CHAPTER 4

EFFECT OF RADIATION ON HEAT TRANSFER OVER AN EXPONENTIALLY STRETCHING SURFACE WITH MHD

4.1 INTRODUCTION

The flow of a viscous incompressible fluid over a stretching sheet has many applications in manufacturing industries and technological process, such as, glass-fiber production, wire drawing, paper production, plastic sheets, metal and polymer processing industries and many others. The study of laminar flow and heat transfer over a stretching sheet in a viscous fluid has been of considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. The study of Magneto Hydro Dynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metalworking processes. The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field.

Since the revolutionary work of Sakiadis (1961a and 1961b) on boundary-layer flow past a moving surface, a great deal of research work has been carried out for the two-dimensional boundary-layer flows. Crane (1970) extended the Sakiadis (1961a and 1961b) flow problem by assuming a stretching boundary. The Crane's problem is one of the flow problems in boundary layer theory that possesses an exact solution. The heat and mass transfer in the flow past a porous stretching surface was discussed by Gupta & Gupta (1977). Brady & Acrivos (1981) proved the existence and uniqueness of the solution for the stretching flow. Three-dimensional flow due to a stretching surface was analyzed by McLeod & Rajagopal (1987). In another paper Wang (1988) extended Crane's problem for a stretching cylinder. Flow problems due to a stretching surface have important applications in geothermal energy recovery, oil recovery and metal extrusion. The transportation of heat in a porous medium has applications in geothermal systems, rough oil mining, soil-water contamination, and bio mechanical problems. Vajravelu (1994) reported steady flow and heat transfer of viscous fluids by considering different heating process in a porous medium.

The above mentioned problems deal with linear stretching of the surface. Magyari & Keller (1999) initiated the work by assuming an exponentially stretching surface. The heat transfer analysis for flow past an exponentially stretching surface was carried out by Elbashbeshy (2001). The same problem by considering the constitutive equation of a visco elastic fluid was investigated by Khan (2006). Due to the applications in electrical power generation, astrophysical flows, solar power technology, space vehicle reentry, and so forth, the heat transfer analysis in the presence of radiation is another important area of research. Raptis (1986) investigated the radiation effects for the flow past a semi-infinite flat plate. The influence of thermal radiation in a visco-elastic fluid past a stretching sheet was illustrated by Chen (2010). Olanrewahu & Abbas (2014a) presented a corrigendum to 'Convection heat and mass transfer in a hydromagnetic flow in presence of thermal radiation and thermal diffusion. Olanrewahu & Hayat (2014b) analysed the effect of buoyancy and transpiration on the flow and heat transfer over a moving permeable surface in presence of radiation.

Sajid & Hayat (2008) discussed the homotopy series solution for the influence of radiation on the flow past an exponentially stretching sheet. Chiam (1998) investigated the heat transfer analysis with variable thermal conductivity in a stagnation point flow towards a stretching sheet. In another paper, the analysis of the effects of variable thermal conductivity was discussed by Chiam (1996). The boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes such as hot rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products as investigated by Magyari & Keller (1999). Magyari et al (2001) also further analysed and reported the analytical and computational solutions when the surface moves with rapidly decreasing velocity using self similarity method.

Magneto hydrodynamic flows have applications in, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of the earth's core. In addition, from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magneto spheres, aeronautical plasma flows, chemical engineering and electronics.

With a general power function form for stretching velocity of the wall, the same problem has been analyzed with surface suction/injection by Ali & Al-Yousef (2002). Magyari & Keller (1999) investigated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Partha et al (2005) studied the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Recently, Sajid et al (2006) extended this problem by investigating the radiation effects on the flow over an exponentially stretching sheet, and solved the problem analytically using the homotopy analysis method. The numerical solution for the same problem was then given by Bidin & Nazar (2009). The study of magneto hydrodynamics has important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field Ganesan & Palani (2004) reported that at high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment as discussed by Seddeek (2002).

In this chapter the numerical solution is presented to study the effect of radiation on heat transfer over an exponentially stretching surface with MHD.

4.2 ANALYSIS

A steady two dimensional flow of an incompressible viscous and electrically conducting fluid caused by a stretching sheet, which is placed in a quiescent ambient fluid of uniform temperature T_∞, is shown in Figure 4.1. A variable magnetic field B(x) is applied normal to the sheet and that the induced magnetic field is neglected, which is justified for MHD flow at small magnetic Reynolds number. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2 u}{\rho} \tag{4.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{4.3}$$

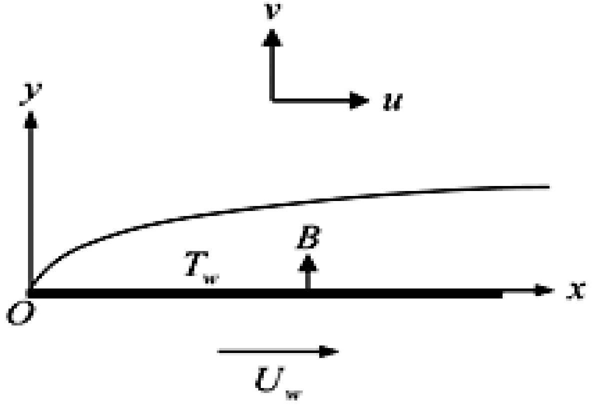

Figure 4.1 Physical model and coordinate system

where u and v are the velocities in the x and y directions, respectively, ρ is the fluid density, υ the kinematic viscosity, k the thermal conductivity, c_p the specific heat, T the fluid temperature in the boundary layer and q_r is the radiative heat flux. The boundary conditions are given by

$$\begin{aligned} &u=U_w=U_0e^{x/L},\ v=0\\ &T=T_w=T_\infty+T_0e^{x/2L} \quad at\ \ y=0\\ &u\to 0, T\to T_\infty \quad as\ \ y\to\infty \end{aligned} \tag{4.4}$$

where U_0 is the reference velocity, T_0 the reference temperature and L is the reference length. Most of the effort in understanding fluid radiation is devoted to the derivation of reasonable simplifications as given by Aboeldahab & El Gendy (2002).

For an optically thick gas, the gas self-absorption rises and the situation become difficult. However, the problem can be simplified by using the Rosseland approximation which simplifies the radiative heat flux as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{4.5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. This approximation is valid at points optically far from the boundary surface, and it is good only for intensive absorption, which is for an optically thick boundary layer as studied by Bataller (2008b). It is assumed that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms gives:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{4.6}$$

Using Equations (4.5) and (4.6), Equation (4.3) reduces to:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2} \tag{4.7}$$

To obtain similarity solutions, it is assumed that the magnetic field B(x) is of the form

$$B = B_0 e^{x/2L} \tag{4.8}$$

where B_0 is the constant magnetic field.

The continuity Equation (4.1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \; and \; v = -\frac{\partial \psi}{\partial x} \tag{4.9}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation as adopted by Sajid & Hayat (2006)

$$\begin{aligned} &u = U_0 e^{x/L} f'(\eta), \quad v = -\left(\frac{\nu U_0}{2L}\right)^{1/2} e^{x/2L}\left(f(\eta) + \eta f'(\eta)\right) \\ &T = T_\infty + T_0 e^{x/2L}\theta(\eta), \quad \eta = \left(\frac{U_0}{2\nu L}\right)^{1/2} e^{x/2L} y \end{aligned} \tag{4.10}$$

where η is the similarity variable, f(η) is the dimensionless stream function, θ(η) is the dimensionless temperature and primes denote differentiation with respect to η. The transformed ordinary differential equations are:

$$f''' + f f'' - 2f'^2 - Mf' = 0 \tag{4.11}$$

$$\left(1 + \frac{4}{3}K\right)\theta'' + \Pr\left(f\theta' - f'\theta\right) = 0 \tag{4.12}$$

in which $M = \frac{2\sigma B_0^2 L}{\rho U_0}$ is the magnetic parameter, $K = \frac{4\sigma^* T_\infty^3}{k^* k}$ the radiation parameter and $\Pr = \frac{\rho \nu c_p}{k}$ is the Prandtl number. The transformed boundary conditions are:

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \quad at \;\; \eta = 0 \\ &f'(\infty) \to 0, \;\; \theta(\infty) \to 0 \qquad\qquad as \;\; \eta \to \infty \end{aligned} \tag{4.13}$$

The skin-friction coefficient and the local Nusselt number which represents the wall shear stress and the heat transfer rate at the surface, respectively are important physical parameters for this type of boundary layer flow.

4.3 SOLUTION OF THE PROBLEM

The set of coupled non-linear governing boundary layer Equations (4.11) and (4.12) together with the boundary conditions (4.13) are solved numerically by using Runge-Kutta Gill method together with the shooting technique. First of all, higher order non-linear differential Equations (4.11) and (4.12) are converted into simultaneous linear differential equations and they are further transformed into the initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge-Kutta Gill method. The step size $\Delta\eta = 0.05$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the local Nusselt number, which is proportional to $-\theta'(0)$ is also sorted out and their numerical values are presented in a tabular form.

Table 4.1. Comparison of values of $-\theta'(0)$ for different values of K, M and Pr with Magyari and Keller (1999)

K	M	Pr	Magyari and Keller (1999)	Present results
0	0	1	0.954782	0.9549
0	0	3	1.869075	1.8692
0	0	5	2.500135	2.5000
0	0	10	3.660379	3.6604

Table 4.2 Comparison of values of $-\theta'(0)$ for different values of K, M and Pr with El-Aziz (2009)

K	M	Pr	El-Aziz (2009)	Present results
0	0	1	0.954785	0.9549
0	0	3	1.869074	1.8692
0	0	5	2.500132	2.5000
0	0	10	3.660372	3.6604

Table 4.3 Comparison of values of $-\theta'(0)$ for different values of K, M and Pr with Bidin and Nazar (2009)

K	M	Pr	Bidin and Nazar (2009)	Present results
0	0	1	0.9548	0.9549
0	0	2	1.4714	1.4716
0	0	3	1.8691	1.8692
1	0	0	0.5315	0.5313

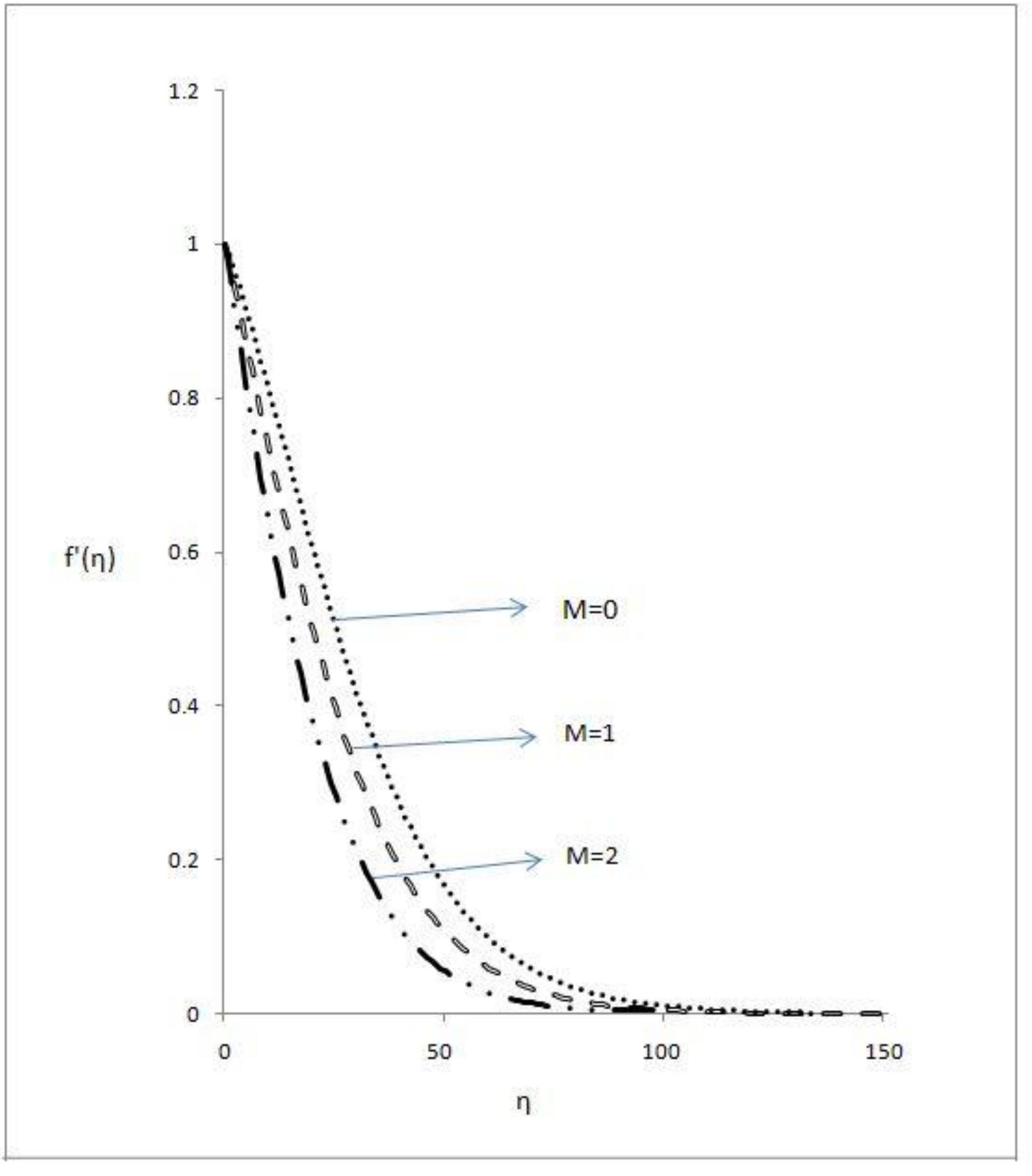

Figure 4.2 Velocity profile for different values of M

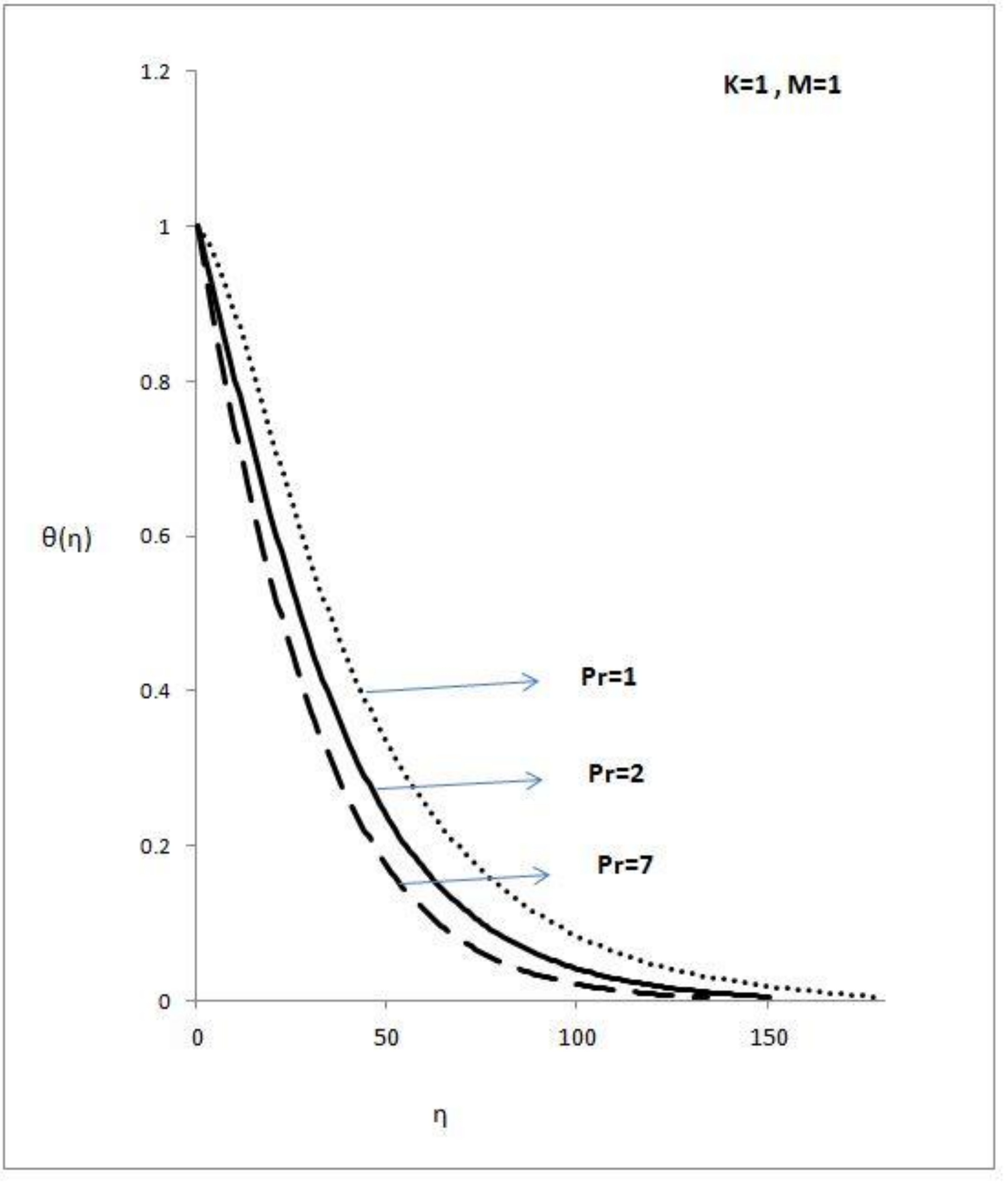

Figure 4.3 Temperature profile for different values of Pr

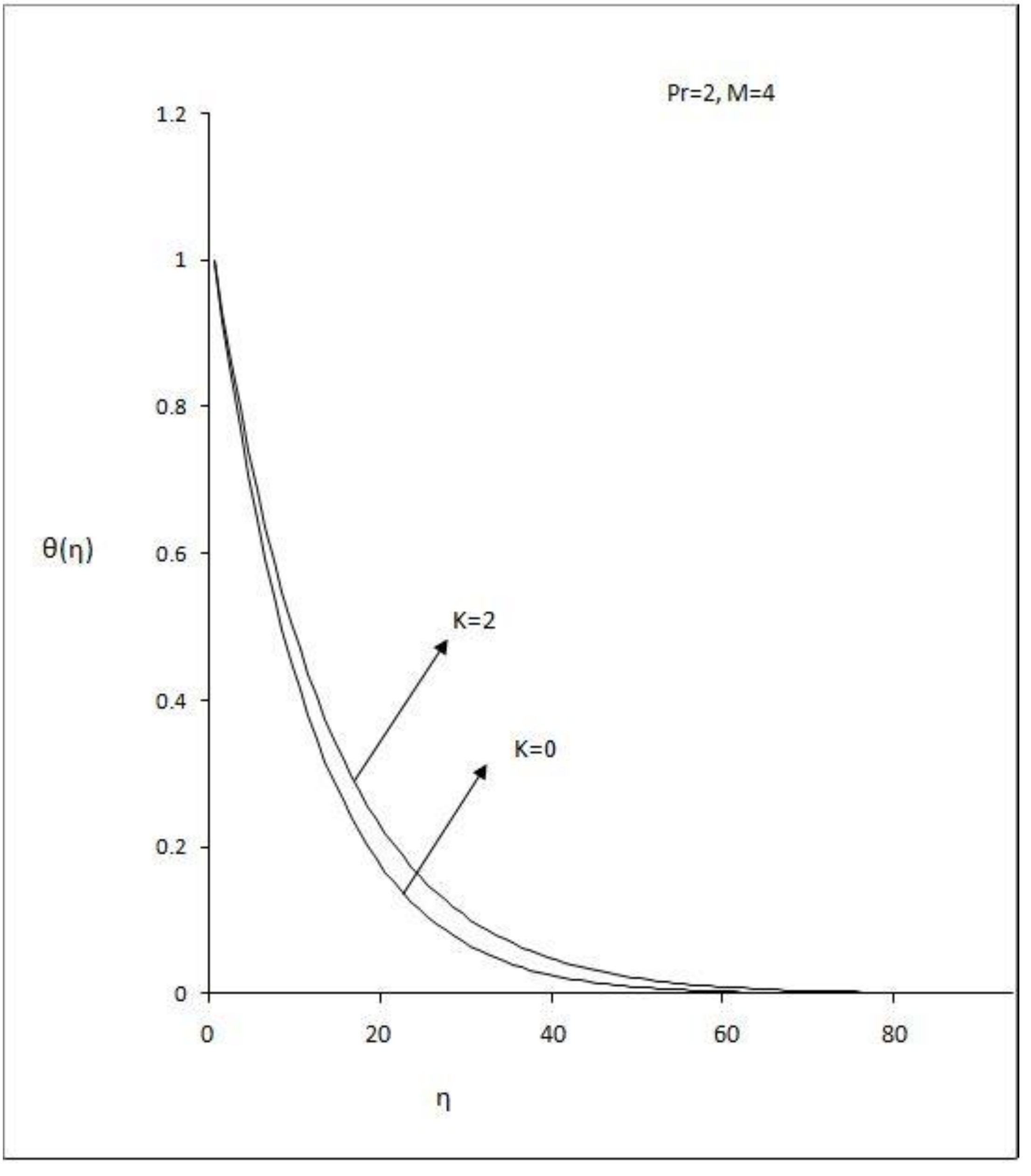

Figure 4.4 Temperature profiles for different values of K

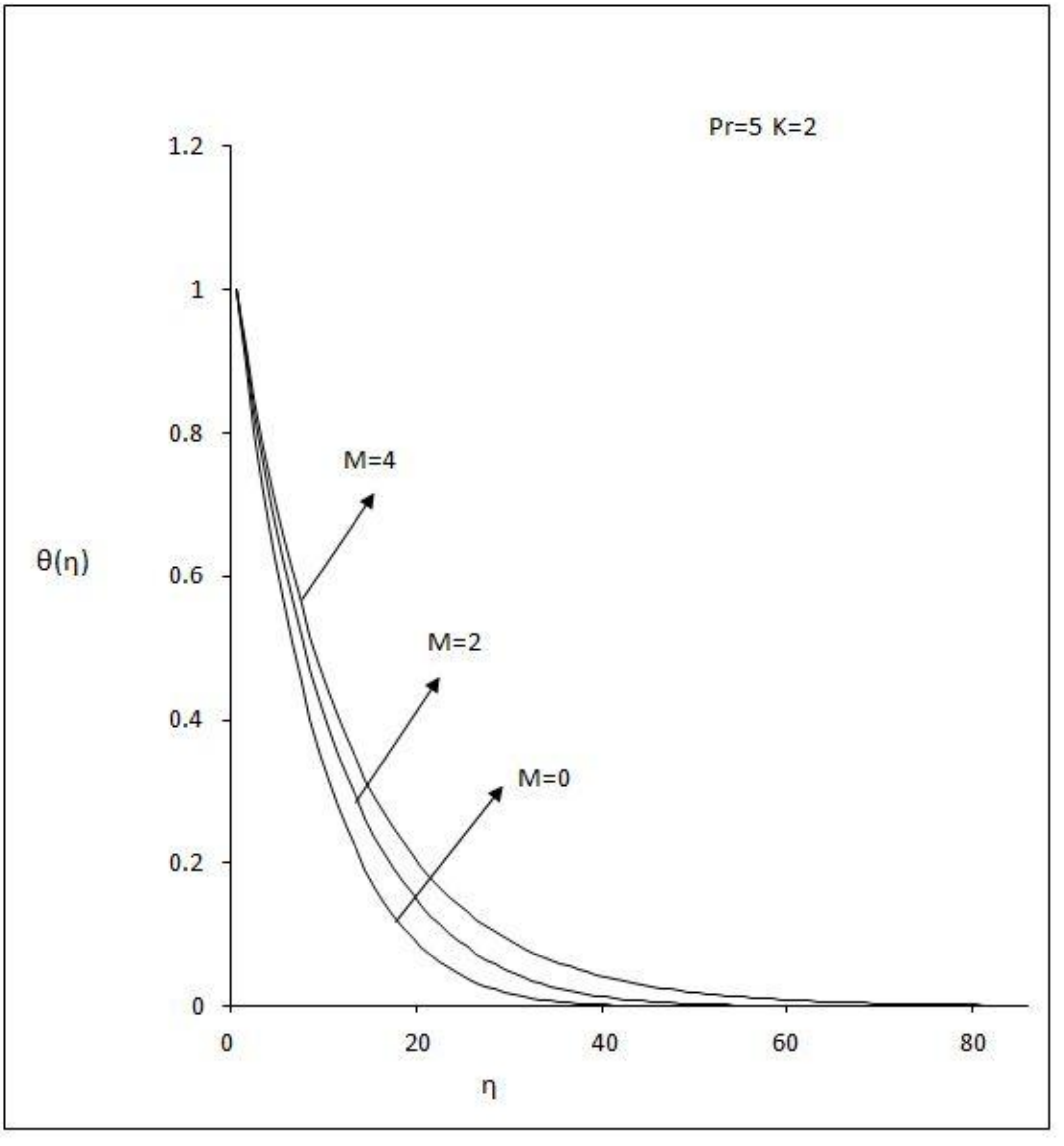

Figure 4.5 Temperature profile for different values of M

4.4 RESULTS AND DISCUSSION

The numerical values due to the effect of magnetic field parameter M, radiation parameter K and Prandtl number Pr on - $\theta'(0)$ is given in the Table 4.1, 4.2 and 4.3. Comparison of the present results with the existing results available in the literature are presented in Table 4.1, 4.2 and 4.3. It can be understood that the present results are in good agreement with the results available in the literature.

The profiles of velocity and temperature are shown in Figures 4.2 to 4.5 respectively, with various values of the parameters. The velocity profiles for different values of the magnetic parameter M presented in Figure 4.2 show that the rate of transport is considerably reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is because the variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to the transport phenomena. We note that the Prandtl number Pr and the radiation parameter K have no influence on the flow field, which is clear from Equation 4.11. The velocity gradient at the surface $f''(0)$ which represents the surface shear stress increases with increasing M. Thus, the magnetic parameter M acts as a controlling parameter to control the surface shear stress.

The temperature profiles for different values of Pr, K and M with other parameters are fixed to unity are presented in Figures 4.3, 4.4 and 4.5, respectively. Figures 4.2 to 4.5 shows that the far field boundary conditions are satisfied asymptotically, thus supporting the accuracy of the numerical results obtained. It is evident from Figures 4.3 to 4.5 that the thermal boundary layer thickness increases as M and K increase, but opposite trends are observed for increasing values of Pr. This results in decreasing manner of the local Nusselt number $-\theta'(0)$, which represents the heat transfer rate at the surface, with increasing M and K but opposite trends are observed for increasing values of Pr. This is because, when Pr increases, the thermal diffusivity decreases and thus the heat is diffused away from the heated surface more slowly and in consequence increase the temperature gradient at the surface

4.5 CONCLUSION

The effect of radiation on heat transfer over an exponentially stretching surface with MHD was investigated. The numerical results obtained agreed very well with previously reported cases available in the literature. It was found that the surface shear stress increases with the magnetic parameter M, while the heat transfer rate increases with Prandtl number Pr, but decreases with both magnetic parameter M and radiation parameter K.