CHAPTER II

EQUATIONS OF MOTION AND ENERGETICS FOR THE PERTURBATIONS

Any atmospheric circulation system, whether it be a small scale convection cell, a cyclone or a large scale zonal wind system is marked by a supply of kinetic energy, and the development of such a system requires either a transformation of some other form of energy into kinetic energy or a transfer of the kinetic energy of some other system into that of developing system. So in order to understand the barotropic and baroclinic instability of the zonal current that is the growth of any superimposed disturbance, it is essential to know in detail the energy exchange between the basic mean state and that of the superimposed disturbance. Various authors have studied energy transformations in the atmosphere with quasigeostrophic models. However, a proper study of the dynamic of incipient perturbations within the Tropics can be done only with nongeostrophic models.

In the following we will study energy processes with primitive equation models in some detail. To discuss energy transformations we will first derive the equations of motion for perturbation from the complete set of primitive equations of motion. In order to incorporate
the variation of coriolis parameter with latitude, (the \(\beta\) effect)
the equations of motion for horizontal flow are replaced by the
vorticity and divergence equations. The equation of vertical motion
is replaced by hydrostatic equation, because we are only interested
in the large-scale atmospheric motions in which the hydrostatic
balance holds with sufficient accuracy. Hence only the kinetic
energy of horizontal motion is examined in what follows. Now the
complete set of primitive equations for adiabatic and frictionless
atmospheric flow, the vorticity equation, the divergence equation,
the hydrostatic equation, the continuity equation and thermodynamic
equation are given below from (2.1) to (2.5) respectively.

\[
\frac{\partial y}{\partial t} + \nabla \cdot \mathbf{v} + \omega \frac{\partial y}{\partial \rho} + \left[ \frac{\partial y}{\partial \rho} \mathbf{v} + \left( \frac{f}{\pi} + \frac{\partial y}{\partial \rho} \right) \nabla \mathbf{v} \right] \cdot \nabla \omega + \frac{\partial y}{\partial \rho} = 0 \quad \text{--- (2.1)}
\]

\[
\frac{\partial D}{\partial t} + \nabla \cdot \mathbf{D} + \omega \frac{\partial D}{\partial \rho} + \frac{D^2}{2} \frac{\partial y}{\partial \rho} + \nabla \omega \cdot \frac{\partial y}{\partial \rho} + 2 \int \frac{\partial y}{\partial \rho} \left( \frac{y}{\pi} + \frac{\partial y}{\partial \rho} \right) + \mathbf{u} \left( \frac{\partial y}{\partial \rho} + \nabla ^2 \mathbf{y} \right) = 0
\quad \text{--- (2.2)}
\]

\[
\frac{\partial \phi}{\partial \rho} = - \frac{R}{\pi} \theta \left( \frac{\partial}{\partial \rho} \right) R |_{\phi} \quad \text{--- (2.3)}
\]

\[
D + \frac{\partial \omega}{\partial \rho} = 0 \quad \text{--- (2.4)}
\]

\[
\frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial \rho} \right) + \nabla \cdot \left( \frac{\partial \phi}{\partial \rho} \right) + S \omega = 0 \quad \text{--- (2.5)}
\]
Here the equations are written in $x, y, p$ and $t$ co-ordinate system. 

$x$ is zonal distance positive eastward, $y$ is meridional distance measured positive northward from the equator, $p$ is pressure as the vertical co-ordinate measured negative upwards from sea level and $t$ is time. Here $u, v$ and $w$ are time rates of change of $x, y$ and $t$ following the motion respectively. $f$ is the geopotential 

$h$ is the potential temperature of air, $S$ is measure of static stability ($-\frac{x}{\theta} \frac{\partial \theta}{\partial p}$). $\alpha$ is specific volume of air, $R$ is the gas constant for dry air, $C_p$ is the specific heat for constant pressure, 

$h_0$ is a standard value of pressure ($h_0 = 1000 \text{ mb}$), $f$ is the variation of coriolis parameter $+f$ with latitude. $\nabla$ is horizontal gradient operator $i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y}$; $J$ is the Jacobian operator.

$$J(u,v) = \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} \frac{\partial u}{\partial x}$$

We can write the horizontal wind as a sum of the nondivergent and the divergent components i.e.,

\[ \mathbf{v} = \nabla \psi + \nabla \chi \]

Here $\psi$ is the stream function and $\chi$ is the velocity potential. This means that vertical component of relative vorticity and the divergence of the horizontal wind can be written respectively:

\[ \frac{\partial \psi}{\partial z} = \mathbf{\nabla}^2 \psi \quad ; \quad D = \mathbf{\nabla}^2 \chi \]

The basic state or the undisturbed motion is assumed to be a zonal flow $U$ satisfying the geostrophic equilibrium with static equilibrium satisfied along the vertical. $U$ can be a function of $y$ and $p$.

So the velocity components in the basic state are:
\[
\bar{v} = -\frac{1}{
\frac{\partial \Phi}{\partial y}} \frac{\partial \Phi}{\partial y} = -\frac{\partial \Psi}{\partial y}
\]
\[
\bar{u} = 0 \quad ; \quad \bar{\omega} = 0
\]

A bar over each quantity denotes a process of averaging with respect to zonal direction. Let us consider the energetics of three types of models.

1. \( v \) is a function of \( y \) and \( \beta \), the perturbations are three dimensional \( x, y, \beta \) and \( t \).

2. \( v \) is a function of \( \beta \) only, and the perturbations are two dimensional \( x, \beta \) and \( t \).

3. \( v \) is a function of \( y \) only, and the perturbations are two dimensional \( x, y \) and \( t \).

We commence with model 1, the most general model of the three.

2.1 Model 1

Let us represent each dependent variable appearing in equations (2.1) - (2.5) by the sum of the zonal mean value and departure from it. For instance,

\[
\begin{align*}
\bar{u} &= u(y, \beta) + u'(x, y, \beta, t) \\
\bar{v} &= v'(x, y, \beta, t) \\
\bar{\omega} &= \omega'(x, y, \beta, t)
\end{align*}
\]

(2.6)
\[
\begin{align*}
\psi &= \tilde{\psi} (y, \beta) + \psi' (x, y, \beta, \lambda) \\
\chi &= \chi' (x, y, \beta, \lambda) \\
\phi &= \tilde{\phi} (y, \beta) + \phi' (x, y, \beta, \lambda)
\end{align*}
\]

Inserting the expressions (2.6) into equations (2.1) - (2.5)
and by averaging this set of equations with respect to \( \lambda \) we will
be having a set of equations representing zonal mean motion.
Subtracting this set of equations from original set of equations
(2.1) - (2.5) we will get equations representing perturbation
motion. The nonlinear terms in the perturbation equations are
dropped out. The instability problem is generally governed by
the energy exchange between zonal mean and eddy motion. The
nonlinear terms will not play any part in this exchange and they
only redistribute the energy among different scales of perturba-
tions. The present study does not include the examination of growth
of perturbation at the expense of another scale of perturbation.
The linearised perturbation equations are shown below as (2.7) -
(2.10)

\[
\begin{align*}
\frac{\partial}{\partial t} (\psi^2) + u \frac{\partial}{\partial x} (\psi^2) + \psi' \left( \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial y^2} \right) - \omega' \frac{\partial}{\partial \beta} \left( \frac{\partial u}{\partial y} \right) + \\
+ (\psi - \frac{\partial u}{\partial y})(\nu^2) - \frac{\partial \chi'}{\partial y} \frac{\partial u}{\partial \beta} = 0 & \quad \cdots (2.7) \\
\frac{\partial}{\partial t} (\chi^2) + u \frac{\partial}{\partial x} (\chi^2) - \frac{\partial \psi'}{\partial x} \frac{\partial \psi}{\partial \beta} + 2 \frac{\partial \chi}{\partial y} \frac{\partial \psi'}{\partial \beta} + \\
+ u' \frac{\partial u}{\partial x} + \nabla^2 \phi' = 0 & \quad \cdots (2.8)
\end{align*}
\]
\[
\frac{\partial}{\partial t} \left( \frac{\partial \Phi'}{\partial p} \right) + u \frac{\partial}{\partial x} \left( \frac{\partial \Phi'}{\partial p} \right) - v' \frac{\partial \Phi}{\partial y} + \bar{S} \omega' = 0 \quad \text{--- (2.9)}
\]

\[
\nabla^2 \chi' + \frac{\partial \omega'}{\partial p} = 0 \quad \text{--- (2.10)}
\]

where

\[
\begin{align*}
\psi' &= -\frac{\partial \psi'}{\partial y} + \frac{\partial \chi'}{\partial x} \quad \text{i.e., } \psi' = \psi_{y} + \psi_{x} \\
\text{and } v' &= \frac{\partial \psi'}{\partial x} + \frac{\partial \chi'}{\partial y} \quad \text{i.e., } v' = \psi_{x} + \psi_{y}
\end{align*}
\]

\(\bar{S}\) is the sonic mean measure of static stability. \(\bar{S}\) may be a function of \(y\) and \(p\). The horizontal variation of static stability parameter \(\bar{S}\) through the equation of thermal wind and gas equation is

\[
\frac{\partial \bar{S}}{\partial y} = -\frac{1}{\Phi} \left[ \frac{\partial}{\partial p} \left( \frac{\partial \Phi}{\partial p} \right) + \frac{(1 - \kappa)}{p} \frac{\partial \Phi}{\partial p} \right]
\]

Here \(\kappa\) is ratio of gas constant for dry air to specific heat at constant pressure, \(R/c_p\).

The above set of four equations (2.7) – (2.10) forms a complete system with four unknown perturbations \(\psi', \chi', \omega', \phi'\). The equations are quite general, since the only assumption so far made is that of hydrostatic balance for the perturbations. The third term in the equation (2.7) is the meridional advection of absolute vorticity of the basic state, by the perturbation. The necessary condition
derived by Kuo (1949) to cause dynamic instability for nondivergent
barotropic zonal current was that the meridional gradient of the
mean absolute vorticity \( \left( \beta - \frac{\partial \bar{u}}{\partial y} \right) \) should change its sign some-
where within the domain. Generally speaking, in the tropical region
the profile of \( \left( \beta - \frac{\partial \bar{u}}{\partial y} \right) \) does not change sign, because of the
large value of \( \beta \) and the small values of \( \frac{\partial \bar{u}}{\partial y} \). Hence barotropic
instability may be of much less significance in the tropics than
elsewhere. The third term in equation (2.9) corresponds to the eddy
meridional advection of basic temperature. The last term in
equation (2.9) corresponds to the vertical advection of basic
temperature by the eddy motion. The relative magnitude of these
two terms (3rd and 4th of (2.9)) will be important when considering
baroclinic instability of the disturbance. Let \( \omega' \) be positive \( \omega' \) be
negative (ascending motion) ahead of the trough, as is usually
observed for the moving troughs in pressure field of middle latitudes,
then ahead of the trough there will be warming due to the third term
and cooling due to the fourth term. We take \( \bar{z} \) (which is very rarely
negative) as positive. The net result of the third and fourth terms
will be either warming or cooling depending upon the dominating term.
The reverse effect occurs ahead of the ridge. Ascent of warm (cold)
air ahead of the trough and descent of cold (warm) air in the rear of
the trough leads to a direct (indirect) thermal circulation. It is
well known, since the time of Margules (1903), that there will be
release of potential energy to increase kinetic energy in the direct
circulation and vice-versa in the indirect circulation. Thus the
relative magnitudes of the third and fourth terms in (2.9) determine
the stability of baroclinic disturbance, just as the relative
magnitudes of $\beta$ and $\frac{\partial}{\partial y}$ in (2.7) determine the stability of baro-

tropic disturbances. The last term in equation (2.7) is the
tilting term. The basic vertical shear constitutes vorticity about
a north-south horizontal axis. This horizontal component of zonal
mean vorticity will be tilted into the vertical by the action of
perturbation vertical velocity field. If $\omega'$ is decreasing toward
$\gamma$ and $\nu$ increases with height, this term makes the perturbation
relative vorticity to increase with time.

Let us now examine what the set of equations (2.7) to (2.10)
implies regarding the energetics of the perturbations. The eddy
kinetic energy over total mass of the atmosphere is defined as

$$K' = \iiint_{\text{globe}} \frac{1}{2} (u'^2 + v'^2) \, dM$$

where $dM = dx \, dy \, dp$.  

The limits of $x, y$ and $p$ are the boundaries of global atmos-

phere.

$$\iiint (\ ) \, dM = \frac{1}{g} \iiint (\ ) \, dx \, dy \, dp$$

$$= \frac{2\pi a}{g} \cos \varphi \iiint (\ ) \, dy \, dp.$$  

Here bar denotes the zonal mean, and $2\pi a \cos \varphi$ is the circumference
of the mean latitude ( $\varphi$ ) circle along which zonal mean has been
performed.
Let 
\[ dm = \frac{2\pi a^2}{3} G \rho \hat{\Phi} \, dy \, dp \]

then 
\[ k' = \iint \frac{1}{2} \left( \hat{u}_1^2 + \hat{u}_2^2 \right) dm \]

And this can be written as
\[ k' = \iiint \frac{1}{2} \left( \nabla \psi' \cdot \nabla \psi' \right) dm + \iiint \frac{1}{2} \left( \nabla \chi' \cdot \nabla \chi' \right) dm + \iiint \int (\psi', \chi') dm \]

The last term expresses a component of kinetic energy due to both \( \psi \) and \( \chi \) fields. However, the integral of any Jacobian vanishes when integrating over the whole mass of the atmosphere. Thus we find
\[ k' = \iiint \frac{1}{2} \left( \nabla \psi' \cdot \nabla \psi' \right) dm + \iiint \frac{1}{2} \left( \nabla \chi' \cdot \nabla \chi' \right) dm \]

The first term represents the eddy kinetic energy due to only non-divergent part of the wind and can be denoted by \( k'_\psi \). The second term represents the eddy kinetic energy due to only divergent part of the wind and can be denoted by \( k'_\chi \).

The total potential energy of the atmosphere, defined as the sum of potential energy and internal energy, is not a good measure of the energy available for conversion into kinetic energy. Lorenz (1955) has overcome this difficulty by defining available potential energy as the difference between the existing total potential energy and the total potential energy when the atmosphere is rearranged adiabatically in such a way that it is stably stratified with isentropes and iso bars parallel. The mathematical expression for eddy
available potential energy over total mass of the atmosphere as

\[ p' = \iint \frac{1}{2} \left( \frac{\partial \Phi}{\partial \phi} \right)^2 \, d\sigma m \]

In order to have the rate of change of \( K_{\psi}, K_{\chi} \) and \( p' \), let us multiply the linearised vorticity (2.7), divergence (2.8) and thermodynamic (2.9) equation with \(-\psi', \chi'\) and \(\frac{1}{5} \frac{\partial \Phi}{\partial \phi}\) respectively. Next every equation is integrated throughout the mass of the atmosphere. All the perturbations are assumed to be periodic in \( x \) direction. It is observed that the divergence of any vector, the Jacobian of any two scalars, and the vertical derivative of any quantity, which is zero at the bottom and top of the atmosphere (here \( \omega = 0 \) at upper and lower boundaries), all vanish when integrated throughout the atmosphere. \( \psi' \) and \( \chi' \) are assumed to be zero at the northern and southern boundaries of the atmosphere. We then find the terms for vorticity equation (2.7), after being analysed as prescribed above.

\[ a) - \iiint \psi' \frac{\partial}{\partial \phi} (\nabla^2 \psi') \, d\sigma m = \frac{\partial K_{\psi}}{\partial \phi} \]

\[ b) - \iiint \psi' \psi \frac{\partial}{\partial \phi} (\nabla^2 \psi') \, d\sigma m = \iiint \frac{\partial u}{\partial y} \bar{u}_{\psi} \psi \, d\sigma m. \]

\[ c) - \iiint \psi' \psi \frac{\partial}{\partial y} (- \frac{\partial u}{\partial y}) \, d\sigma m = \iiint \frac{\partial \bar{u}}{\partial y} \psi \, d\sigma m. \]

\[ d) - \iiint \psi' \omega \frac{\partial}{\partial \phi} (- \frac{\partial u}{\partial y}) \, d\sigma m = \iiint \frac{\partial \bar{u}}{\partial y} \omega \psi \, d\sigma m. \]

\[ e) - \iiint \psi' \bar{u} \, d\sigma m = - \iiint \bar{u} \psi' \, d\sigma m \]

--- (2.11)
also from analysis of the divergence equation (2.9) we have:

\[
\nabla \cdot \text{div} \mathbf{F} = \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial x} \right)
\]

By examining the terms \(a, b, c\), it reveals that there are three sets of terms each with some form but opposite sign. For instance, whenever the vertical direction of vorticity is included in the model, we have to use the tilting term too.

\[
\text{Energy redistribu}^2 \text{tion and absence of any one of these terms in the system energetically inconsistent and spurious errors may be expected.}
\]

\[
\begin{align*}
\text{(I)} & - \iiint x' \frac{\partial u}{\partial x} \, dV \\
\text{(II)} & - \iiint x' \frac{\partial v}{\partial x} \, dV \\
\text{(III)} & - \iiint x' \frac{\partial w}{\partial x} \, dV
\end{align*}
\]
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e) \[-\iint\int \chi' \, \frac{\partial u}{\partial y} \cdot \frac{\partial u'}{\partial x} \, dM = \iint\int \frac{\partial u}{\partial y} \, \nu' \chi' \, dM + \iint\int \frac{\partial u}{\partial y} \, \left( \nu' \chi + \nu' \chi' \right) \, dM + \iint\int \frac{\partial u}{\partial y} \, \left( \nabla \nu' \cdot \nabla \chi' \right) \, dM.\]

f) \[-\iint\int \chi' \, u' \, \beta \, dM = -\iint\int \beta \, \nu' \chi' \, dM.\]

g) \[-\iint\int \chi' \, \nabla^2 \phi' \, dM = -\iint \phi' \, \frac{\partial \phi'}{\partial p} \, dM.\]

---(2.12)

We can also mention the combinations of the terms with the same form and opposite sign from this set (2.12). There are two combinations, (i) the term (b) and a half of the magnitude of the first term of (e), and (ii) the term (f) and the second term of (e). So if Coriolis parameter is a variable the terms both \( \beta \nabla \nu' \) and \( \nu' \beta \) should be included in the divergence equation.

Similarly zonal advection of divergence and Jacobian term should be included together in the model. So whenever we make modelling for atmospheric study we must pay attention to this consistency aspect of the problem. Similarly from the analysis of the thermodynamic equation (2.9), we obtain

\[a) \iint\int \frac{1}{\beta} \, \frac{\partial \phi'}{\partial p} \, \frac{\partial}{\partial y} \left( \frac{\partial \phi'}{\partial p} \right) \, dM = \frac{\partial \phi'}{\partial y} \]

\[b) \iint\int \frac{1}{\beta} \, \frac{\partial \phi'}{\partial p} \, u \, \frac{\partial}{\partial x} \left( \frac{\partial \phi'}{\partial p} \right) \, dM = 0\]

\[c) \iint\int \frac{1}{\beta} \, \frac{\partial \phi'}{\partial p} \, \left( \frac{\partial}{\partial y} \phi' \right) \, dM = -\iint\int \frac{1}{\beta} \, \phi' \, \frac{\partial u}{\partial y} \, \frac{\partial \phi'}{\partial p} \, dM.\]
An equation for the rate of change of eddy kinetic energy of non-
divergent motion can be obtained by replacing each term of equation
(2.7) by the corresponding energetical term from the set of expres-
sions (2.11). In otherwords from (2.11) the right hand side of the
expression (a) is equal to the sum of remaining right hand side
terms with negative sign. Similarly an equation for the rate of
change of eddy kinetic energy due to divergent part of wind can be
obtained by making use of equation (2.8) and the set of expressions
(2.12).

\[
\frac{\partial k_{x'}}{\partial t} = - \mathcal{I}_H \left[ k_{x'}, \bar{K} \right] - \mathcal{I}_V \left[ k_{x'}, \bar{R} \right] + \mathcal{I} \left[ k_{x'}, k_{x'} \right] \quad \cdots (2.14)
\]

\[
\frac{\partial k_{x'}}{\partial t} = - \mathcal{I}_H \left[ k_{x'}, \bar{K} \right] - \mathcal{I}_V \left[ k_{x'}, \bar{R} \right] - \mathcal{I} \left[ k_{x'}, k_{x'} \right] + \mathcal{I} \left[ P', k_{x'} \right] \quad \cdots (2.15)
\]

Making use of equation (2.9) and the set (2.15) we can obtain an
equation for the rate of change of eddy available potential energy

\[
\frac{\partial \bar{P}}{\partial t} = \frac{1}{2} \left[ \bar{P}, P' \right] - I \left[ P', k_{x'} \right] \quad \cdots (2.16)
\]

Here I denotes the interaction between two forms of energy represented
in the brackets. This essentially means that a term appears commonly
in the two kinds of energy equations with the same form but with
opposite signs. Thereby a gain in one kind of energy is a loss in
another kind of energy by the same amount due to this interaction term.
Thus these terms can be considered as representing a transformation.
of energy from one kind to another, and so usually referred as
energy transformation functions.

Here

\[ I_H \left[ k'_\psi, \bar{r} \right] = \iint \frac{\partial u}{\partial y} \bar{u}'_\psi \psi' \, dm \quad \ldots \quad (2.17) \]

\[ I_V \left[ k'_\psi, \bar{r} \right] = \iint \frac{\partial u}{\partial p} \bar{u}'_\psi \omega' \, dm \quad \ldots \quad (2.18) \]

\[ I_H \left[ k'_\chi, \bar{r} \right] = \iint \frac{\partial u}{\partial y} \left( \bar{u}'_\chi \psi' \chi' + \bar{u}'_\psi \psi' \chi' + \bar{u}'_\chi \psi' \right) \, dm \quad (2.19) \]

\[ I_V \left[ k'_\chi, \bar{r} \right] = \iint \frac{\partial u}{\partial p} \bar{u}'_\chi \omega' \, dm \quad \ldots \quad (2.20) \]

\[ I \left[ k'_\chi, k'_\psi \right] = -\iint \bar{\omega} \left( \bar{\psi}' \psi \chi' \right) \, dm \quad \ldots \quad (2.21) \]

\[ I \left[ \bar{p}', k'_\chi \right] = \iint \bar{\omega'} \bar{\psi}' \, dm \quad \ldots \quad (2.22) \]

\[ I \left[ \bar{p}', p' \right] = \iint \frac{1}{\bar{p}} \bar{u} \frac{\partial u}{\partial p} \bar{\psi} \frac{\partial \psi}{\partial p} \, dm \quad \ldots \quad (2.23) \]

In the above equality (2.21) \( \bar{\omega} \) is the absolute verticality in basic state
\[ \bar{\omega} = \left( \frac{-\partial u}{\partial y} \right) \]

The suffixes \( H \) and \( V \) are used here for convenience of distinguishing horizontal and vertical processes. In the above expressions (2.17) to (2.23) the notation is such that the energy transformed from left to right quantity in a bracket, if the transformation function takes a positive sign.

The expression (2.17) denotes the interaction of kinetic energy between the zonal mean component and the eddy \( \psi \) component. This
term is related to the meridional eddy transfer of momentum only due to nondivergent component of wind. (2.19) denotes another aspect of the interaction of kinetic energy between the zonal mean component and the eddy component, essentially relating to the meridional eddy transfer of momentum due to component of wind. In the same way (2.18) represents the redistribution of kinetic energy between and through the vertical transfer of momentum due essentially to eddy component. The interaction (2.20) is related to the vertical transport of momentum due to eddy component. On an overall view, it is very interesting to note the following:

\[ \frac{\bar{u} v}{x} = \frac{u' \bar{v}}{x} + \frac{u' \bar{v}}{x} + \frac{u' \bar{v}}{x} + \frac{u' \bar{v}}{x} \]

and

\[ \frac{\bar{u} \omega}{x} = \frac{u' \bar{v}}{x} + \frac{u' \bar{v}}{x} \]

we find from (2.17)-(2.20) that a part of total transport of momentum either meridionally or vertically contributes only to the interaction between and and another part associates only with the interaction between and. The insight into these processes have become possible because of representation of eddy kinetic energy by two parts and Another interesting term is the interaction (2.21) between eddy kinetic energy due to component of wind and component of wind. In a way this is redistribution of eddy kinetic energy among and components of motion. This process is the main link between rate of change of and Without knowing the behaviour of

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* This word 'momentum' here as well as the later parts of the discussion denotes 'eddy zonal momentum'.
under different distributions of $u$ with $\psi$ and with other basic state parameters, it is difficult to visualise pictorially this interaction term. However, the following discussion may be tentatively accepted. For the same magnitude and sign of $\nabla \psi \cdot \nabla \chi$, the production, if $\nabla \psi \cdot \nabla \chi > 0$, (or dissipation if $\nabla \psi \cdot \nabla \chi < 0$) of eddy kinetic energy of nondivergent motion is more with constant basic cyclonic shear rather than with constant basic anticyclonic shear.

The interaction process (2.22) between eddy available potential energy and eddy kinetic energy due to $\chi$ component of wind (but not due to $\psi$ component of wind) is quite well known. This is the process by which the conversion of potential into kinetic energy takes place through the rising of warm air and the simultaneous sinking of cold air*. (2.23) is the interaction between eddy and basic available potential energy. This is mainly through the meridional transport of sensible heat accompanied with the basic meridional temperature gradient or basic vertical shear.

*In the quasigeostrophic model with the absence of $\chi$ component of horizontal wind, this particular interaction will take place between eddy kinetic energy and eddy available potential energy. Here, the model being more general and with feasibility of representing eddy kinetic energy into two components, it revealed that eddy kinetic energy due to divergent part only plays role in this transformation.
The interactions between different forms of energy will be schematically represented as an energy diagram in Fig. 2.1

Fig. 2.1

The two directional arrows in the above diagram depict the transformation eitherway from one form of energy to another. In order to fix the direction of transformation of one form of energy to other in our model at particular instant of time, we must know the magnitude and sign of each interaction term.

Summing up the equations (2.14) and (2.15) we get:

\[
\frac{\partial k'}{\partial t} = - I_\mu \left[ k', k' \right] - I_\nu \left[ k', k' \right] + I \left[ P', k' \right] \quad \cdots (2.24)
\]

Rewriting (2.16)

\[
\frac{\partial P'}{\partial t} = I \left[ P, P' \right] - I \left[ P', k' \right] \quad \cdots (2.25)
\]
Where

\[ I_{u'} [k', \tilde{k'}] = \iint \frac{\partial u}{\partial y} \cdot \tilde{u} \, \text{d}m \quad \text{--- (2.26)} \]

\[ I_{v'} [k', \tilde{k'}] = \iint \frac{\partial u}{\partial \tilde{p}} \cdot \tilde{u} \, \text{d}m \quad \text{--- (2.27)} \]

\[ I \quad [\tilde{p}', \tilde{k'}] = \iint \frac{\partial}{\partial \tilde{p}} \omega \, \text{d}m \quad \text{--- (2.28)} \]

\[ I \quad [\tilde{p}, \tilde{p}'] = \iint \frac{1}{\tilde{a}_{\tilde{p}}} \cdot f \frac{\partial u}{\partial \tilde{p}} \cdot \omega \, \text{d}m \quad \text{--- (2.29)} \]

(2.24) represents the rate of change of total eddy kinetic energy.

The interaction of kinetic energy between zonal and eddy is expressed by (2.28). The process of transformation is through the total meridional transport of zonal momentum by the disturbance accompanied with the meridional shear of the basic zonal wind.

(2.27) denotes another contribution to the interaction of kinetic energy between zonal mean and eddy, through the process of total vertical transport of zonal momentum by the disturbance accompanied with the vertical shear of the basic zonal wind. The meridional \((\tilde{u}, \tilde{v})\) and vertical \((\tilde{u}, \tilde{\omega})\) transports of zonal momentum by the disturbance are known to be the main cause for the maintenance of the zonal mean currents in the atmosphere. (2.28) and (2.29) are already mentioned before as (2.22) and (2.23) respectively and their importance for baroclinic instability will be revealed in the discussion of the model (2). We shall summarize the energy processes in the present model (1) in Fig. 2.2.
From the law of conservation of energy the sum of the four forms of energy depicted in Fig. 2.2 should be conserved for all the time. With the calculation of each interaction term, it becomes possible to estimate energy growth rate of any form of energy in the atmosphere. Eliassen (1961) with his two level quasigeostrophic model computed some of the energy interaction terms and growth rate of eddy kinetic energy. In the numerical experiment with the primitive equations of motion applied over nine levels, Smagorinsky, Manabe and Holloway (1965) measured the time values of the interaction terms and discussed the energy processes in detail.

2.2 Model 2

Basic zonal current is function of $\beta$ only and all the perturbations are functions of $\lambda, \beta$ and $\frac{d}{dt}$. This sort of model is generally known as pure baroclinic model. To this particular model the linearised vorticity, divergence thermodynamic energy and continuity perturbation equations from (2.7) - (2.10) become:
\[
\frac{\partial}{\partial t} \left( \frac{\partial^2 \psi}{\partial x^2} \right) + u \frac{\partial}{\partial x} \left( \frac{\partial^2 \psi}{\partial x^2} \right) + \frac{\partial^2 \psi}{\partial x^2} = 0 \quad \text{(2.30)}
\]

\[
\frac{\partial}{\partial t} \left( \frac{\partial^2 \chi'}{\partial x^2} \right) + u \frac{\partial}{\partial x} \left( \frac{\partial^2 \chi'}{\partial x^2} \right) - \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\partial x} + \frac{\partial^2 \beta}{\partial x^2} = 0 \quad \text{(2.31)}
\]

\[
\frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial \beta} \right) + u \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial \beta} \right) - \frac{\partial \psi}{\partial \beta} + \frac{\partial \psi}{\partial \beta} = 0 \quad \text{(2.32)}
\]

\[
\frac{\partial^2 \chi'}{\partial x^2} + \frac{\partial \psi}{\partial \beta} = 0 \quad \text{(2.33)}
\]

\[\text{Hence,} \quad \chi' = \frac{\partial \chi'}{\partial x} \quad \text{and} \quad \psi' = \frac{\partial \psi}{\partial x}\]

Because of the independent nature of the disturbance with respect to \( \psi \) in the present model, we arrive at the following observations:

(i) \( \chi' \) is related only to the velocity potential and \( \psi' \) is determined by the stream function alone, and (ii) a comparison between the set of equations (2.7) - (2.10) and (2.30) - (2.33) will reveal that

(a) in the vorticity equation the meridional and vertical advects of basic vorticity, the tilting term, and the divergence term with basic vorticity as coefficient vanish in the set (2.30) - (2.33),

and (b) in the divergence equation the Jacobian term automatically goes out.

The missing of the above mentioned terms is not in any way related to the energetical inconsistency of the model. The consistency of the model is inherent as will be seen in the following.

The rate of change of eddy kinetic energy due to \( \psi \) and \( \chi \) components of wind and the rate of change of eddy available potential energy can be written as below:
\[ \frac{\partial k'_{\psi}}{\partial t} = I [ k'_{\psi}, k'_{\psi}] \quad \ldots (2.34) \]
\[ \frac{\partial k'_{\psi}}{\partial t} = -I [ k'_{\psi}, \varphi] - I [ k'_{\psi}, k'_{\psi}] + I [ \bar{p}, k'_{\psi}] \quad \ldots (2.35) \]
\[ \frac{\partial \varphi'}{\partial t} = I [ \bar{p}, \varphi] - I [ \bar{p}, k'_{\psi}] \quad \ldots (2.36) \]

As usual I stands for interaction between two forms of energy.

Here
\[ I [ k'_{\psi}, k'_{\psi}] = -\int \overline{\nu \psi} \overline{\nu} \, dm \quad \ldots (2.37) \]
\[ I [ k'_{\psi}, \varphi] = \int \overline{\varphi} \, \overline{\nu} \, dm \quad \ldots (2.38) \]
\[ I [ \bar{p}, k'_{\psi}] = \int \overline{\omega} \, \frac{\partial \varphi'}{\partial \bar{p}} \, dm \quad \ldots (2.39) \]
\[ I [ \bar{p}, \varphi] = \int \overline{\varphi} \, \frac{\partial \varphi'}{\partial \bar{p}} \, dm \quad \ldots (2.40) \]

where \( dm \) now becomes \( \frac{2 \pi a}{y} \cos \varphi \, dy \Delta y \).

Here \( \Delta y \) is unit thickness along \( y \).

The energy diagram is shown in Fig. 2.3.

![Fig. 2.3](image_url)

The chief difference between this energy diagram (Fig. 2.3) and that of (Fig. 2.1) is the absence of interaction between \( k'_{\psi} \) and \( \varphi \). This is because of the \( y \) independent nature of the disturbance and of the basic current by which horizontal momentum
transfer is not possible. Both from Model 1 and Model 2 we can see the conversion process for eddy available potential energy is with eddy kinetic energy due to divergent part of the wind ($u'_x$) only. The conversion is mainly through the process of vertical circulation in the zonal plane. The redistribution of eddy kinetic energy between nondivergent part and divergent part is closely associated with the effect of earth's rotation.

By adding (2.34) and (2.35) we arrive at the rate of change of eddy kinetic energy.

\[
\frac{\partial \mathbf{k'}}{\partial t} = I[K', \mathbf{K}] + I[P', K'] \quad \text{--- (2.41)}
\]

\[
\frac{\partial P'}{\partial t} = I[\mathbf{P}', P'] - I[P', K'] \quad \text{--- (2.42)}
\]

Here

\[
I[K', \mathbf{K}] = \int \frac{\partial u}{\partial \phi} \frac{u'}{u'} \, dm \quad \text{--- (2.43)}
\]

\[
I[\mathbf{P}', P'] = \int \frac{\partial \mathbf{P}}{\partial \phi} \frac{\mathbf{P}'}{\mathbf{P}'} \, dm \quad \text{--- (2.44)}
\]

\[
I[P', K'] = \int \frac{\partial \mathbf{Q}'}{\partial \phi} \, dm \quad \text{--- (2.45)}
\]

We summarize energy processes of Model 2 in Fig. 2.4

![Diagram](image)

**Fig. 2.4**

The interaction process of $K'$ with $\mathbf{K}$ in this model is only through vertical transport of eddy momentum due to $x$ component, unlike vertical transport of total $u'_w$ and $u'_x$ eddy zonal momentum in the counterpart of Model 1. We shall measure all interaction terms in (2.37) - (2.40) and (2.43) - (2.45) theoretically and
present in the Chapter V. The energy interaction terms for a purely baroclinic model with the primitive equations were computed by Murakami (1967) for a stationary disturbances induced by topography and diabatic heating. From the above equation (2.41) and (2.42) and from the energy diagram (Fig. 2.4) we can easily understand that the meridional transport of sensible heat by the disturbance and the process of vertical circulation \( \left( \frac{\partial \Phi}{\partial p} \right) \) in the zonal plane are the two chief factors that are interlinked in the eddy and basic energy distributions of the model. To understand the nature of these two terms in the energy processes let us proceed as described below:

\[
\eta \psi' = A(\beta) \sin \mu x + B(\beta) \cos \mu x
\]

\[
= \tilde{\eta} \psi(\beta) \sin \left[ \mu x + \delta \phi \right]
\]

where \( \tilde{\eta} \) is amplitude of the perturbation wave

\[
\tilde{\eta} \psi = \sqrt{A^2 + B^2}
\]

and \( \delta \) is phase angle of the wave

\[
\delta = \tan^{-1}\left( \frac{B}{A} \right)
\]

To make physical explanation simple, let us introduce the geostrophic approximation for the perturbations, i.e., \( \psi' = \frac{\phi'}{f} \).

Sensible heat transfer (defined as the zonal average) can be related to variation of phase angle of the disturbance in the vertical as

\[
\frac{\partial \Phi}{\partial p} = f \frac{1}{\mu} \frac{\partial \mu}{\partial p} - \frac{2}{\nu} \frac{\partial \delta}{\partial p}
\]

\[\text{(2.46)}\]
When the axis of the disturbance tilts toward west with decreasing pressure $\frac{\partial S}{\partial p} < 0$, the sensible heat must be northward $\frac{\partial v}{\partial p} < 0$. The reverse is the case for the eastward tilt of the disturbance.

We now imagine a case that the sensible heat transfer is northward due to westward tilt of the disturbance. Then referring to equation (2.35) and identity (2.40) the term $\iint \frac{1}{\Sigma} \frac{\partial S}{\partial p} \frac{\partial \Phi^f}{\partial p} \text{d}m$ becomes positive, because $\frac{\partial v}{\partial p}$ is negative and $\frac{\partial u}{\partial p}$ is negative ($v$ increases with height). Principally this makes eddy available potential energy to increase with time at the expense of basic available potential energy. The transformation is accomplished by the northward transport of the sensible heat by the disturbance from warmer to colder regions. Therefore this process of northward transport of sensible heat should make the basic meridional temperature gradient weak and thereby reduce the basic vertical shear of the zonal current with time. To know fully what this change of the basic shear will bring about to the vertical structure of the disturbance, consider thermodynamic equation (2.32)

$$\frac{\partial}{\partial t} \left( - \frac{\partial \Phi^f}{\partial p} \right) = -u \frac{\partial}{\partial x} \left( - \frac{\partial \Phi^f}{\partial p} \right) + \psi' \frac{\partial u}{\partial p} - \omega \omega' \quad (2.32)$$

The first term on the right side will not contribute to the rate of change of eddy available potential energy. Hence, just for this part of discussion we will omit the first term in the above equation. The remaining two terms on the right side will oppose each other in causing the change of temperature with time. As usually observed for moving troughs in pressure field of the middle latitudes, $\psi'$ can be positive and $\omega'$ can be negative (ascending motion)
ahead of the trough. Since $u$ increases with height ($\frac{\partial u}{\partial p} < 0$), the second term on right side of (2.32) causes warming ahead of the trough. Because $\xi$ is positive, the third term causes cooling ahead of the trough. So warming or cooling ahead of the trough depends on the net effect of these two terms. Exactly the reverse will happen ahead of the ridge. Let us consider first the case of strong vertical shear ($\frac{\partial u}{\partial p} < 0$). The second term dominates over the third resulting warming ahead of the trough and cooling ahead of the ridge. Thus we find that the sensible heat transfer should be northward ($\frac{\partial \phi}{\partial p} < 0$) and then the axis of the disturbance at this stage tilts west with increasing height. Moreover we find a direct circulation because of existence of ascending of warm air (ahead of trough) and descending of cold air (ahead of ridge). Accordingly the kinetic energy of the disturbance grows with time due to the effect of release of eddy available potential energy. The loss of eddy available potential energy due to this process is supplied by the transfer of available potential energy from the zonal mean component, because the first term of (2.42) is positive due to the northward sensible heat transfer. As discussed earlier the northward transport of sensible heat will reduce the basic meridional temperature gradient and thereby the basic vertical shear diminishes. The vertical shear will decrease gradually as the northward sensible heat transfer continues. Then the relative magnitude of second and third term of (2.32) will change in such a way the third term dominates the second so that we find cooling ahead of the trough and
warming ahead of the ridge, resulting \( \frac{\partial \theta}{\partial p} > 0 \) and \( \frac{\partial \omega}{\partial p} < 0 \).

This southward transfer of sensible heat contributes to recover the basic meridional temperature gradient (the vertical shear \( \frac{\partial v}{\partial p} \) becomes strong again), and thus the zonal mean available potential energy increases at the expense of the eddy available potential energy. The loss of eddy available potential energy will be partly compensated by the indirect circulation (\( \frac{\partial \omega}{\partial p} < 0 \)) through conversion of eddy kinetic energy into eddy available potential energy.

Referring to the above qualitative discussion, we now understand that the change of the basic zonal current will be accomplished by the change of physical properties of the disturbance, such as the change of \( \frac{\partial \theta}{\partial p} \), \( \frac{\partial \omega}{\partial p} \) and so on. In this study, however, we assumed the basic zonal current to be independent of time, as it is usually done by most of the theoretical workers. One of the reasons for this is to avoid the difficulty in the complex nature of the mathematical treatments of the eigen value problem which will be presented in the following chapter. Due to this assumption, however, serious limitation may be forced on the change of the structure of the perturbation, as can easily be appreciated by the preceding discussion. In other words, we are going to investigate the nature of the disturbance which specifically associates with the basic zonal flow of infinite capacity of energy - that is, no matter what energy interaction may occur between the zonal current and the disturbance, the energy for the basic zonal flow remains the same. Since the zonal
mean kinetic energy and the eddy kinetic energy are of the same order of magnitude, as was observationally pointed out by Murakami (1960), Saltzman (1962), Teweles (1963) and Wiin-Nielsen (1965), the assumption of infinite energy capacity of the basic flow cannot be applied to the actual atmosphere. The assumption of time independence of $u$ can be made only when we investigate the change of the physical properties of the disturbance at the initial stage of the development of the disturbance. On the other hand, we know from many instability theories developed so far that the physical properties of the disturbance thus obtained were very similar at the initial stages to those we observe in the atmosphere. Therefore it will still be of interest to investigate the nature of the disturbance, as an eigen value problem, or as an initial value problem (one time step) in which the basic state is not a function of time. In this thesis we present the eigen value problem in Chapter III.

2.3 Energy cycle for the Monsoon Circulation:

It is very interesting to extend the above discussion of energy cycle to the case of zonal currents and perturbations of the summer monsoon circulation over South East Asia. During the summer monsoon season (June to September) the lower troposphere of the Indian region (south of the monsoonal trough line i.e. $20^\circ$N) is dominated by a broad westerly flow with series of depressions forming over the north Bay of Bengal and moving inland in a west-northwesterly direction. The upper troposphere over South India and adjoining areas of the north Indian ocean is featured with the tropical easterly
jet stream, which attains its maximum strength between 14 and 16 km. (at 15°N) and may have speeds in excess of 120 knots. In the lower and middle troposphere the thermal gradient is positive, with warmer regions north of 25°N and a decrease of temperature southwards. This particular thermal pattern makes the lower tropospheric westerlies decrease with height and become easterlies (just about 500 mb level) increasing with height aloft attaining a maximum winds at about the 150 mb level. So for over a sufficiently good depth of the atmosphere basic fields $\frac{\partial T}{\partial y}$ and $\frac{\partial u}{\partial p}$ are positive. Now the study of energy interaction between perturbation and the basic flow can be made, visualising the depression in the lower westerly flow or a wavy flow in upper easterly stream as a perturbation. The basic zonal wind shear being positive, the second term on the right hand side of equation (2.52) i.e ( $v^' \frac{\partial P}{\partial y} \frac{\partial u}{\partial p}$ ) causes cooling ahead of the trough and warming ahead of the ridge. From the $\omega'$ equation following Kuo (1955), $\omega' \frac{\partial T}{\partial y}$ we can say the regions of warm air advection and that of cold air advection are associated with ascending and descending motions respectively. In the monsoon case $\frac{\partial T}{\partial y}$ being positive, ahead of the ridge (trough) of the lower westerly wave ascending (descending) motion occurs. This supports general occurrence of rainfall in a large region on the west-southwest section of a monsoon depression. A recent case study of the monsoon depression by Rao and Rajamani (1967) reveals that vertical velocity field is having ascending motion on the southwestern quarter of the depression.

* (an exact $\omega'$ equation for quasigeostrophic two layer model is presented in Chapter IV)
Thus referring again to the thermodynamic equation (2.32), with \( \omega \) positive, the changes of temperature produced by vertical motion in these disturbances are of opposite kind to those produced by advection term in agreement with the previous discussion. If the advection term dominates the \( \omega \) term in thermodynamic equation, a situation of direct thermal circulation arises with warm air (ahead of the ridge) ascending and cold air (ahead of the trough) descending. Moreover, we find that the sensible heat transfer should be southward and then from equation (2.46) the axis of the disturbance tilts towards east with increasing height. Accordingly the kinetic energy of the disturbance grows with time due to the effect of release of eddy available potential energy. This particular loss of eddy available potential energy is replenished by the transformation of zonal mean available potential energy, because the first term of (2.42) is positive due to southward transfer of sensible heat and decrease of westerly wind with height. These processes thus favour the strengthening of the perturbation found in the westerly monsoonal regime of the lower troposphere. However, this continuous southward transport of sensible heat weakens the basic thermal gradient and thereby the zonal wind shear. Then the relative magnitudes of second and third term will change in such a way that the third term of (2.32) dominates over the second term. Then the whole energy processes mentioned above will reverse in such a way that disturbance will become weak and the perturbation energy will be fed into the zonal mean system. However, it is difficult to discuss the life cycle of a monsoon depression without invoking nonadiabatic heat.
and frictional effects.

2.4 Model 5:

In this model, the basic zonal current is a function of \( \phi \) only and all the perturbations are depending upon \( x, y \) and \( t \). This type of model is known as the barotropic model.

Because of the assumption of vertical independence of the disturbance and the basic current, all the terms which are related to the derivative with respect to \( \phi \) in the set of equation (2.7) - (2.10) will drop out, giving

\[
\frac{\partial}{\partial t} (\nabla \phi) + u \frac{\partial}{\partial x} (\nabla \phi) + \nabla \left( \frac{\partial \phi}{\partial y} \right) = 0 \tag{2.47}
\]

\[
- \nabla \cdot \vec{u} + \frac{\partial u}{\partial y} \frac{\partial \phi'}{\partial x} + u' \frac{\partial \phi'}{\partial y} + \nabla \phi' = 0 \quad \text{--- (2.48)}
\]

\[
\nabla \cdot \vec{u}' = 0 \quad \text{--- (2.49)}
\]

where \( u' = - \frac{\partial \phi'}{\partial y} \) and \( \phi' = \frac{\partial \phi}{\partial y} \),

since \( \frac{\partial \phi'}{\partial y} \) vanishes in this model, together with the assumption that \( \phi' \) is zero at the top and bottom of the atmosphere, the vertical \( \phi' \)-velocity \( \phi' \) becomes zero at all levels. And only possible solution of equation (2.49) with the assumption of vanishing \( \phi' \) at the horizontal boundaries is that \( \phi' \) vanishes everywhere. Accordingly the horizontal wind component can be described only by stress function. The thermodynamic equation will not be used in this model because \( \phi' \) is independent of \( \phi \) and \( \phi' \) is zero.
The equation (2.48) is nothing but linearised form of the balance equation first introduced by Charney (1955). We can determine either \( \psi' \) from \( \phi' \) or \( \phi' \) from \( \psi' \). The rate of change of \( \psi' \) can be determined by (2.47), which is the linearised barotropic vorticity equation.

As the divergent component of wind is absent, the kinetic energy due to \( x \) - component also vanishes, i.e., \( k' = k'_{\psi} \). The rate of change of eddy kinetic energy is obtained by multiplying both sides of (2.47) by \( -\psi' \) and integrating throughout the horizontal domain. The result is

\[
\frac{\partial k'}{\partial t} = -I[k', \bar{k}'] \quad (2.50)
\]

where

\[
I[k', \bar{k}'] = \int \frac{\partial u}{\partial y} \overline{u'v'} \, dm \quad (2.51)
\]

in which

\[
dm = \frac{2\pi \Delta \phi}{y} \bar{\phi} \, dy \, A_p
\]

Here \( A_p \) is unit thickness along \( p \) - direction.

The only possible energy interaction is between the eddy and zonal mean kinetic energy. The interaction process is through the meridional eddy momentum transfer accompanied with the meridional shear of the basic current. To understand more thoroughly this interaction process, we shall rewrite (2.51). With the assumption that the momentum transfer at the northern and southern boundaries is zero (or simply \( \psi' \) is zero at the boundaries), we find

\[
I[k', \bar{k}'] = -\int u \frac{\partial}{\partial y} \overline{(u'u')} \, dm \quad (2.52)
\]

Let us now denote the meridional mean of \( U \) by \( \overline{U} \), and the departure
from it by \( u'' \). Then we obtain

\[
I[\kappa', \kappa] = - \int u'' \frac{\partial}{\partial y} (\overline{uu'}) \, dx
\]

(2.53) reveals that \( \overline{u} \) does not contribute at all to the interaction process because there is no net convergence of eddy momentum transfer across the boundaries. \( u'' \) is positive where the zonal current \( u \) is strong and negative where \( u \) is weak. Therefore if there occurs the convergence of eddy momentum transfer \( \frac{\partial}{\partial y} (\overline{uu'}) < 0 \) in the region where \( u'' \) is positive and the divergence of eddy momentum transfer in the region where \( u'' \) is negative, \( I[\kappa', \kappa] \) turns out to be positive, indicating that kinetic energy transforms from \( \kappa' \) into \( \kappa \). That is, the eddy kinetic energy tends to decrease, the disturbance is in barotropic stable state. On the other hand, if \( u'' \) and \( \frac{\partial}{\partial y} (\overline{uu'}) \) correlate positively, \( I[\kappa', \kappa] \) becomes negative and hence \( \kappa' \) increases - that is, the disturbance is barotropically unstable. In this way the direction of kinetic energy transfer is entirely determined by the distribution of the eddy momentum transfer relative to the zonal wind profile.

The eddy momentum transfer is related to the tilt of the axis of the disturbance in horizontal plane. To show this, we express the \( \psi' \) - field in the form

\[
\psi' = \overline{\psi}(y) \sin \left[ \mu x + \xi(y) \right]
\]

The notations are the same as in Model 2.

Then we obtain

\[
\overline{uu'} = - \frac{\partial \psi'}{\partial x} \frac{\partial \psi'}{\partial y} - \frac{1}{\mu} \overline{u^2} \frac{\partial \xi}{\partial y}
\]
The eddy momentum transfer is positive (northward) if the axis of the disturbance tilts towards east with increasing latitude. The larger the tilt the more the northward momentum transfer. When the axis of the disturbance is from northwest to southeast, the eddy momentum transfer is found to be southward.

The tilt of the axis of the disturbance is determined by the vorticity equation (2.47). A qualitative explanation is as follows. Suppose that the disturbance has no horizontal tilt of the axis (from north to south) at the initial time. The term \( \beta v \) is positive ahead of the trough. This indicates that the vorticity tends to decrease ahead of the trough; the trough will move westward with time due to the \( \beta \) - effect. Since \( \beta \) is larger in the lower latitude, the westward movement of trough is more predominant in the lower than in the higher latitudes, resulting that the orientation of the axis of the trough is northeast - southwest. Likewise the wedge also tends to tilt in the same orientation. This means that due to the term there occurs northward momentum transfer by the disturbance. Moreover the lower the latitude, the larger is the momentum transfer. As a result of this, a convergence of the eddy momentum transfer occurs in the middle latitudes where we usually encounter the westerly jet stream. From (2.50) and (2.51) we arrive at an important conclusion that due to the \( \beta \) - term there occurs a transformation of kinetic energy from the eddy to the zonal mean component, indicating the \( \beta \) - term has a stabilising effect. Next we shall consider the term \( \left( -\frac{\partial u}{\partial y} \right) v' \), which will be called here as the shear term. Near the jet stream
centre \(-\frac{\partial u}{\partial y}\) is positive and away from the jet stream it is negative.

Thus due to the shear term, the trough (wedge) tends to move westward from the initial position in the region where \(-\frac{\partial u}{\partial y}\) is positive (the maximum westward movement at the jet centre where \(-\frac{\partial u}{\partial y}\) is maximum), and eastward away from the jet stream axis where \(-\frac{\partial u}{\partial y}\) may be negative. At the inflection point at which \(-\frac{\partial u}{\partial y}\) vanishes, the trough (wedge) will stay at the same location as initial.

Accordingly we find the shear term has such effect as to generate the southward momentum transfer to the south of the jet stream centre and the northward momentum transfer to the north. So there occurs a divergence of eddy momentum transfer near the jet stream axis where \(u''\) is positive. Referring (2.55) it may be mentioned that due to the shear term the eddy kinetic energy will grow by receiving kinetic energy from the zonal mean component, i.e., the shear term has an instabilising effect. We shall next consider the combined effect \((\beta - \frac{\partial u}{\partial y})u'\). If the meridional profile of \(u\) is so smooth that the magnitude \(|\frac{\partial u}{\partial y}|\) is always smaller than \(\beta\), i.e., \((\beta - \frac{\partial u}{\partial y}) > 0\), no chance exists that the disturbance could be unstable. Where the \(u\) profile is very sharp so that the shear term dominates the \(\beta - \frac{\partial u}{\partial y}\) term, \((\beta - \frac{\partial u}{\partial y})\) could vanish somewhere and thus there exists a chance of amplification of disturbance. Due to the above discussion it may be informed that in order to have an actual instability of disturbance, the region at which \((\beta - \frac{\partial u}{\partial y})\) is negative should be comparatively extensive as compared with the region of positive \((\beta - \frac{\partial u}{\partial y})\).

The qualitative discussion so far made is almost the same as more sophisticated and exact Kuo's (1949) argument on the barotropic
instability. Anyhow the necessary condition of the barotropic instability is that \( \beta - \frac{3\nu}{\beta' x^2} \) should vanish somewhere in the region considered. We are interested in this thesis about the phenomena of instability in Tropics. We know \( \beta \) is large and \( \frac{3\nu}{\beta' x^2} \) is usually small in these latitudes. From the knowledge of the above discussion we infer that the barotropic instability would be a rare occurrence near the tropics.

Following almost the same discussion as we made for the model 2, we can discuss qualitatively the time change of the basic zonal current with reference to the change of horizontal axis of the disturbance by using the barotropic vorticity equation (2.47). However, such discussion will not be reproduced here. We only emphasise again that the same precaution which we arrived at in the model 2 for an assumption of time independence of \( \nu \) should be taken care of here too.