4.1. Introduction

The reliability in a broader sense has been defined as follows: reliability indicates the probability implementing specific performance or function of products and achieving successfully the objectives within a time schedule under a certain environment, Wang, Z.H. (1992). In general, a higher priority is placed on quality control rather than reliability in the process of manufacturing. Nonetheless, high quality is not equivalent to high reliability. For example, a certain component, which has passed quality control procedure in conformity to the specifications, may lead to problems when operating with other components. This involves reliability design that is related to electrical or mechanical interface compatibility among spare parts. With the rapid technological progress and increasing complexity of system structure, any failure of any component may lead to system malfunction or serious damage. For instance, a weapon system is a precise and sophisticated system that comprises several sub-systems, components and spare parts. Failure of even a single element will likely have adverse impact upon the operability of the weapon system, and thus may become threat to the national security.

A repairable system is a system that can be repaired to operate normally in the event of any failure such as computer network, manufacturing system, power plant or fire prevention system. Availability comprises “reliability” and “recovery part of
unreliability after repair”, indicating the probability that repairable systems, machines or components maintain the function at a specific moment. It is generally expressed as the operable time over total time.

In recent years, reliability and availability have expanded their influence in various industries and fields, thus serve as an integral quality element in the organization system and manufacturing process. To maintain the reliability of sophisticated systems to a higher level, the system’s structural design or system components of higher reliability shall be required, or both of them are performed simultaneously, Henley et al. (1985). The system structure is virtually designed under the limitations such as weight, volume or other technologies, so the reliability cannot be further improved. In this case, replacing highly reliable components can improve the system reliability. While improving the reliability of systems and components, the associated cost also increases. Thus, it is a very important topic for decision-makers to fully consider both, the actual business and the quality requirements.

A Series-parallel system indicates sub-systems in which several components are connected in parallel, and then in series, or sub-systems where several components are connected in series, and then in parallel. A series-parallel system can be improved by four methods, (1) use more reliable components; (2) increase redundant components in parallel; (3) utilize both #1 and #2; and (4) enable repeatedly the allocation of entire system framework. For the framework of series-parallel system, it is very difficult to find out an optimal solution under multiple constraint conditions Painton and Campbell (1995) solved the reliability optimization problem related to
personal computer design. They regarded a personal computer as a series-parallel system of several components, each of which has three optional packages.

The *selective maintenance operation* is an optimal decision-making activity for systems consisting of several equipments under limited maintenance duration. The main objective of the selective maintenance operation is to select the most important equipment or subsystem to maintain. It also has to determine the appropriate maintenance actions in order to minimize the sum of production losses due to the system failures and the maintenance cost during the next working time. Such kind of problems can be encountered for equipments that perform sequences of tasks and can be repaired only during intervals of tasks. Such cases occur in military equipment production lines in which maintenance actions are carried out on weekends, vehicles are maintained between two deliveries and computer systems are maintained at night, etc.

Rice et al. (1998) were the first to deal with the selective maintenance problem. They modeled a maintenance decision-making problem for a special type of series–parallel system, considering $M$ subsystems in series, the $i^{th}$ subsystem consisting of $N_i$, $i = 1,2,\ldots,M$ identical parallel components. They considered constant fault rate for each component, i.e. they assumed an exponential distribution for the component life, and considered only single maintenance mode. Cassady, Pohl, et al. (2001) and Cassady, Murdock et al. (2001) improved upon the Rice et al. (1998) model by assuming Weibull distribution for component life and considered multiple maintenance modes, including minimal repair (MR) and replacement of faulted
components, and preventive maintenance (PM) for the functioning components. Lust et al. (2009) also proposed a variant model. They set up a selective maintenance optimal model for the general serial–parallel system containing multiple components, to maximize system reliability after the maintenance action, and presented a solution algorithm by integrating the heuristic method with tabu search (TS).

In this chapter we assume that the system comprises two types of subsystems. One is the type of subsystems in which the components are very sensitive to the functioning of the whole system and, therefore, on deterioration these should be replaced by new ones. Let these subsystems range from 1 to \( s \). The other type of subsystems are those in which the components after deterioration can be repaired and then replaced. Let such subsystems range from \( s + 1 \) to \( m \). In fig. 1 the group \( X \) consists of the \( s \) subsystems with sensitive components which on failure are replaced by new ones and \( Y \) the remaining \( (m – s) \) subsystems in which the components can be repaired.

Figure 1-Parallel components in Repairable and Replaceable Subsystems.
4.2. Definitions and Notations

Every industrial and engineering organization depends upon the effective performance of repairable and replaceable components of the system. A repairable component of a system can be defined as a component which after deterioration can be restored to an operating condition by some maintenance action. On the other hand, a replaceable component is the one which after failure is replaced by a new one.

We consider a system which requires to perform a sequence of identical missions after every given (fixed) period. The system consists of several subsystems where each subsystem can work properly if at least one of its components is operational.

Thus we are working under the following two assumptions

Assumption 1: all the component states in a subsystem are independent

Assumption 2: the reliability, the cost and the weight of each component within a subsystem are identical

Let $r_i$ denote the probability that a component of a subsystem $i$ survives the mission given that the component is functioning at the start of the mission, and let $n_i$ denote the number of components in subsystem $i$ all in the functioning state at the start of the mission. Since group X of the system is a series arrangement of the subsystems, its reliability can be defined as

$$R_i = \prod_{i=1}^{s} \left\{ 1 - (1 - r_i)^{n_i} \right\}, \quad i = 1, 2, ..., s \quad (4.1)$$

Similarly for group Y composed of $(m - s)$ independent subsystems $(s + 1, s + 2, ..., m)$ connected in series, the reliability can be defined as
\[ R_2 = \prod_{i=s+1}^{m} \{ 1 - (1 - r_i)^{n_i} \} \quad i = s + 1, \ldots, m \]  

(4.2)

Since the system is a series arrangement of these two groups X and Y, the complete system reliability can be defined by

\[ R = \prod_{i=1}^{2} R_i, \quad i = 1,2 \]  

(4.3)

We will use the following notations in our formulation of the problem:

\( k_i = \) Total number of failed components in the subsystem \( i, i = 1,\ldots,m \) at the end of a mission

\( d_i = \) Number of failed components to be replaced or repaired in subsystem \( i, i = 1,\ldots,m \), prior to the next mission.

\( d = (d_1,\ldots,d_m) \)

\( a_i = \) Number of new components available for replacement in \( i^{th} \) subsystem of group X; \( i = 1,2,\ldots,s \).

\( t_i = \) Time units required for repairing and then replacing a failed component in the \( i^{th} \) subsystem of group Y; \( i = s + 1,\ldots,m \)

\( T_0 = \) Total time available for repairing / replacing the failed components in the system between two missions

\( c_i' = \) Cost units required for replacing a failed component by a new one in the subsystem \( i \) (of group X)

\( c_i'' = \) Cost units required for repairing and then replacing a failed component in the
4.3. Selective Maintenance

Ideally, all the failed components in each subsystem of group X are replaced by new ones prior to the beginning of the next mission/run. In a similar way, ideally all the failed components in the subsystems of group Y are repaired and then replaced prior to the beginning of the next mission/run. However, due to the constraints on the cost and time it may not be possible to repair and replace all the failed components in the system.

The time required for repairing and then replacing all the failed components in the $i^{th}$ subsystem of group Y is given by

$$t_i k_i, \ i = s + 1, \ldots, m$$

(4.4)

The maintenance time available for repairing and then replacing the failed components between two missions is $T_0$ time units.

Let us assume here that there is a separate server/team for the repairs of the failed components in the subsystems of group Y. If $T_0 < t_i k_i$ for at least one $i, i = s + 1, \ldots, m$ then all the failed components can not be repaired and replaced prior to the beginning of the next mission.

Further, the cost required for replacing the failed components by new ones in group X is

$$C_i = \sum_{i=1}^{s} c_i k_i$$

(4.5)

and the cost required for replacing the failed components after repairs in group Y is
\[ C_2 = \sum_{i=s+1}^{m} c_i k_i \]  

(4.6)

Therefore, the total cost required to repair and/or replace all the failed components in the system prior to the next mission by adding (4.5) and (4.6) is obtained as 

\[ C = C_1 + C_2 \]

Suppose that the total maintenance cost available for repairing and then replacing of failed components between two missions is \( C_0 \) cost units. If \( C_0 < C \), then all the failed components can not be repaired and/or replaced prior to the beginning of next mission. In such cases, a method is needed to decide which failed components should be repaired and replaced prior to the next mission and the rest be left in a failed condition. This process is referred to as Selective Maintenance.

In the selective maintenance the number of components available for the next mission in the \( i^{th} \) subsystem will be

\[ (n_i - k_i) + d_i \quad i = 1, 2, ..., m. \]  

(4.7)

### 4.4. Formulations of the Problem in various situations

Now we present three mathematical programming models in different prospects for the decision-makers.

**Model 1**: let the decision maker want to maximize the system reliability within the limited available budget and given maintenance time between two missions. From (4.3) and (4.7), the reliability of the system to be maximised is given by

\[ R = \prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i - k_i + d_i} \right\}. \]  

(4.8)
Since the total cost of replacing the components should not exceed $C_0$, we have

\[ \sum_{i=1}^{s} c_i^j d_i + \sum_{i=s+1}^{m} c_i^j d_i \leq C_0 . \]  

(4.9)

The maximum tolerable time between two missions (Spent in the repairs of the components in various subsystems of group Y simultaneously by separate servers/teams) is given by $T_0$. Thus we should have the $(m-s)$ constraints.

\[ t_i d_i \leq T_0, i = s+1,...,m . \]  

(4.10)

For replacement of failed components in the $i^{th}$ subsystem of group X only $a_i$ new components are available. This imposes the constraints:

\[ 0 \leq d_i \leq a_i \text{ and integer, } i = 1,..,s . \]  

(4.11)

If $a_i > k_i$ for some $i$, i.e. if the no. of failed items is less than the no. of available items then the upper bound $a_i$ for $i^{th}$ subsystem should be replaced by $k_i$ in (4.11).

Finally for group Y subsystems we should have

\[ 0 \leq d_i \leq k_i , \text{ and integer } i = s+1,...,m . \]  

(4.12)

The mathematical programming formulation of the problem is to maximize (4.8) under the constraints (4.9) to (4.12). We have to find the decision variables $d_i, i = 1,...,s, s+1,...,m$ by solving the following NLIP problem:
Allocation of components for a System Reliability

\[
\text{Maximize } R = \prod_{i=1}^{m} \left\{ \left[ 1 - (1-r_i) \right]^{r_i - k_i + d_i} \right\} \\
\text{Subject to}
\sum_{i=1}^{s} c_i' d_i + \sum_{i=s+1}^{m} c_i'' d_i \leq C_0 \\
t_i d_i \leq T_0, i = s + 1, \ldots, m \\
0 \leq d_i \leq a_i \text{ integer, } i = 1, \ldots, s \\
0 \leq d_i \leq k_i \text{ integer, } i = s + 1, \ldots, m
\] (4.13)

**Model 2:** If the decision maker wants to minimize the total cost for the required reliability \( R^* \), say, of the system then the problem is formulated as follows:

\[
\text{Minimize } C = \sum_{i=1}^{s} c_i' d_i + \sum_{i=s+1}^{m} c_i'' d_i \\
\text{Subject to}
\prod_{i=1}^{m} \left\{ \left[ 1 - (1-r_i) \right]^{r_i - k_i + d_i} \right\} \geq R^* \\
t_i d_i \leq T_0, i = s + 1, \ldots, m \\
0 \leq d_i \leq a_i \text{ integer, } i = 1, \ldots, s \\
0 \leq d_i \leq k_i \text{ integer, } i = s + 1, \ldots, m
\] (4.14)

**Model 3:** Let the decision makers want to minimize the total maintenance time for pre-determined reliability requirement \( R^* \) and the given cost \( C_0 \). For this situation we assume that there is a single team for the repairs of the components of various subsystems of group Y. This means that the repairing of the components in the various subsystems of group Y is in series. Then the problem can be formulated as
4.5. Numerical illustration

Consider a system having the group X consisting of 3 subsystems and also the group Y consisting of 3 subsystems. The available time between two missions for repairing and replacing the components is 10 time units. Let the given maintenance cost of the system be 680 units. The other parameters for the various subsystems are given in table 4.1.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Subsystem</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_i$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>$n_i$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$r_i$</td>
<td>0.8</td>
<td>0.75</td>
<td>0.8</td>
<td>$r_i$</td>
<td>0.8</td>
<td>0.75</td>
<td>0.8</td>
</tr>
<tr>
<td>$a_i$</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>$t_i$</td>
<td>4</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$c_i'$</td>
<td>120</td>
<td>105</td>
<td>120</td>
<td>$c_i'$</td>
<td>50</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>$k_i$</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>$k_i$</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 4.1:** The number of failed components and the respective cost and time etc. in the various subsystems.
Model 1: The nonlinear programming problem (NLPP) (4.13) corresponding to the model 1 is obtained as

\[
\text{Maximise } Z(d_1, ..., d_6) = \\
\left[ (1 - 0.8)^{(1+d_1)} \right] \times \left[ (1 - 0.75)^{(2+d_2)} \right] \times \left[ (1 - 0.8)^{(3+d_3)} \right] \times \left[ (1 - 0.75)^{(4+d_4)} \right] \times \left[ (1 - 0.8)^{(1+d_5)} \right] \times \left[ (1 - 0.75)^{(1+d_6)} \right] \\
\text{Subject to} \\
120d_1 + 105d_2 + 120d_3 + 50d_4 + 40d_5 + 50d_6 \leq 680 \\
4d_4 \leq 10, 5d_5 \leq 10, 3d_6 \leq 10 \\
0 \leq d_1 \leq 2, 0 \leq d_2 \leq 2, 0 \leq d_3 \leq 1, \text{and integer} \\
0 \leq d_4 \leq 3, 0 \leq d_5 \leq 2, 0 \leq d_6 \leq 3 \text{ and integer.}
\]

Note that the upper bound for \( d_2 \) is kept as 2 because only two components got failed in subsystem 2 although the available new components are 3.

The above nonlinear programming problem is solved by using LINGO computer program. The optimal Solution obtained after 264 iterations is as follows:

\[
d_1 = 2, d_2 = 1, d_3 = 0, d_4 = 2, d_5 = 2, d_6 = 3 \text{ with Max } Z = 0.9248
\]

So in subsystems 1, 2, and 3 we replace 2, 1, and 0 component respectively while in the subsystems 4, 5 and 6 we repair and then replace 2, 2 and 3 components respectively.

Model 2: let us assume that the minimum reliability required for the system is 0.96. The nonlinear programming problem (NLP) in which we minimize the cost under the reliability and time restraints is obtained from (4.14) as
\[ Min \ Z = 120d_1 + 105d_2 + 120d_3 + 50d_4 + 40d_5 + 50d_6 \]

Subject to

\[ Z(d_1, \ldots, d_6) \geq 0.96 \]

\[ 4d_4 \leq 10, 5d_5 \leq 10, 3d_6 \leq 10 \]

\[ 0 \leq d_1 \leq 2, 0 \leq d_2 \leq 2, 0 \leq d_3 \leq 1, \text{ and integer} \]

\[ 0 \leq d_4 \leq 3, 0 \leq d_5 \leq 2, 0 \leq d_6 \leq 3 \text{ and integer.} \]

The optimal Solution for the above problem is obtained as.

\[ d_1 = 2, d_2 = 2, d_3 = 1 \text{ and } d_4 = 2, d_5 = 2, d_6 = 2 \text{ with minimum maintenance cost } = 850 \]

**Model 3:** In model 3 is we minimize the maintenance time between two missions with the upper limit for reliability of the system as 0.96 and limited maintenance cost for the system as 850. The corresponding NLP problem from (4.15) is

\[ Min \ Z = 4d_4 + 5d_5 + 3d_6 \]

Subject to

\[ Z(d_1, \ldots, d_6) \geq 0.96 \]

\[ 120d_1 + 105d_2 + 120d_3 + 50d_4 + 40d_5 + 50d_6 \leq 850 \]

\[ 0 \leq d_1 \leq 2, 0 \leq d_2 \leq 2, 0 \leq d_3 \leq 1, \text{ and integers.} \]

\[ 0 \leq d_4 \leq 3, 0 \leq d_5 \leq 2, 0 \leq d_6 \leq 3 \text{ and integers.} \]
The optimal Solution obtained for this problem is obtained as

\[ d_1 = 2, d_2 = 2, d_3 = 1, d_4 = 2, d_5 = 2, d_6 = 2 \] with minimum maintenance time = 24.