u'_i + R'_{i-1} + B'_i + B'_{i+1}, C'B_{i-1} + B'_i + B'_{i+1} \right\}
\] (3.6)
For the sequences \( S \) and \( S' \) it is obvious that

\[
CA_{i-1} = C'A_{i-1}, CB_{i-1} = C'B_{i-1}
\]

\[
X_i = X_{i+1}'(X = A \text{ or } B); l_i = l_{i+1}'; t_i = t_{i+1}' ; u_i = u_{i+1}'
\]

\[ (3.7) \]

Writing Eq. (3.4) for \( p = i + 1 \) and using Eq. (3.7), we get

\[
\max \{CA_{i-1} + A_i + A_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i + B_{i+1}, CA_{i-1} + A_i + \\
l_i + t_i + u_i + R_{i-1} + B_i + B_{i+1} + CB_{i-1} + B_i + B_{i+1}\} < \max \{CA_{i-1} + A_i + A_{i+1} + l_i + t_i + u_i + R_{i-1} + B_i + CB_{i-1} + B_i + B_{i+1} \}
\]

\[ (3.8) \]

Subtracting \( CB_{i-1} + B_i + B_{i+1} \) from both sides, the inequality Eq. (3.8) reduces to

\[
\max \{CA_{i-1} + A_i + A_{i+1} + l_{i+1} + t_{i+1} + u_i + R_i + B_{i+1}, CA_{i-1} + A_i + \\
l_i + t_i + u_i + R_{i-1} + B_i + B_{i+1}\} < \max \{CA_{i-1} + A_i + A_{i+1} + l_i + t_i + u_i + R_{i-1} + B_i, CA_{i-1} + A_i + l_{i+1} + t_{i+1} + u_{i+1} + R_i + B_{i+1} + B_i \}
\]

\[ (3.9) \]

Further subtracting

\[ CA_{i-1} + A_i + A_{i+1} + l_i + t_i + u_i + l_{i+1} + t_{i+1} + u_{i+1} + R_{i-1} + R_i + B_i + B_{i+1} \]

from each side, we get

\[
\max \{-l_i - t_i - u_i - R_{i-1} - B_i, -A_{i+1} - l_{i+1} - t_{i+1} - u_{i+1} - R_{i+1}\} < \max \{-l_{i+1} - t_{i+1} - u_{i+1} - R_i - B_{i+1}, -A_i - l_i - t_i - u_i - R_{i-1}\}
\]

\[ (3.10) \]

\[
\min \{A_i + l_i + t_i + u_i + R_{i-1}, B_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i\} < \min \{A_{i+1} + l_{i+1} + t_{i+1} + u_{i+1} + R_i, B_i + l_i + t_i + u_i + R_{i-1}\}
\]

\[ (3.11) \]
3.3.1 Algorithm Finding an Optimal Sequence

Our problem can be represented in tableau form as follows:

<table>
<thead>
<tr>
<th>Item $i$</th>
<th>Machine $A$ ($A_i$)</th>
<th>$l_i$</th>
<th>$t_i$</th>
<th>$u_i$</th>
<th>$r_i$</th>
<th>Machine $B$ ($B_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A_1$</td>
<td>$l_1$</td>
<td>$t_1$</td>
<td>$u_1$</td>
<td>$r_1$</td>
<td>$B_1$</td>
</tr>
<tr>
<td>2</td>
<td>$A_2$</td>
<td>$l_2$</td>
<td>$t_2$</td>
<td>$u_2$</td>
<td>$r_2$</td>
<td>$B_2$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
</tr>
<tr>
<td>$n$</td>
<td>$A_n$</td>
<td>$l_n$</td>
<td>$t_n$</td>
<td>$u_n$</td>
<td>$r_n$</td>
<td>$B_n$</td>
</tr>
</tbody>
</table>

where $A_i$, $B_i$ are the service times on machines $A$ and $B$ respectively. $l_i$, $u_i$ are respectively the loading and unloading times of $i^{th}$ item to be transported. $t_i$ is the transportation time of item $i$ in transporting it from machine $A$ to $B$ and $r_i$ is the returning time of the transport agent from machine $B$ to $A$ after delivering the $i^{th}$ item on machine $B$.

The result of above theorem gives the following procedure for an optimal sequence:

Step 1: Assume there are two fictitious machines ($G$ and $H$) in place of $A$ and $B$ respectively. Assume that the service times for these fictitious machines are given by $G_i$ and $H_i$ where

$$G_i = A_i + l_i + t_i + u_i + R_{i-1}, \quad H_i = B_i + l_i + t_i + u_i + R_{i-1}$$

Step 2: Applying Johnson’s (1954) rule to the fictitious machine times $G_i$ and $H_i$ constructed in step 1, we obtain the optimal sequence.
3.4 Example  Let a machine tandem queuing problem be given in the following tableau form:

<table>
<thead>
<tr>
<th>Item ( i )</th>
<th>Machine ( A ) ( (A_i) )</th>
<th>( l_i )</th>
<th>( t_i )</th>
<th>( u_i )</th>
<th>( r_i )</th>
<th>( R_{i-1} )</th>
<th>Machine ( B ) ( (B_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

**Solution:** Let \( G \) and \( H \) be two fictitious machines representing \( A \) and \( B \) respectively and \( G_i \) and \( H_i \) be the service times of \( G \) and \( H \). Then our reduced problem is

<table>
<thead>
<tr>
<th>Item ( i )</th>
<th>Machine ( G ) ( (G_i = A_i + l_i + t_i + u_i + R_{i-1}) )</th>
<th>Machine ( H ) ( (H_i = B_i + l_i + t_i + u_i + R_{i-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>22</td>
</tr>
</tbody>
</table>

By Johnson’s rule to the above reduced times, the optimal sequence is 1, 5, 3, 4, 2. The minimum total production time is calculated as follows:

\[
Y_{i-1} \quad \text{represents the time at which the transport agent returns to machine} \ A \ \text{to take the next item.}
\]

\[
Y_{i-1} = CA_i + l_i + t_i + u_i + r_i
\]
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<table>
<thead>
<tr>
<th>Item $i$</th>
<th>Machine $A$</th>
<th>$l_i$</th>
<th>$t_i$</th>
<th>$u_i$</th>
<th>$r_i$</th>
<th>$Y_{i-1}$</th>
<th>Machine $B$</th>
<th>Idle Time $A$</th>
<th>Idle Time $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-7</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>-</td>
<td>14-23</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>7-15</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>28-34</td>
<td>0</td>
<td>05</td>
</tr>
<tr>
<td>3</td>
<td>15-24</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>39-43</td>
<td>0</td>
<td>05</td>
</tr>
<tr>
<td>4</td>
<td>24-43</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>43</td>
<td>52-59</td>
<td>0</td>
<td>09</td>
</tr>
<tr>
<td>2</td>
<td>43-59</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>55</td>
<td>69-74</td>
<td>15</td>
<td>10</td>
</tr>
</tbody>
</table>

The total processing time of all the items through the system (i.e., total production time) is 74hrs. Idle time for machine $A$ is 15 hrs. for $B$ it is 43 hrs. & for the agent is 11 hrs. So machine $A$ is busy 79.72% of time, $B$ is busy 41.89% of time and the transport agent is busy 85.13% of time.