PREFACE

The numerical approximation of solutions of ordinary differential equations played an important role in Numerical Analysis and still continues to be an active field of research. This is mainly due to the pressure of needs to model mathematically real world phenomena. Most of the phenomena in nature can be successfully described by means of ordinary or partial differential equations. The analytical solution of these differential equations may or may not be known. Numerical techniques for solving differential equations occurring in scientific and engineering problems are not only feasible but also very desirable. One of the reasons is that numerical methods can give the solution when analytical methods fail.

The present thesis titled “Numerical Solution of Two Point Boundary Value Problems” is devoted to the study of the non polynomial splines consisting of five chapters including introduction. The choice of the contents has been made keeping in view, their utility and relevance to the problems of numerical solution of the differential equations. We have developed the non polynomial spline function approximation methods to obtain numerical solution of ordinary differential equations. A brief description of the contents of each chapter is as follows:

Chapter 1 contains general introduction of two point boundary value problems and the numerical techniques to solve the differential equations. Some useful properties of band matrices are given that are concerned with the convergence of spline methods. A brief introduction of singularly perturbation problems, fourth order two point
boundary value problem and system of fourth order boundary value problem is also given.

In Chapter 2, the difference scheme using adaptive cubic spline is presented for solving a self-adjoint singularly perturbed two point boundary value problem of the form

$$Lu = \varepsilon u'' - q(x)u = r(x),$$

$$u(a) = \alpha_0, \quad u(b) = \alpha_1,$$

where, $q(x), r(x)$ are smooth and bounded real functions, $\alpha_0$ and $\alpha_1$ are given constants and $\varepsilon$ is a parameter such that $0 < \varepsilon \ll 1$. Our scheme leads to a tri-diagonal linear system. The convergence analysis is given which shows the method is second and fourth order convergent depending upon the choice of parameters $A_1, A_2, A_3$ and $A_4$. Numerical illustrations are given to verify the theoretical analysis of our methods.

In Chapter 3, parametric quintic spline is used for the solution of singularly perturbed two point boundary value problems. Such kind of problems arises in various fields of engineering and science, for example elasticity, fluid dynamics, optimal control theory, hydrodynamics etc. Methods of orders two, four and six are obtained which lead to five-diagonal linear system. The convergence analysis is also given.

In Chapter 4, we develop a class of methods based on non-polynomial quintic spline at off-step points for the solution of linear fourth order boundary value problems. Spline relations and boundary conditions are developed and convergence analysis of
the method is discussed. Numerical examples are given to illustrate the superiority of our methods over other exiting methods. Such kind of problems arises in plate deflection theory. Elastic beam is one of the most commonly used elements in structures of aircrafts, building ships and bridges. Deflection of beam under certain load can be modeled in the form of fourth order two point boundary value problems.

In Chapter 5, an exponential quintic spline at mid knots is used for approximating the solution of system of fourth order boundary value problems associated with obstacle, unilateral and contact problems. These problems can be characterized by a sequence of boundary value problems using penalty function technique. The present technique develops a class of methods of order two, four and six. Two numerical examples are considered for the numerical illustration of the proposed method. It is shown that the method developed in this chapter is more efficient than the other finite difference, collocation and spline methods.

At last, a comprehensive and brief bibliography has been given which covers the references of the present thesis.