CHAPTER 5

A NUMERICAL APPROACH WITH VARIABLE TEMPERATURE BOUNDARY CONDITIONS OVER A CONTINUOUS MOVING PLATE

5.1 INTRODUCTION

Heat and mass transfer plays an important role in drying, filtration processes, saturation of porous materials by chemicals, solar energy collectors, nuclear reactors, in manufacturing industries for the design fins, steel, rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, space craft design, satellites, combustion and furnace design, material processing, energy utilization, temperature measurement, remote sensing for astronomy and space exploration, food processing and cryogenic Engineering, as well as numerous agricultural, health and military application. Effect of heat and mass transfer plays vital role, in space craft design, in nuclear reactors, pollution of environment etc. Flow through porous media have numerous engineering problems, for example, in the study of underground water resources ,the movement of oil and natural gas through oil reservoirs, purification of crude oil, pulp. The study of convection flow with mass transfer along a vertical plate is receiving considerable attention to many researchers because of its vast application in the field of cosmic and geophysical science. Processes involving coupled heat and mass transfer occur frequently in nature. It occurs not only due to temperature difference, but also due to concentration difference or the combination of these two. Quite often, there exist certain industrial processes involving continuous surfaces that move steadily through an otherwise quiescent ambient environment for which a correct assessment of the axial temperature and concentration variation of the material are given relevant importance. Flow through porous media have numerous engineering problems, for example, in the underground water resources, rainwater harvesting, the movement of oil and natural gas through oil and sandstone reservoirs, purification of crude oil, paper and pulp industry, membrane separation process, the flow of blood. The purpose of present study is to investigate the heat and mass transfer effect on flow past an exponentially accelerated infinite vertical plate with variable temperature along with mass diffusion.

The boundary layer flow past a continuously moving semi-infinite plate was first studied by Sakiadis (1961a, 1961b and 1961c). The axisymmetric case was considered by Koldenhof (1965) whereas the heat transfer aspect was considered by Tsou et al (1967). The case of flat plate, and continuously moving cylinders, was discussed by Pechoc (1967), Bourne & Elliston (1970), Rotte & Beck (1969) and Karmis & Pechoc (1978).The viscous dissipation effects were not considered. These problems are important in technology, it is necessary to study the effects of variable plate temperature on the temperature field. Such a phenomenon has been studied in case of stationary plate by Tifford & Chu (1949). Heat transfer in flow past a continuous moving plate with variable temperature was studied by Soundalgekar & Ramana Murty (1980) It has been shown in Schlichting (1968) that similarity solution to energy equation does not exist when viscous dissipation and variable plate temperature are considered simultaneously. Hence without considering viscous dissipation effects, the problem is considered here on taking into account the variable temperature of the plate.

Motivated by the above mentioned investigations and applications, in this chapter the numerical solution is presented to study the mass transfer in flow past a continuous moving plate with variable temperature.

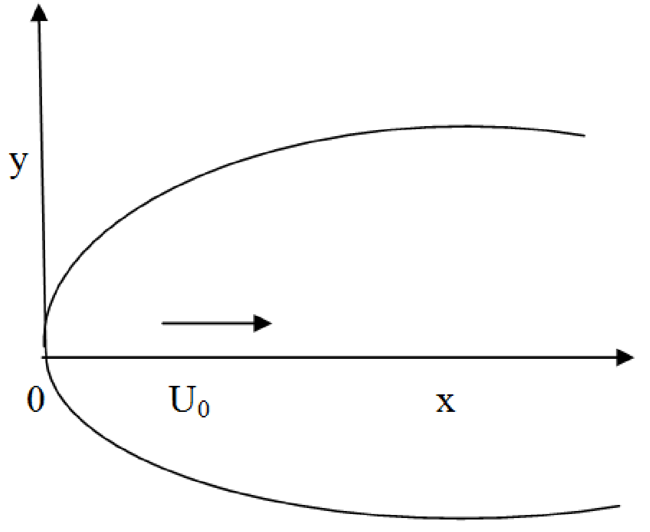

Figure 5.1 Schematic representation of the boundary layer on a continuous moving surface

5.2 ANALYSIS

The plate is assumed to be moving in a stationary fluid which is incompressible and viscous. The x-axis is taken along the moving plate and the y-axis is taken normal to the plate. If u and v are the velocity components along x and y-axis respectively, then under usual boundary layer approximation, the flow and the mass transfer are given by the system of boundary layer equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{5.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{5.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{5.4}$$

Here υ is the kinematic viscosity, α the thermal diffusivity, T is the temperature of the fluid, C is the concentration of the fluid and D is the coefficient of the mass diffusivity.

The boundary conditions are

$$u = U_0, \quad v = 0, \quad T=T_w, \quad C = C_w \quad \text{at } y = 0$$

$$u = 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{at } y \to \infty \tag{5.5}$$

It is assumed that the plate temperature varies as

$$T_w(x) - T_\infty = Ax^n \tag{5.6}$$

Introducing the usual similarity transformations

$$\eta = y\sqrt{\frac{U_0}{\nu x}}$$
$$\psi = \sqrt{\nu x U_0}\; f(\eta) \tag{5.7}$$

The velocity components are given by

$$u=\frac{\partial \psi}{\partial y} \quad \textit{and} \quad v=-\frac{\partial \psi}{\partial x} \tag{5.8}$$

Equations (5.1) -(5.4) reduce to

$$f''' + \frac{1}{2} f f'' = 0 \tag{5.9}$$

$$\theta'' - \Pr n f' \theta + \frac{1}{2} \Pr f \theta' = 0 \tag{5.10}$$

$$\varphi'' + \frac{1}{2} Sc f \varphi' = 0 \tag{5.11}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}; \qquad \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Where (5.12)

$$\Pr = \frac{\mu c_p}{k}; \qquad Sc = \frac{\nu}{D}$$

and the primes denote differentiation with respect to η.

The transformed boundary conditions are given by

At $\eta = 0$; $f = 0$; $f' = 1$; $\theta = 1$; $\phi = 1$

As $\eta \to \infty$; $f'(\infty) = 0$; $\theta(\infty) = 0$; $\phi(\infty) = 0$ (5.13)

Numerical solutions of Equations (5.9) to (5.11) for n = 0 have been given by Tsou et al (1967) for an incompressible fluid with different Prandtl numbers. The solutions and numerical results for n = 0, 1 and 2 and for Sc = 0.22, 0.6, 0.78 are given. The concentration profiles are shown in Figure 5.2 and it depicts that there is a fall in concentration of fluid with increasing Sc or n.

The rate of heat and mass transfer, in terms of the Sherwood number is given by

$$Sh = \frac{xM_w}{D_m \Delta C} = -\left(\mathrm{Re}\right)^{\frac{1}{2}} \varphi'(0)$$

$$\text{i.e., } Sh\left(\mathrm{Re}\right)^{-\frac{1}{2}} = -\varphi'(0) \qquad (5.14)$$

where M_w is the mass flux at the wall and are given by

$$M_w = -D_M \left(\frac{\partial C}{\partial y}\right)_{y=0} = -D_M \Delta C \sqrt{\frac{U_0}{\nu x}}\, \varphi'(0) \qquad (5.15)$$

here $\Delta C = C_w - C_\infty$.

Table 5.1 Numerical values of - θ′(0) for various values of Pr and n

Pr \ n	0	1	2
0.7	0.3508	0.8028	1.1211
2	0.6831	1.4683	2.0024
10	1.6808	3.4505	4.6431

Table 5.2 Numerical values of - ϕ′(0) for various values of Sc and n

Sc \ n	0	1	2
0.22	0.1456	0.1456	0.1456
2	0.68325	0.68325	0.68325
5	1.1538	1.1538	1.1538

Table 5.3 to 5.8 shows the viscous boundary layer thickness and thermal boundary layer thickness.

Table 5.3 Viscous boundary layer thickness (η_m) for different values of Sc, n and Pr=0.25

Pr=0.25						
n	Sc					
	0.25	0.5	0.75	1	1.25	1.5
0.25	3.1875	2.625	2.1875	1.8125	1.5625	1.3125
0.5	4.5625	3.875	3.3750000	3.0000	2.6875000	2.3750000
0.75	5.2500000	4.5000000	3.9375000	3.5625000	3.2500000	3.0000000
1	5.5625	4.7500000	4.1875000	3.8125000	3.5000000	3.1875000
1.25	5.6250000	4.7500000	3.8125000	3.8125000	3.5625000	3.3125000
1.5	5.5625	4.6875000	4.1875000	3.7500000	3.5000000	3.2500000

Table 5.4 Thermal boundary layer thickness (η_t) for different values of Sc, n and Pr=0.25

Pr=0.25						
n	Sc					
	0.25	0.5	0.75	1	1.25	1.5
0.25	5.8125	5.5	5.25	4.9375	4.6875	4.5
0.5	5.3125000	4.9375000	4.6250000	4.3750000	4.1875000	4.0000000
0.75	4.7500000	4.2500000	3.8750000	3.6875000	3.4375000	3.3125000
1	4.2500000	3.7500000	3.4375000	3.1875000	3.0000000	2.8125000
1.25	4.0000000	3.4375000	3.4375000	2.8750000	2.6875000	2.5625000
1.5	3.6875000	3.1875000	2.8750000	2.6250000	2.5000000	2.3125000

Table 5.5 Viscous boundary layer thickness (η_m) for different values of Sc, n and Pr=0.5

Pr=0.5						
n	Sc					
	0.25	0.5	0.75	1	1.25	1.5
0.25	3.25	2.6875	2.1875	1.875	1.5625	1.375
0.5	4.3125	3.5625	3.0625	2.6875	2.4375	2.25
0.75	4.6875	3.8125	3.3125	2.9375	2.6875	2.4375
1	4.75	3.875	3.375	3	2.75	2.5
1.25	4.625	3.75	3.3125	2.9375	2.75	2.5
1.5	4.5	3.625	3.1875	2.8125	2.625	2.4375

Table 5.6 Thermal boundary layer thickness (η_t) for different values of Sc, n and Pr=0.5

Pr=0.5						
n	Sc					
	0.25	0.5	0.75	1	1.25	1.5
0.25	6.1875	5.9375	5.625	5.3125	5.0625	4.8125
0.5	5.125	4.6875	4.375	4.125	4	3.8125
0.75	4.6875	3.75	3.4375	3.1875	3	2.875
1	3.75	3.1875	2.875	2.625	2.5	2..375
1.25	3.4375	2.875	2.5625	2.3125	2.1875	2.0625
1.5	3.1875	2.625	2.3125	2.125	1.9375	1.8125

Table 5.7

Viscous boundary layer thickness (η_m) for different values of Sc, n and Pr=0.2

Pr=0.2						
n	Sc					
	0.25	0.50	0.75	1.00	1.25	1.50
0.25	3.1250000	2.5000000	2.1250000	1.7500000	1.5000000	1.2500000
0.50	4.6250000	4.0000000	3.5000000	3.0625000	2.6875000	2.3750000
0.75	5.5625000	5.0000000	4.5000000	4.0625000	3.6875000	3.3125000
1.00	6.3750000	5.8125000	5.3125000	4.8750000	4.5000000	4.1250000
1.25	7.0000000	6.5000000	6.0000000	5.5625000	5.1875000	4.8750000
1.50	7.5625000	7.0625000	6.6250000	6.1875000	5.8750000	5.5000000

Table 5.8

Thermal boundary layer thickness (η_t) for different values of Sc, n and Pr=0.2

Pr=0.2						
n	Sc					
	0.25	0.5	0.75	1.00	1.25	1.5
0.25	5.6250000	5.2500000	6.3125000	4.7500000	4.5625000	4.3125000
0.50	5.3750000	5.0000000	4.6875000	4.4375000	4.1875000	4.0000000
0.75	5.1875000	4.6250000	4.3125000	4.0625000	3.8125000	3.6250000
1.00	4.6875000	4.3125000	4.0625000	3.8125000	3.5625000	3.3750000
1.25	4.5000000	4.1250000	3.8125000	3.6250000	3.3750000	3.2500000
1.50	4.3125000	3.9375000	3.6875000	3.4375000	3.2500000	3.1250000

5.3 RESULTS AND DISCUSSION

An increase in Pr or n leads to an increase in the value of the Nusselt number. An increase in Sc or n leads to a fall in the concentration and a rise in the value of the Sherwood number. The values of viscous boundary layer thickness η_m and thermal boundary layer thickness η_t are given in Tables 5.3 to 5.8, for various values of Sc, Pr, n. It is clearly understood that as the Schmidt number Sc increases, the viscous boundary layer thickness η_m decreases. Also it is clearly understood from the Figure 5.2 to 5.4 that as Pr increases the thermal boundary layer thickness η_t decreases irrespective of n and Pr. As n increases the viscous boundary layer thickness η_m increases, this is true for $0 \leq n \leq 1$. For $n > 1$ for some cases of Pr it is decreasing. As n increases the thermal boundary layer thickness η_t decreases irrespective of Pr and Sc. As Pr increases the viscous boundary layer thickness η_m decreases and as Pr increases the thermal boundary layer η_t decreases irrespective of n and Sc.

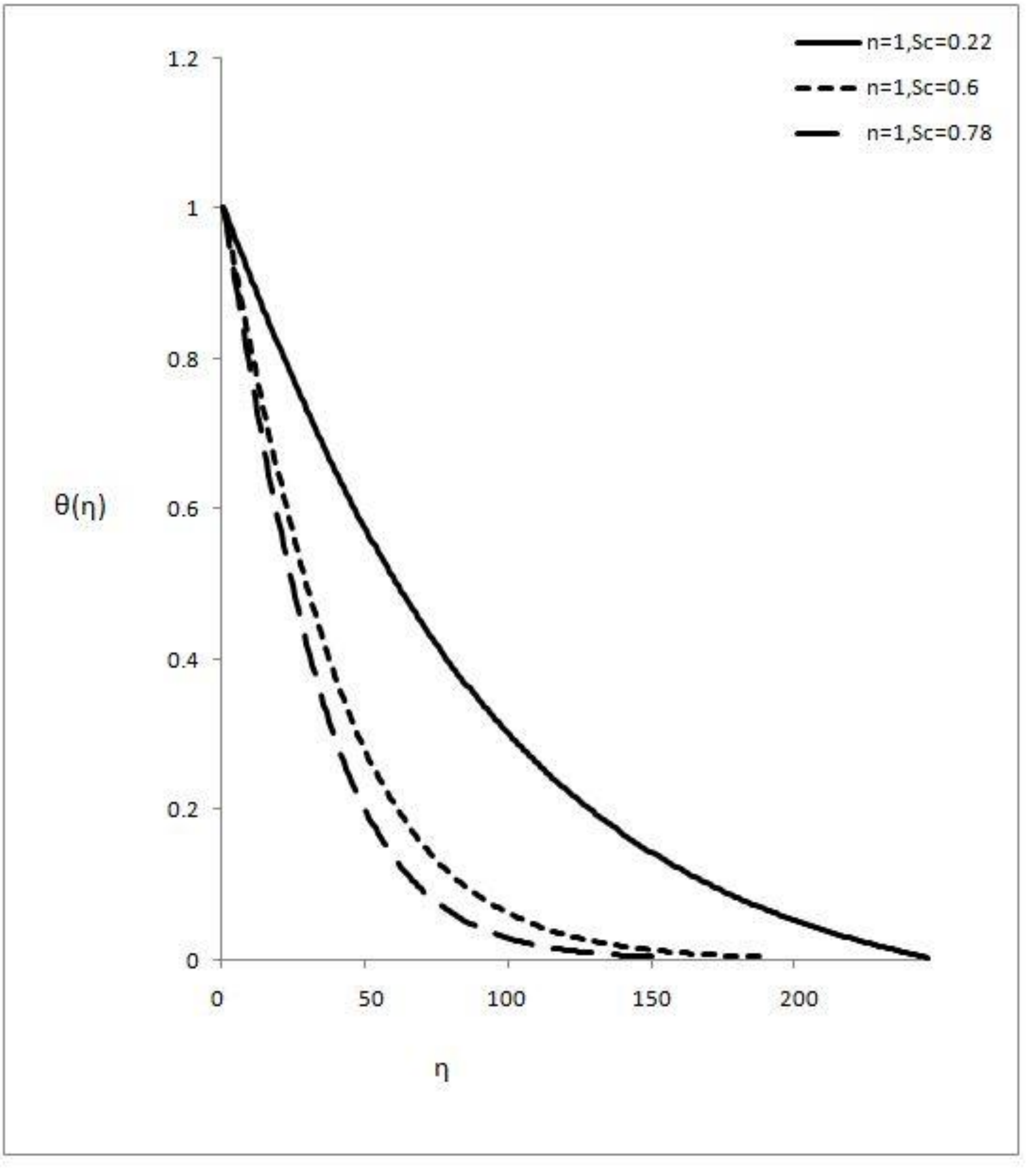

Figure 5.2 Temperature profiles with Pr = 0.7 and n = 1

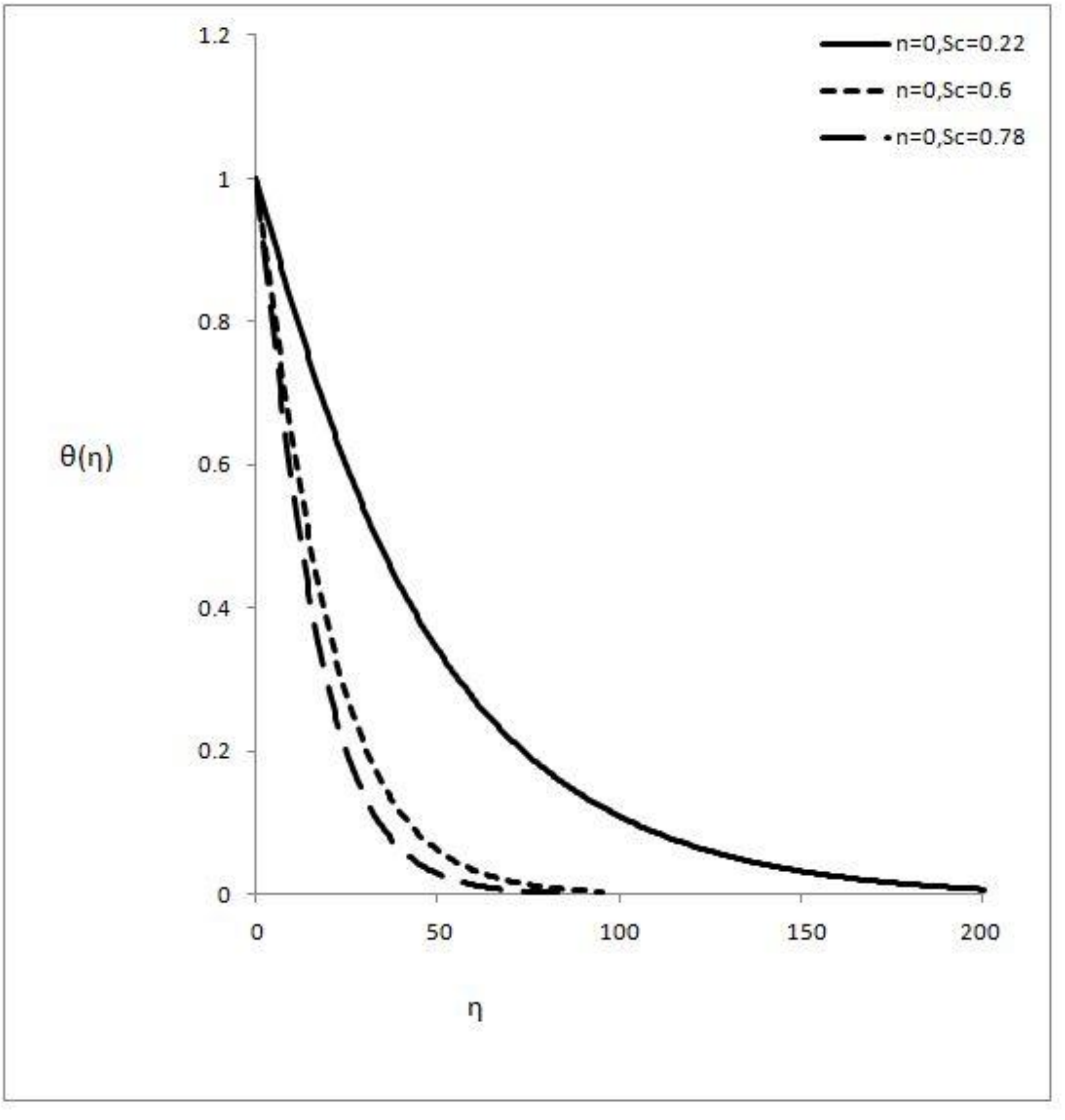

Figure 5.3 Temperature profiles with Pr = 0.7 and n = 0

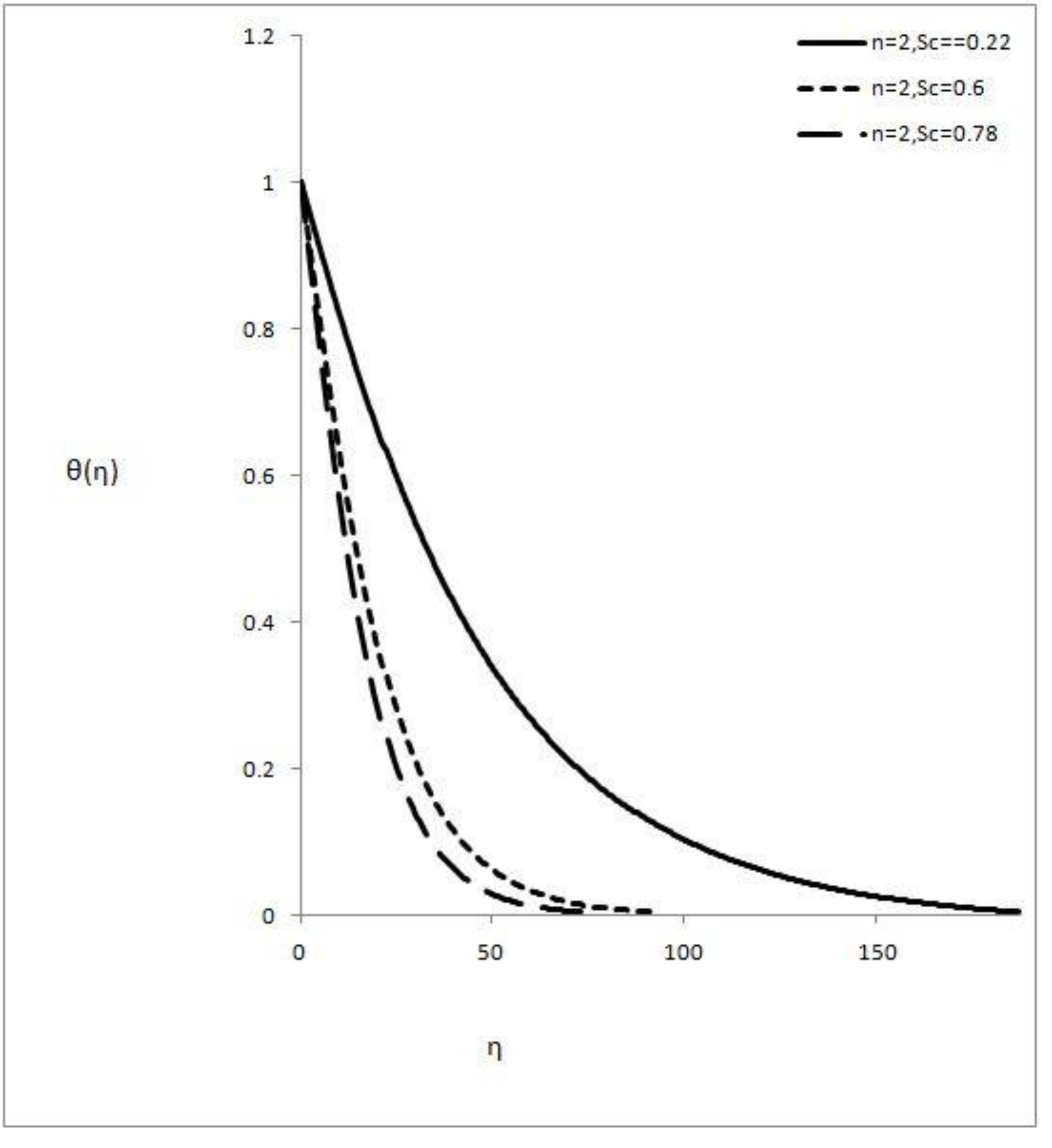

Figure 5.4 Temperature profiles with Pr = 0.7 and n = 2

5.4 CONCLUSION

Under the conditions mentioned, it can be concluded that increase in Prandtl number leads to increase in Nusselt number and thickness of the viscous boundary layer. However, the thickness of the thermal boundary layer decreases with increase in Prandtl number.

Also, an increase in Schmidt number leads to increase in Sherwood number, whereas the viscous boundary layer decreases. An increase in ‘n’ leads to increase in Nusselt number, Sherwood number and thickness of the thermal boundary layer.