CHAPTER 3

FREE CONVECTION OF HEAT TRANSFER IN FLOW PAST A SEMI-INFINITE FLAT PLATE IN TRANSVERSE MAGNETIC FIELD WITH HEAT FLUX

3.1 INTRODUCTION

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications are found in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion, of the earth's core. Also, it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

The study of boundary layer heat flow and mass transfer over an inclined plate has generated much interest among various researchers in astrophysical, renewable energy systems and also hypersonic aerodynamic area for a number of decades. In recent years MHD flow problems have become a considerable interest in view of its significant applications in industrial manufacturing processes such as plasma studies, petroleum industries, Magneto Hydro Dynamics power generator cooling of nuclear reactors, boundary layer control in aerodynamics. The heat removal strategies in many engineering applications such as cooling of electronic components rely on natural convection heat transfer, due to its simplicity, minimum cost, low noise, smaller size and reliability. In most natural convection studies, the base fluid has a low thermal conductivity, which limits the heat transfer enhancement. However, the continuing miniaturization of electronic devices requires further heat transfer improvements from an energy saving viewpoint. Many authors have studied the effects of magnetic field on mixed, natural and forced convection heat and mass transfer problems. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. This problem has also an important bearing on metallurgy where Magneto Hydro Dynamic (MHD) techniques have recently been used. Raptis & Singh (1983) studied the effects of uniform transverse magnetic field on the free convection flow of an electrically conducting fluid past an infinite vertical plate for the classes of impulsive and uniformly accelerated motion of the plate.

Investigation of Magneto Hydro Dynamic flow for an electrically conducting fluid past a heated surface has attracted the interest of many researchers in view of its important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. This study has been largely concerned with the flow and heat transfer characteristics in various physical situations. However, few studies have been carried out to examine the effect of geometric complexity, such as irregular surfaces, on the convection heat transfer which is because complicated boundary conditions or external flow fields are difficult to deal with. However, the prediction of heat transfer from an irregular surface is of fundamental importance, and is encountered in several heat transfer devices, such as flat-plate solar collectors and flat-plate condensers in refrigerators. Moreover, surfaces are sometimes intentionally roughened to enhance heat transfer for the presence of rough surfaces disturbs the flow and alters the heat transfer rate. However, all of the previous studies considered only the case of a flat plate or simple two-dimensional bodies, and few have been done on wavy surfaces. The study of boundary layer flow of heat and mass transfer over an inclined plate has generated much interest from astrophysical, renewable energy systems and also hypersonic aerodynamics researchers for a number of decades. Many authors have studied the effects of magnetic field on mixed, natural and forced convection heat and mass transfer problems. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing.

The study of flow and heat transfer in fluid past a porous surface has attracted considerable scientific attention based on applications, chemical engineering, where boundary-layer control, transpiration cooling and gaseous diffusion are important. Equally important is the study of heat generation or absorption in moving fluids for problems involving chemical reactions and those concerned with dissociating fluids. Specifically, the effects of heat generation may alter the temperature distribution, consequently affecting the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers.

Combined buoyancy-generated heat and mass transfer due to temperature and concentration variations, in fluid- saturated porous media, have several important applications in variety of engineering processes including heat exchanger devices, petroleum reservoirs, chemical catalytic reactors, solar energy porous wafer collector systems, ceramic materials, migration of moisture through air contained in fibrous insulations and grain storage installations and the dispersion of chemical contaminants through water-saturated soil, super convecting geothermic etc. Due to the fast growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy suppliers to telephone switch boards and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen and helium) and medium molecular weight.

The fluid mechanical properties of the penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymer liquids like polyethylene oxide and poly-isobutylene solution in cetane, having better electromagnetic properties are normally used as a cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product.

Hossain et al (1999) studied the effects of viscous and Joule heating on the flow of viscous incompressible fluid past a semi-infinite plate in the presence of a uniform transverse magnetic field. The combined effects of forced and natural convection heat transfer in the presence of a transverse magnetic field from vertical surfaces are also studied by many researchers. Chen (2010) investigated the momentum, heat and mass transfer characteristics of MHD natural convection flow over a permeable, inclined surface with variable wall temperature and concentration, taking into consideration the effects of Ohmic heating and viscous dissipation. Seddeek (2002) analyzed the effect of heat radiation and variable viscosity and magnetic field in the case of unsteady flow. Abdelkhalek (2005) investigated the effects of mass transfer on steady two-dimensional laminar MHD mixed convection flow.

Lai & Kulacki (1990) used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. Magneto Hydro Dynamics flows have applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of the earth’s core. In addition, from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magneto spheres, aeronautical plasma flows, chemical engineering and electronics. An excellent summary of applications is given by Huges & Young (1966) and Raptis (1986) studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical plate embedded in a porous medium. Helmy (1998) analyzed MHD unsteady free convection flow past a vertical porous plate embedded in a porous medium. Elbashbeshy (1997) studied heat and mass transfer along a vertical plate in the presence of a magnetic field.

In all the studies mentioned above, the heat due to viscous dissipation is neglected. Gebhart (1962) has shown the importance of viscous dissipation heat in free convection flow in the case of isothermal and constant heat flux at the plate. Soundalgekar (1972) analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Cookey et al (2003) investigated the influence of viscous dissipation and radiation in unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction.

In this chapter the numerical solution is presented for free convection of heat transfer in flow past a semi-Infinite flat plate in a transverse magnetic field with heat flux. The governing equations are solved numerically using the Runge-Kutta Gill method with shooting technique.

3.2 ANALYSIS

A steady, two dimensional, incompressible flow of a viscous fluid on a continuous flat surface, issuing from a slot and moving with a constant velocity U_0 in a fluid at rest, in the presence of a transverse magnetic field of strength B_0 are considered. Let the x-axis be taken along the sheet in the direction of motion of the sheet and y-axis normal to it with velocity components u and v directed along their axes respectively. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field are negligible in comparison with the applied magnetic field. If σ is the electrical conductivity of the fluid then the flow and heat transfer are given by the following equations:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \sigma\frac{{B_0}^2 u}{\rho} - \frac{\upsilon}{k'}u \tag{3.2}$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3.3}$$

The boundary conditions are

$$
\begin{aligned}
&u = U_0, \quad v = 0, \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k} \quad at \quad y = 0 \\
&u \to 0, \quad T \to T_\infty, \qquad\qquad as \quad y \to \infty
\end{aligned}
\tag{3.4}
$$

Here U_0 is the uniform velocity of the plate and q_w is the heat flux per unit area. u,v are the velocity components in x, y directions respectively, ρ the density of the fluid, k' the permeability of the porous medium, T the temperature of the fluid, υ the kinematic viscosity, $\alpha = k/\rho c_p$ the thermal diffusivity, c_p the specific heat at constant pressure, k the thermal conductivity of the fluid, B_0 the magnetic induction.

The Equations (3.2) and (3.3) are coupled, parabolic and nonlinear partial differential equations and hence the analytical solution is not possible. Therefore the numerical technique is employed to obtain the required solution. Numerical computations are greatly facilitated by non-dimensionalization of the equations. Proceeding with the analysis, we introduce the following similarity transformations and dimensionless variables which will convert the partial differential equation from two independent (x, y) variables to a system of coupled, non-linear ordinary differential equations in a single variable η. i.e. coordinate normal to the plate.

In order to write the governing equations and the boundary conditions in dimensionless form the non-dimensional quantities are introduced by the stream function ψ, defined by

$$
u = \frac{\partial \psi}{\partial y} \; and \; v = -\frac{\partial \psi}{\partial x} \tag{3.5}
$$

which satisfies the equation of continuity. Assuming f and θ to be the functions of η only and making the following substitution,

$$\psi = \sqrt{\nu x U_0}\, f$$
$$\eta = y\sqrt{\frac{U_0}{\nu x}}$$
$$v = \frac{1}{2}\sqrt{\frac{\nu U_0}{x}}\left(\eta f' - f\right)$$
$$u = U_0 f'$$
$$\theta(\eta) = \frac{k\left(T - T_\infty\right)}{q_w}\sqrt{\frac{U_0}{\nu x}}$$
$$\Pr = \frac{\mu c_p}{k} \text{ (Prandtl number)}$$
$$M = \frac{m^2}{\mathrm{Re}} \text{ (Magnetic field parameter)}$$
$$m^2 = \frac{\sigma B_0^2 x^2}{\mu} \text{ (m is the Hartman number)}$$
$$\mathrm{Re} = \frac{U_0 x}{\nu} \text{ (Reynolds number)}$$
$$K = \frac{\nu x}{k' U_0} \text{ (Permeability parameter)}$$

the boundary layer Equations (3.1) - (3.3) become

$$f''' + \frac{1}{2} f f'' - M f' - K f' = 0 \tag{3.6}$$

$$\theta'' + \frac{\Pr}{2}\left(f\theta' - f'\theta\right) = 0 \tag{3.7}$$

With boundary conditions

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -1 \quad && at\ \eta = 0 \\ &f'(\infty) = 0, \quad \theta(\infty) = 0 && as\ \eta \to \infty \end{aligned} \tag{3.8}$$

Equations (3.6) and (3.7) subject to the boundary conditions (3.8) were solved numerically using Runge-Kutta Gill method on a computer for different values of magnetic parameter for air using shooting method. The effects of variation of M and K on temperature profiles have been plotted. Numerical values of $f''(0)$ and $-\theta'(0)$ for different values of M, K and Pr have been calculated and they are presented in a tabular form. The skin-friction coefficient and Nusselt number are important physical parameters for this type of boundary layer flow.

Knowing the velocity field, the skin-friction coefficient at the plate can be obtained, which in non-dimensional form is given by

$$C_f = 2(\mathrm{Re})^{\frac{-1}{2}} f''(0) \tag{3.9}$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form, in terms of the Nusselt number, is given by

$$Nu = (\mathrm{Re})^{\frac{-1}{2}} \theta'(0) \tag{3.10}$$

3.3 SOLUTION OF THE PROBLEM

The set of coupled non-linear governing boundary layer Equations (3.6) and (3.7) together with the boundary conditions as given in Equation (3.8) are solved numerically by using Runge-Kutta Gill method together with the shooting technique. First of all, higher order non-linear differential Equations (3.6) to (3.7) are converted into simultaneous linear differential equations and they are further transformed into the initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge-Kutta Gill method. The step size

$\Delta\eta$ = 0.05 is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient and the Nusselt number, which are respectively proportional to $f''(0)$ and - $\theta'(0)$ are also sorted out and their numerical values are presented in a tabular form. Extensive calculations have been performed to obtain the flow and temperature fields for a wide range of parameters $0.5 \leq M \leq 2$, $0.5 \leq K \leq 2$ and $0.5 \leq Pr \leq 10$.

3.4 RESULTS

Table 3.1 shows numerical values of magnetic field parameter effects on $f''(0)$ and - $\theta'(0)$. The profiles for velocity and temperature are shown in Figure 3.1-3.5 respectively with various values of the parameters.

Table 3.1 Effect of magnetic field parameter on $f''(0)$ and - $\theta'(0)$

M	K	Pr	$f''(0)$	$-\theta'(0)$
0.5	0.5	0.71	0.4438	1.6466
1	0.5	0.71	1.0828	2.1758
2	0.5	0.71	1.4733	2.8471
0.5	1	0.71	1.0828	2.1729
0.5	2	0.71	1.4734	2.8314
0.5	0.5	1	0.4444	1.3312
0.5	0.5	1.25	0.4444	1.1647
0.5	0.5	5	0.4444	0.5370
0.5	0.5	10	0.4444	0.3721

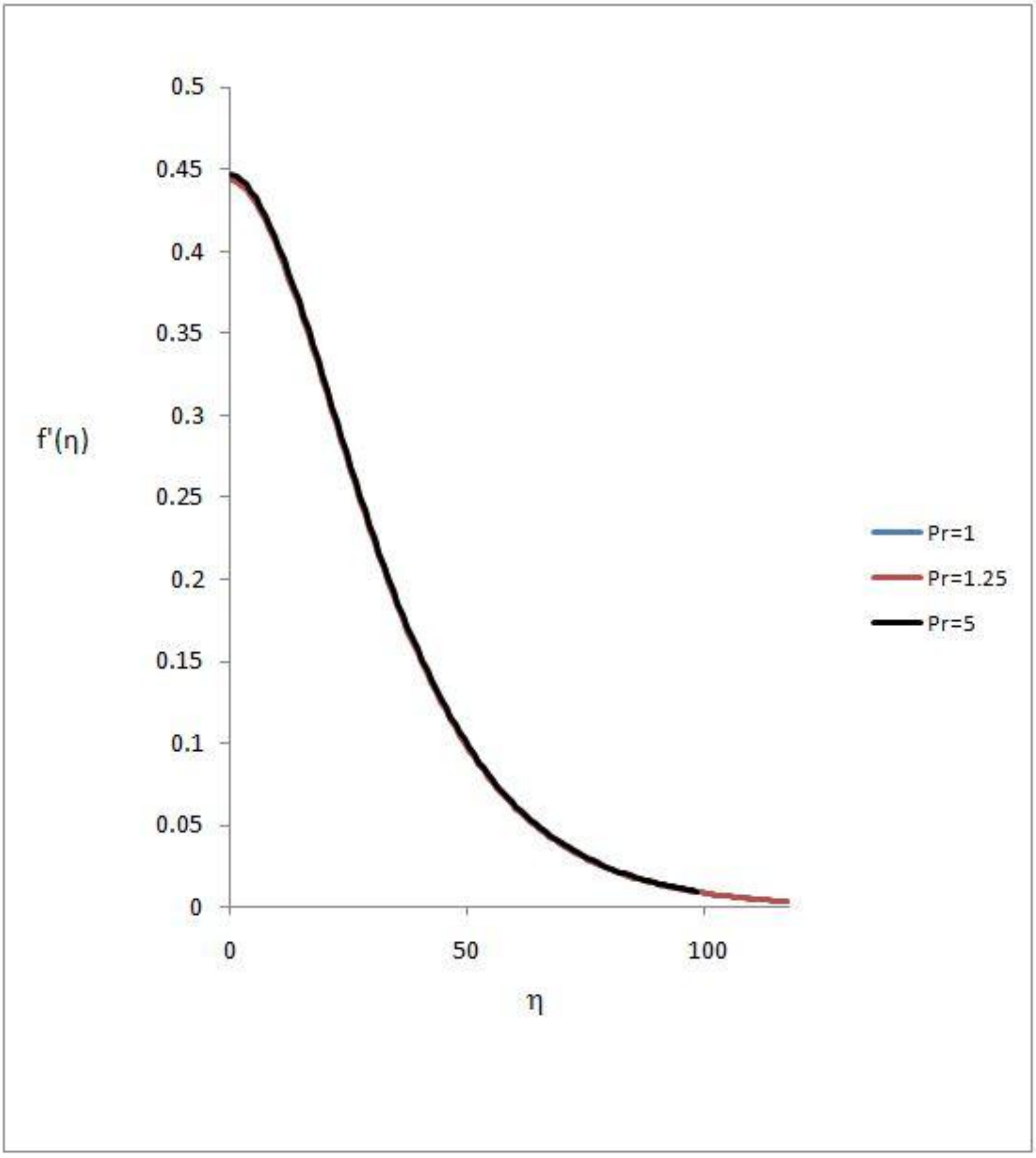

Figure 3.1 (a) Effect of Prandtl number Pr on non- dimensional velocity f′

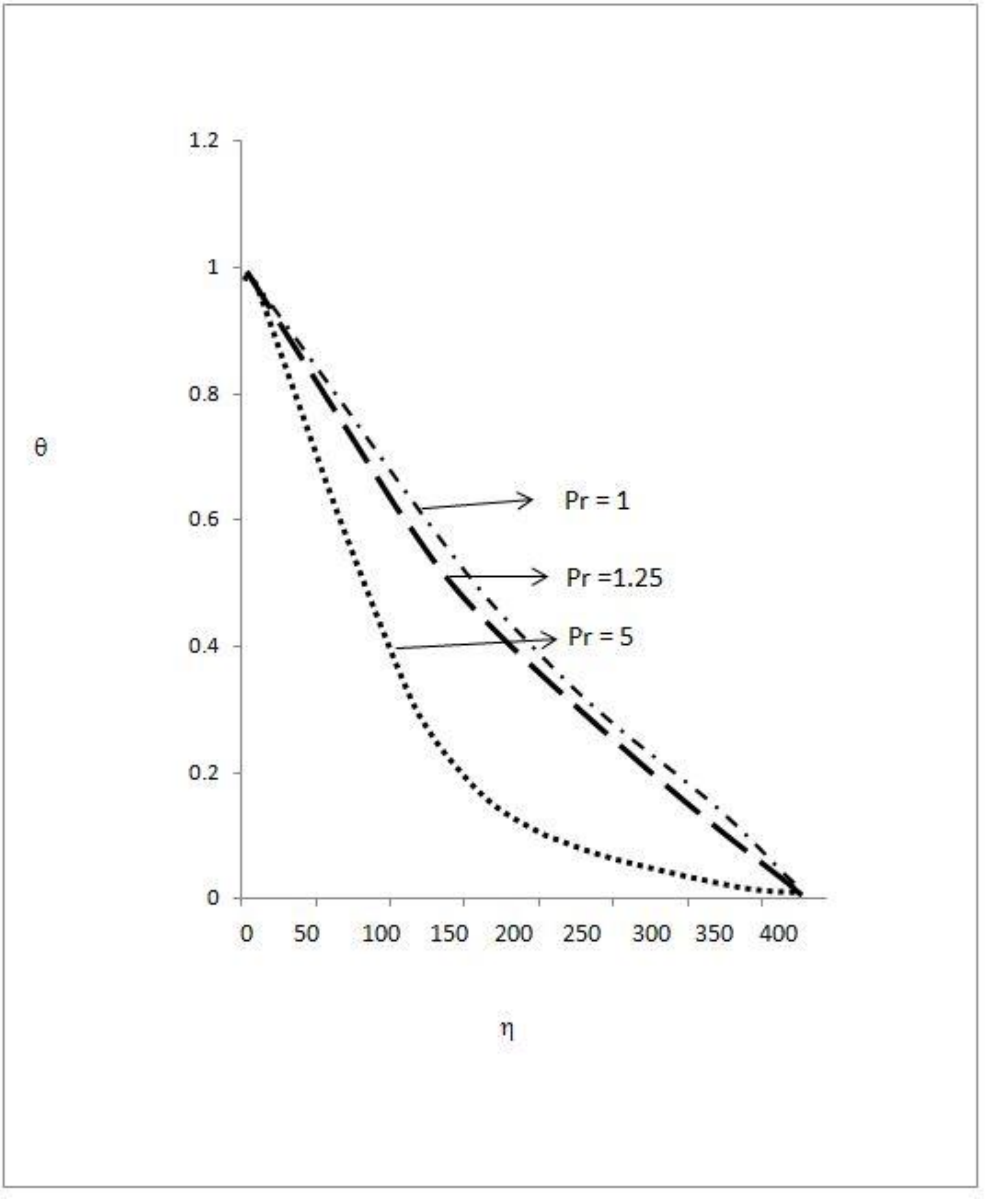

Figure 3.1(b) Effect of Prandtl number Pr on non-dimensional temperature θ

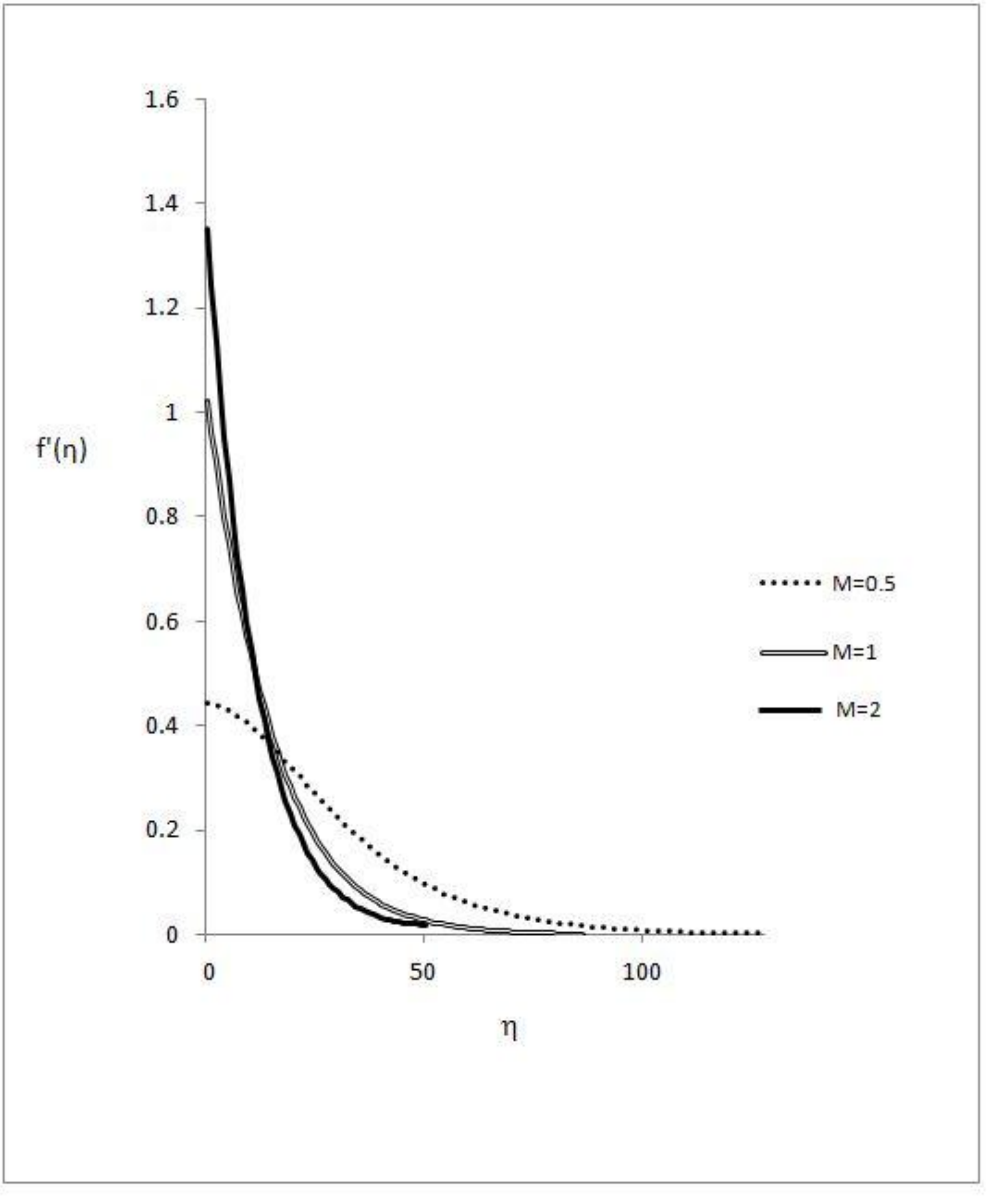

Figure 3.2 Effect of magnetic field parameter M on non- dimensional velocity f '

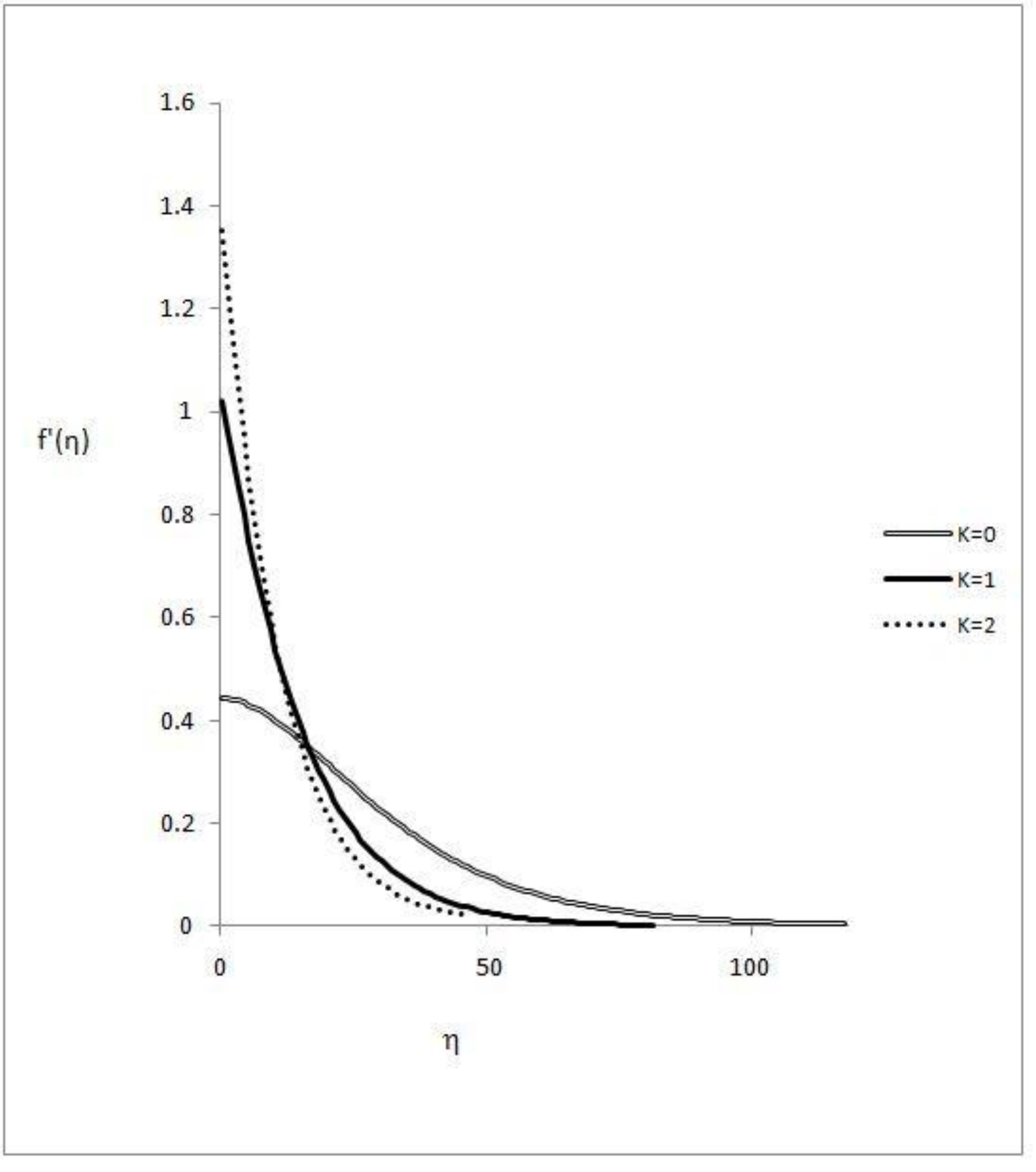

Figure 3.3 Effect of permeability parameter K on non-dimensional velocity f '

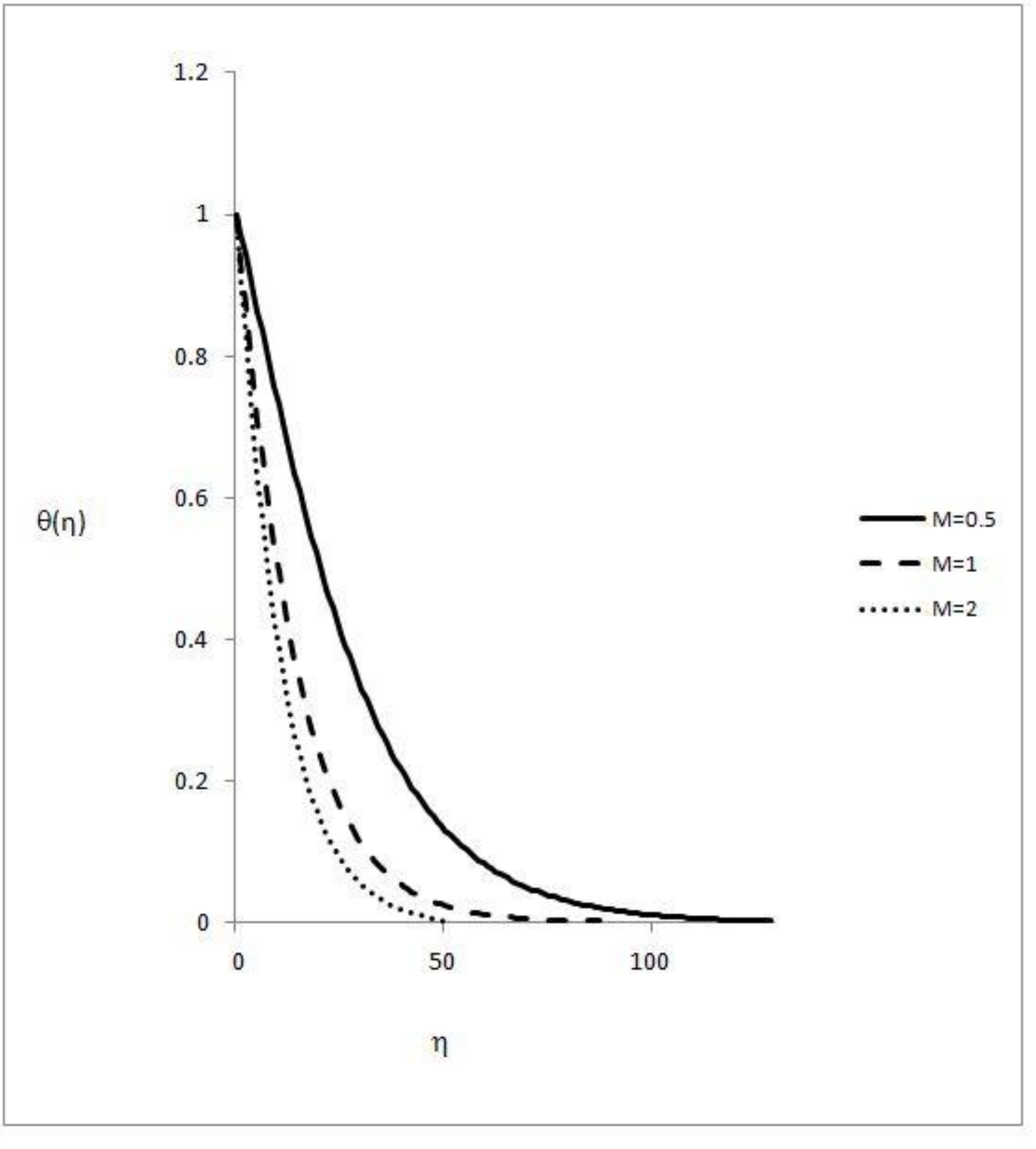

Figure 3.4 Effect of magnetic field parameter M on non-dimensional temperature θ

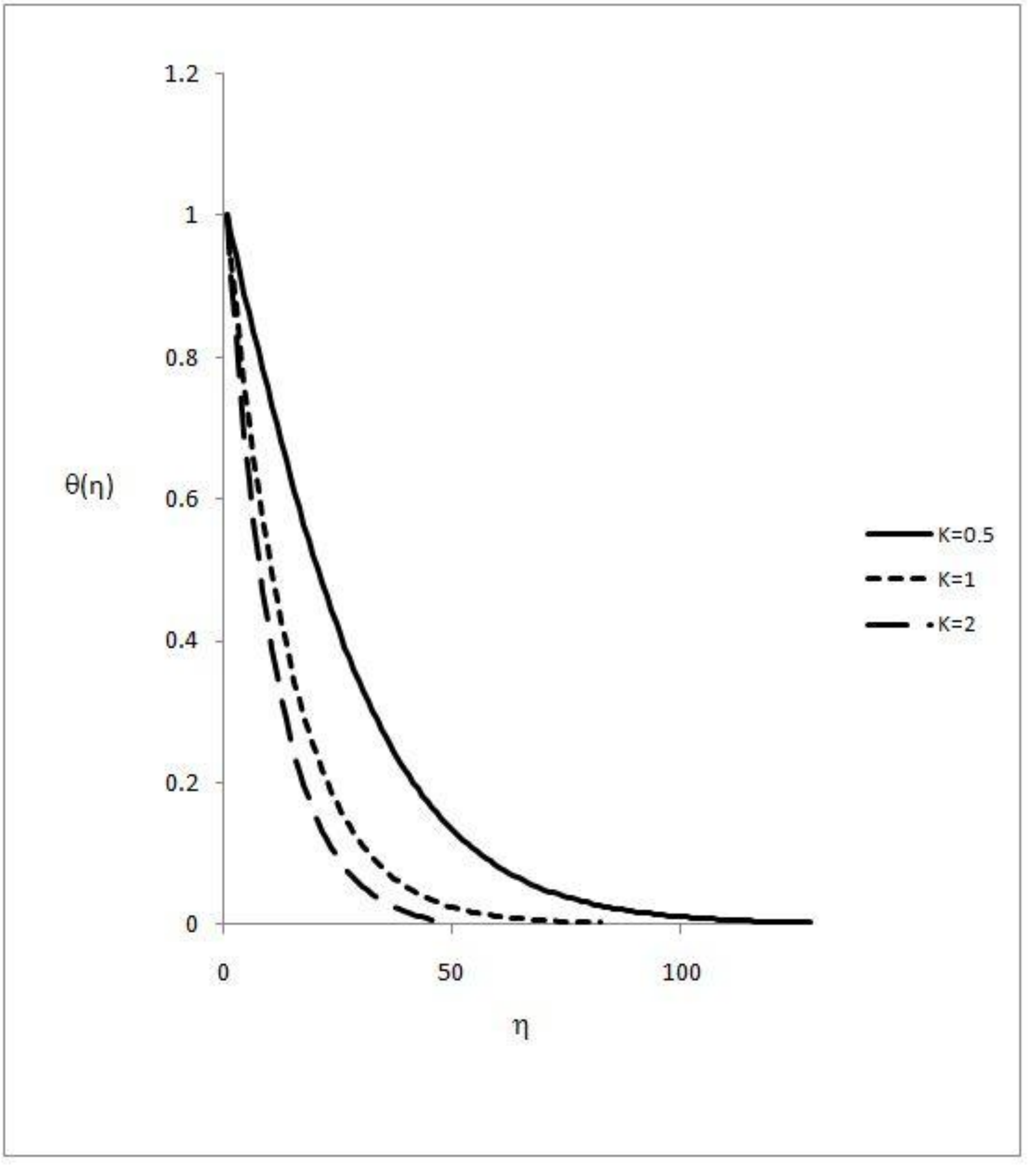

Figure 3.5 Effect of permeability parameter K on non-dimensional temperature θ

3.5 DISCUSSION

As a result of the numerical calculations, the dimensionless velocity and temperature distributions for the flow under consideration are obtained and their behavior has been discussed for variations in the governing parameters viz., magnetic field parameter M, permeability parameter K, Prandtl number Pr.

Figure 3.1 (a) and Figure 3.1 (b) illustrate the velocity and temperature profiles for different values of the Prandtl number Pr. From Figure 3.1 (a) and Figure 3.1 (b), it is observed that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average velocity within the boundary layer. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing temperature. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Hence, in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

Figure 3.2 presents the velocity profile for various values of magnetic field parameter M while all other parameters are kept at some fixed values. An increase in Magnetic field parameter M results in increase in velocity profile. It is observed that from Figure 3.3 the increase in the permeability parameter K increases the velocity profile.

The influence of magnetic field parameter M on the temperature is presented in Figure 3.4. It is observed that there is a decrease in the temperature as the magnetic field parameter M increases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free convection flow. Also, as M increases, the peak value of the temperature decreases rapidly near the porous plate and then decays smoothly.

Figure 3.5 presents typical temperature profiles in the boundary layer for various values of the permeability parameter K while all other parameters are kept at some fixed values. An increase in K will therefore increase the resistance of the porous medium which will tend to decelerate the flow and reduce the velocity. The fluid temperature decreases as the permeability parameter K increases. Also, as K increases, the peak values of the temperature decrease rapidly near the plate and decreases smoothly.

The effects of various parameters on the skin friction coefficient and Nusselt number are shown in the Table 3.1. It is observed from the Table 3.1 that as M and K increases, there is a fall in the skin-friction coefficient but there is an increase in Nusselt number. As the Prandtl number increases, there is no change in the skin-friction coefficient, but there is a decrease in the Nusselt number.

3.6 CONCLUSION

Using the similarity transformation a set of ordinary differential equations has been derived from the conservation of mass and momentum in the boundary layer. These nonlinear, coupled differential equations have been solved physically by using valid boundary conditions and through Runge-Kutta Gill method together with shooting technique. The conclusions of the study are as follows:

- The velocity decreases with an increase in the magnetic parameter and permeability parameter.

- A positive increase in Prandtl number is shown to reduce the velocity and temperature in the flow.
- The temperature decreases with an increase in magnetic parameter and permeability parameter.
- An increase in M and K leads to fall in the skin-friction coefficient but there is an increase in Nusselt number