CHAPTER 2

BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A CONTINUOUS SURFACE IN THE PRESENCE OF HYDROMAGNETIC FIELD

2.1 INTRODUCTION

A porous medium means a material consisting of a solid matrix with an interconnected void. The solid matrix is either rigid or it undergoes small deformation. The interconnectedness of the void (the pores) allows the flow of one or more fluids through the material. In single-phase flow the void is saturated by a single fluid. In two-phase flow a liquid and a gas share the void space. In a natural porous medium, the distribution of pores with respect to shape and size is irregular. Examples of natural porous media are beach sand, sandstone, limestone, rye bread, wood and the human lung.

In recent years, the problems of free convective heat transfer flows through a porous medium under the influence of a magnetic field have been attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. Flow through a porous medium have numerous engineering and geophysical applications, for example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. In view of these applications, many researchers have studied MHD free convective heat and mass transfer flow in a porous medium.

The study of hydromagnetic boundary layers on stretching surface has attracted considerable interest due to its wide applications especially in engineering and industrial processes. Numerous investigations have been conducted on the Magneto Hydro Dynamic (MHD) flows and heat transfer. MHD was initially known in the field of astrophysics and geophysics and later become a very important in engineering and industrial processes. For example, MHD can be found in MHD accelerators and generators, electric transformers, power generators, refrigeration coils, pumps, meters, bearing, petroleum production and metallurgical processes which involve cooling of continuous strips or filaments. In metallurgical processes, the rates of cooling and stretching of the strips can be controlled by drawing the strips in an electrically conducting fluid subject to a magnetic field, so that a final product of desired characteristics can be achieved. Hasanpour et al (2010) investigated numerically the MHD mixed convective flow in a lid-laden cavity filled with porous medium using Lattice Boltzmann method and reported that the fluid circulation within the cavity is reduced by increasing magnetic field strength and the heat transfer depends on the magnetic field strength and the Darcy number. Chamkha (1999) and Aboeldahab (2005) considered MHD problem in three-dimensional flow, while Ishak et al (2008) studied the effect of a uniform transverse magnetic field on the stagnation-point flow over a stretching vertical sheet. Different aspects of MHD flow and heat transfer due to a stretching surface by considering vertical sheet, stretching cylinder and moving extensible surface were examined. Many processes in engineering occur at high temperatures and the full understanding of the effect of radiation on the rate of heat transfer is necessary in the design of equipment. The effect of radiation on the boundary layer flow was studied by

Elbashbeshy & Dimian (2002), and Hossain et al (1999). Further, the radiation effect is also considered by Bataller (2008a) in the study of boundary layer flow over a static flat plate (Blasius flow) and Cortell (2008) in the study of boundary layer flow over a moving flat plate (Sakiadis flow) in a quiescent fluid. The problems of Bataller (2008a) and Cortell (2008) have been extended by Ishak (2009) and found the existence of dual solutions when the plate and the fluid move in the opposite directions. Ferdows et al (2011) found that radiation increases the skin friction coefficient and the rate of heat transfer in natural convection flow over inclined porous surface.

Stagnation point flow has become an interacting area of research due to its varied applications both in industrial and scientific arena such as extrusion of polymers, cooling of metallic plates, aerodynamics, plastic extrusion, glass blowing and fiber spinning etc. The two dimensional flow of a fluid near a stagnation point is a classical problem in fluid dynamics. The plane and axisymmetric flow near a stagnation point on a surface have attracted many investigations during the past several decades because of its wide applications such as cooling of electronic devices by fans, cooling of nuclear reactors and many hydrodynamic processes.

Raptis et al (2004) have presented the steady forced convection flow through a porous medium bounded by a semi-infinite plate when the fluid is viscous and the free stream velocity is not constant. The combined forced and free convection in stagnation flow becomes important as the buoyancy forces owing to the temperature differences between the surface and the free stream is large. The flow of incompressible viscous fluid past a continuously moving semi-infinite plate by considering variable viscosity and variable temperature was studied by Soundalgekar et al (2004).

Rahman & Sattar (2006) have investigated the effect of heat generation or absorption on convective flow of a micro polar fluid past a continuously moving vertical porous plate in presence of a magnetic field. The chemical reaction effect on heat and mass transfer flow along a semi-infinite horizontal plate has been studied by Anjalidevi & Kandaswamy (1999) and later it was extended for Hiemenz flow by Seddeek et al (2007) and for polar fluid by Patil & Kulkarni (2008). Salem & Abd El-Aziz (2008) have reported the effect of hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation or absorption.

Ibrahim et al (2008) studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. A detailed numerical study has been carried out for unsteady hydromagnetic natural convection heat and mass transfer with chemical reaction over a vertical plate in rotating system with periodic suction by Parida et al (2011). Rajeswari et al (2009) have investigated chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through a vertical porous surface in presence of suction. Mahdy (2010b) has studied the effect of chemical reaction and heat generation or absorption on double diffusive convection from vertical truncated cone in a porous media with variable viscosity. Pal & Talukdar (2010) have studied perturbation analysis of unsteady magneto hydrodynamic convective heat and mass transfer in boundary layer slip flow past a vertical permeable plate with a thermal radiation and chemical reaction. Further the effect of thermal radiation, heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate in presence of transverse magnetic field was investigated by Makinde & Ogulu (2008). The analysis of MHD mixed-convection interaction with thermal radiation and higher order chemical reaction is carried out by Makinde (2011a). Aziz (2009) theoretically examined a similarity solution for a laminar thermal boundary layer over a flat plate with a convective surface boundary condition. It has been reported that a similarity solution is possible if the convective heat transfer along with the hot fluid on the lower surface of the plate is inversely proportional to the square root of the axial distance. Recently, the combined effects of an exponentially decaying internal heat generation and a convective boundary condition on the thermal boundary layer over a flat plate are investigated by Olanrewaju et al (2012a). In their study, authors have neglected the Sherwood effect. Similar analysis has been carried out by Makinde (2010; 2011b) without heat source and with heat source, neglecting chemical reaction effect. There has been considerable interest in studying the effect of chemical reaction and heat source effect on the boundary layer flow problem with heat and mass transfer of an electrically conducting fluid in different geometry by Bakr (2011), Bisht et al (2011) & Noor et al (2012). The problem of flow and heat transfer over a moving surface has drawn considerable attention and a very good amount of literature has been generated on this problem by Fang (2003) and Ishak et al (2008). Kumari & Nath (2001) discussed the problem of MHD boundary layer flow of a non-Newtonian fluid over a continuously moving surface with a parallel free stream. Recently, Jat & Chaudhary (2010) studied the flow of incompressible viscous conducting fluid past a continuous moving surface in the presence of transverse magnetic field. Olanrewaju et al (2012b) studied the effect of variable viscosity and magnetic field on heat transfer to a continuous flat plate. Further, Olanrewaju (2012c) studied the effect of Internal heat generation on hydromagnetic Non Darcy flow over a stretching sheet in presence of thermal radiation and omic dissipation.

In this chapter the approximate numerical solution is presented to study the effects on both momentum and heat transfer problem with viscous dissipation and Joule heat transfer for an electrically conducting fluid past a continuously moving plate in the presence of a uniform transverse magnetic field.

2.2 ANALYSIS

Consider the two-dimensional steady flow of an electrically conducting, viscous, incompressible fluid past a continuously moving surface with uniform velocity U in the presence of uniform transverse magnetic field of strength B_0. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The x-axis is taken along the surface and y-axis normal to it as shown in Figure 2.1.

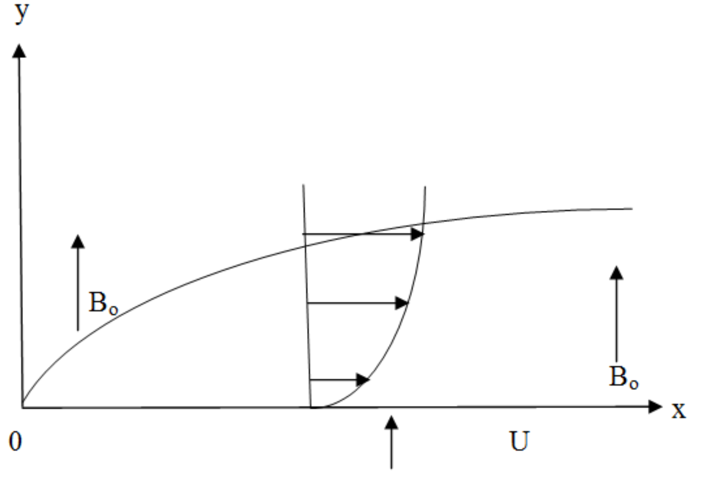

Figure 2.1 Coordinate system for continuously moving surface

The fluid properties are assumed to be isotropic and constant. Therefore, under the usual boundary layer approximations, the governing equations of motion are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma_e B_0^{\ 2}u}{\rho} \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma_e B_0{}^2 u^2}{\rho c_p} \tag{2.3}$$

where ρ is the density, μ the coefficient of viscosity, σ_e the electrical conductivity, k thermal conductivity of the fluid and c_p the specific heat at constant pressure. The other symbols have their usual meanings.

The corresponding boundary conditions are:

$y = 0$; $u = U$; $v = 0$; $T = T_w(x)$

$y \to \infty$; $u = 0$; $T = T_\infty$; (2.4)

The continuity Equation (2.1) is satisfied by introducing the stream function ψ(x, y), such that

$$u=\frac{\partial \psi}{\partial y}, \qquad v=-\frac{\partial \psi}{\partial x} \tag{2.5}$$

The momentum and energy Equations (2.2) and (2.3) can be transformed to the corresponding ordinary differential equations by introducing the following similarity transformations

$$\psi=\sqrt{Ux\upsilon_\infty}\; f(\eta)$$

(2.6)

$$\eta=y\sqrt{\frac{U}{\upsilon_\infty x}} \tag{2.7}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{2.8}$$

where υ_∞ is a reference kinematic viscosity.

It will be assumed that the temperature difference between the moving surface and the free stream varies as Ax^n.

$$T_w(x)-T_\infty=Ax^n \tag{2.9}$$

where A is a constant, n is exponent, and x is measured from the leading edge of the surface.

The momentum and energy Equations (2.2) and (2.3) after some simplifications reduce to the following forms:

$$f''' - \left(\frac{1}{\theta-\theta_r}\right)\theta' f'' - \left(\frac{\theta-\theta_r}{2\theta_r}\right) f f'' + \mathrm{Re}_m{}^2\left(\frac{\theta-\theta_r}{\theta_r}\right) f' = 0 \tag{2.10}$$

$$\theta'' - n\Pr f'\theta + \frac{\Pr}{2} f\theta' - \Pr Ec\left(\frac{\theta_r}{\theta-\theta_r}\right) f''^2 + \mathrm{Re}_m{}^2 \Pr Ec f'^2 = 0 \tag{2.11}$$

The corresponding boundary conditions are

$$\eta = 0; \quad f(0)=0; \quad f'(0)=1; \quad \theta(0)=1$$

$$\eta \to \infty; \quad f'=0; \quad \theta=0 \tag{2.12}$$

$$\mathrm{Re}_m = B_0\sqrt{\frac{\sigma_e x}{\rho U}}$$ (Local Magnetic Field)

$$Ec = \frac{U^2}{c_p(T_w-T_\infty)}$$ (Eckert number)

$$\theta_r = \frac{T_r-T_\infty}{T_w-T_\infty}$$ (Dimensionless reference temperature, constant)

It is important to note that θ_r is negative for liquids (Pr > 1.0) and positive for gases (Pr < 1.0). The Equations (2.10) and (2.11) constitute a Non-linear coupled boundary value problem prescribed at two boundaries, the analytical solution of which is not feasible. Therefore, these equations have been solved numerically by computer using shooting techniques together with the Runge-Kutta Gill method with a step size of 0.01. The corresponding velocity and temperature profiles are shown in Figures in the following section 2.3.

2.3 RESULTS AND DISCUSSION

The profiles of velocity and temperature are shown in Figures 2.2 to Figure 2.9. It is seen from Figure 2.2 that the velocity decreases as n and Re_m, the power-law index of the surface temperature variation (exponent) and the magnetic parameter increases with the Prandtl number Pr = 0.02 and the θ_r = 2 respectively. θ_r is the parameter that characterizes the influence of viscosity. From Figures 2.3 and 2.4, it is understood that the velocity profiles are almost identical for different values of n and Re_m. It is clear from Figure 2.5 that the velocity decreases as Magnetic parameter Re_m increases.

Figures 2.6 to 2.9 show the temperature profiles for different values of n and Re_m. It is seen from Figure 2.6 that as n increases as the temperature decreases in the fixed value of Re_m. Figures 2.7 and 2.8 reveal that the temperature decreases as n and Re_m increases. Figure 2.9 shows that the temperature decreases as the magnetic parameter increases with Pr = 0.02, n =0.3 and Ec =0.

Figure 2.10 gives the values of $-\theta'(0)$, the heat transfer rate for various values of n and Re_m. From the figure it is observed that as Re_m increases as the heat transfer rate $-\theta'(0)$ decreases, but as n increases the heat transfer rate increases. So the parameters n and Re_m have considerable influence on the heat transfer rate $-\theta'(0)$.

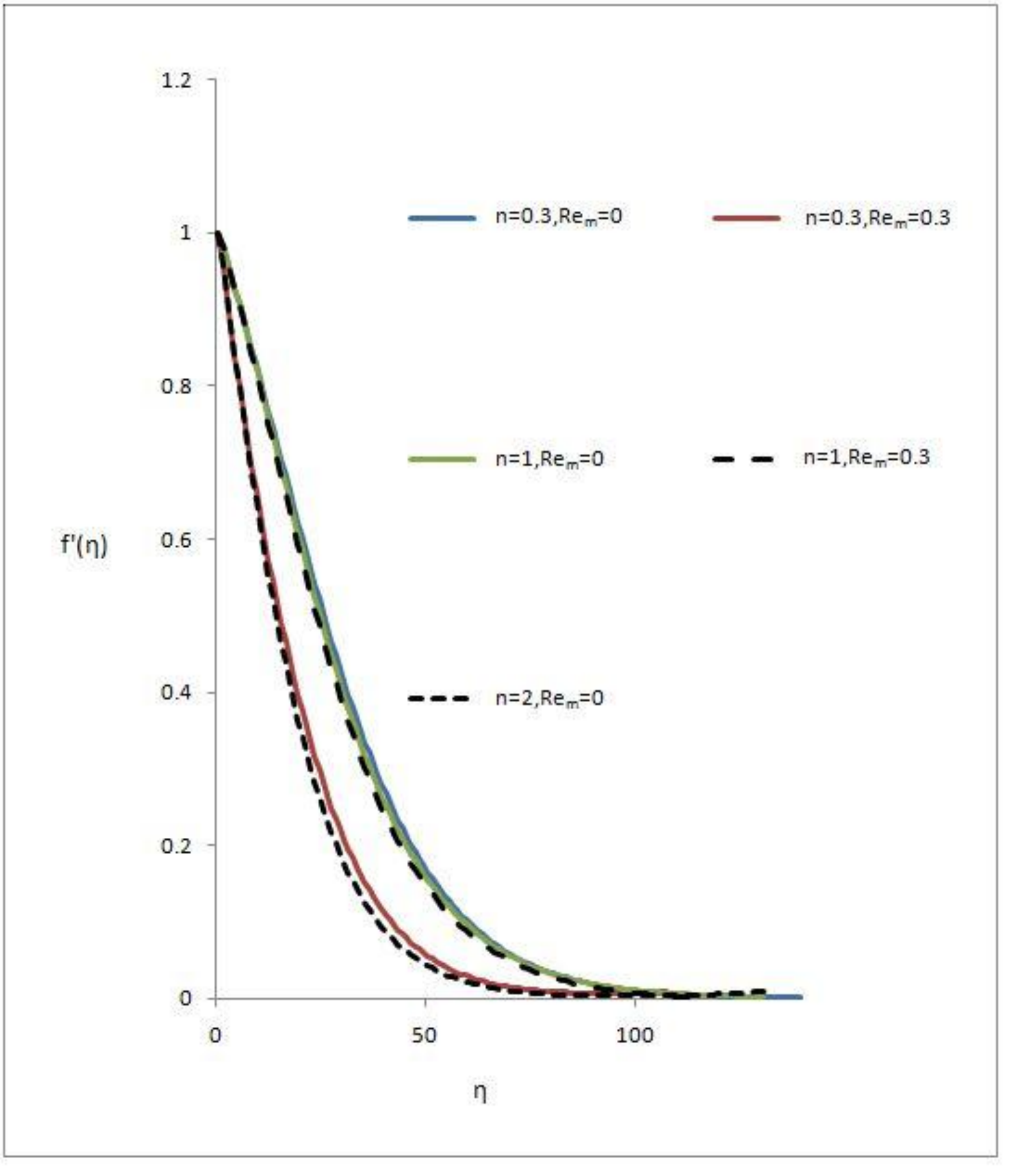

Figure 2.2 Velocity profile against η for different values of n and Re_m with Pr=0.02 and θ_r = 2.0

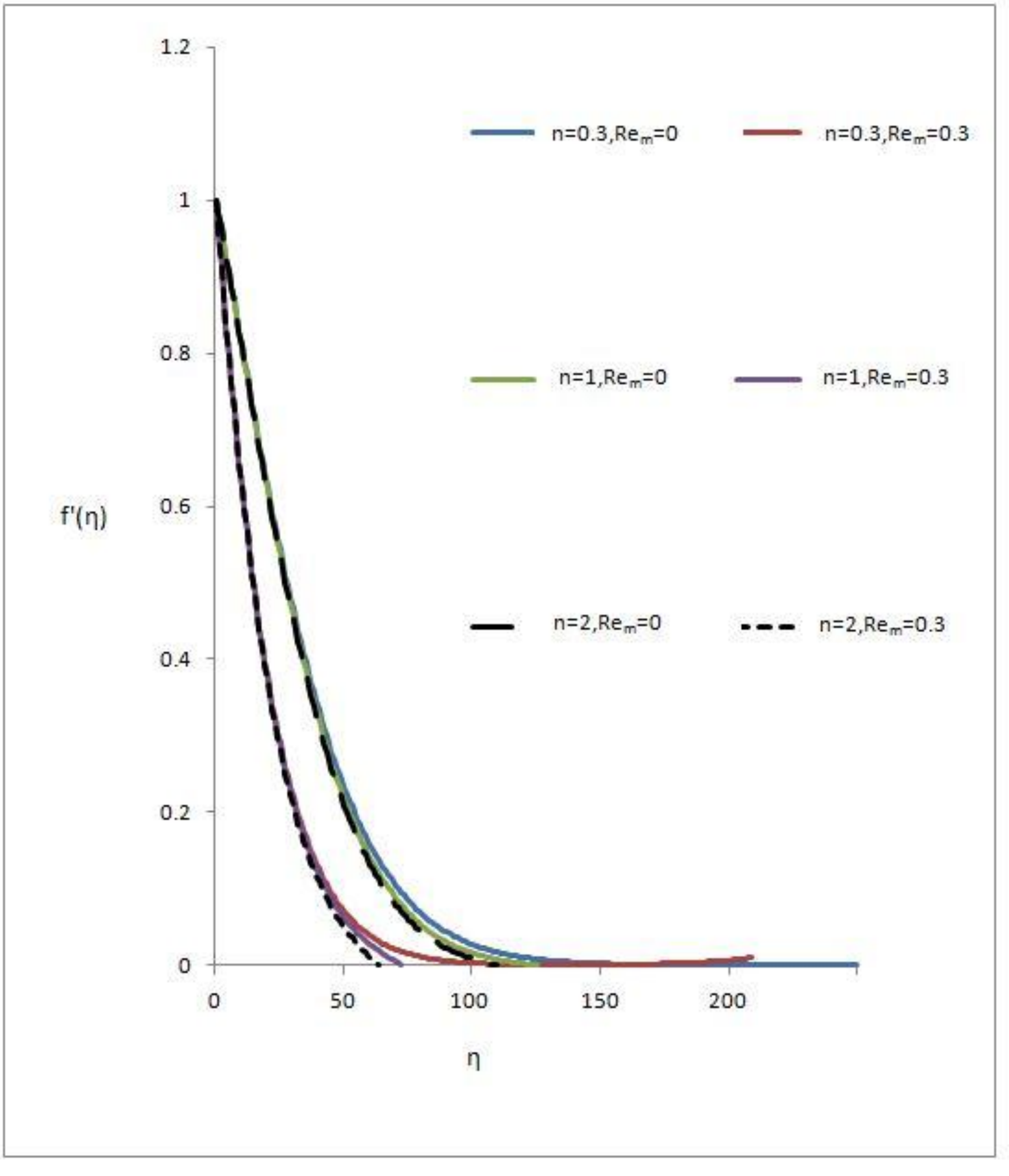

Figure 2.3 Velocity profile against η for different values of n and Re_m with Pr =0.02, Ec = 0 and θ_r=2.0

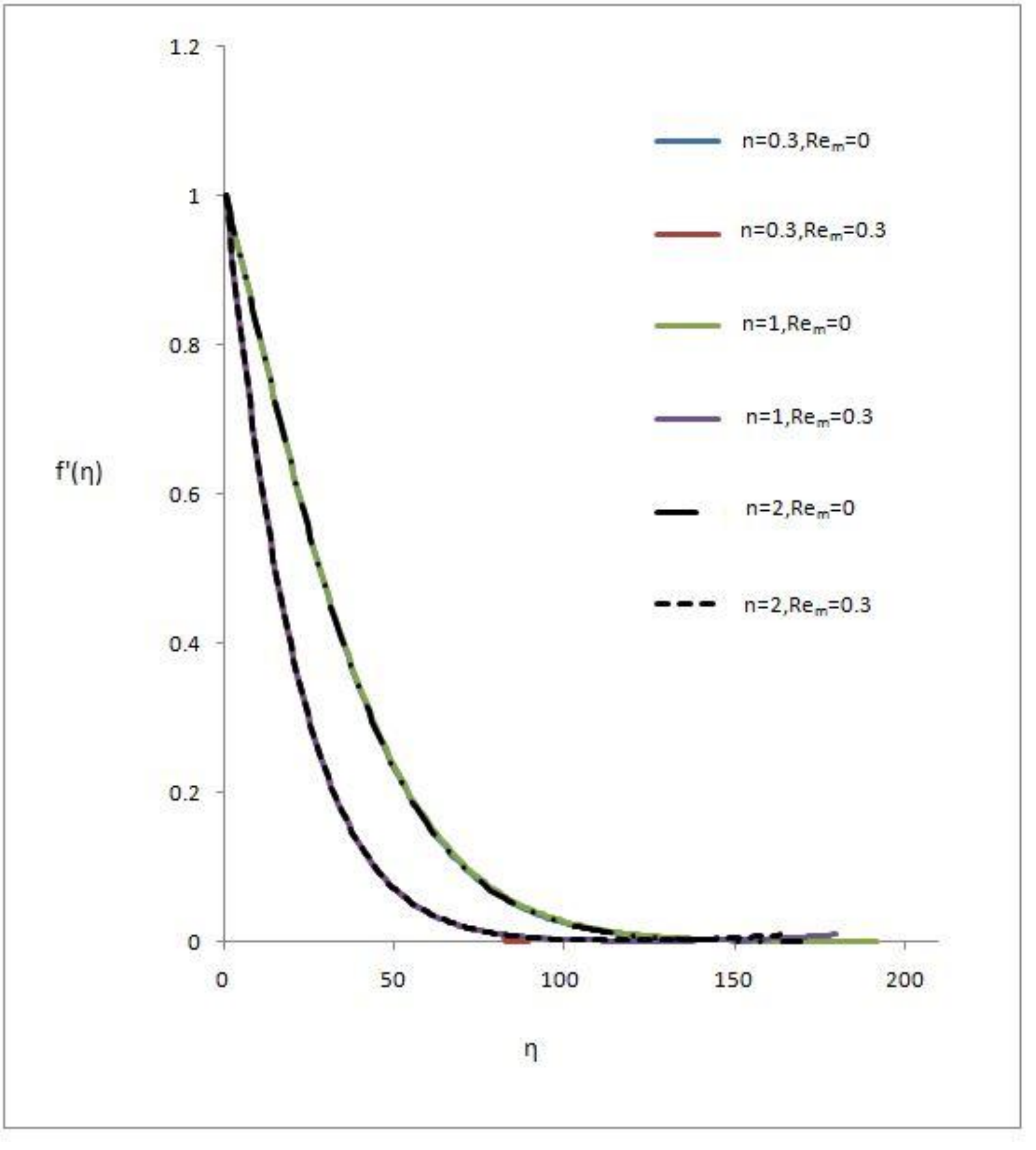

Figure 2.4 **Velocity profile against η for different values of n and Re_m with $Pr = 0.02$, $Ec = 0.5$ & $\theta_r = 2.0$**

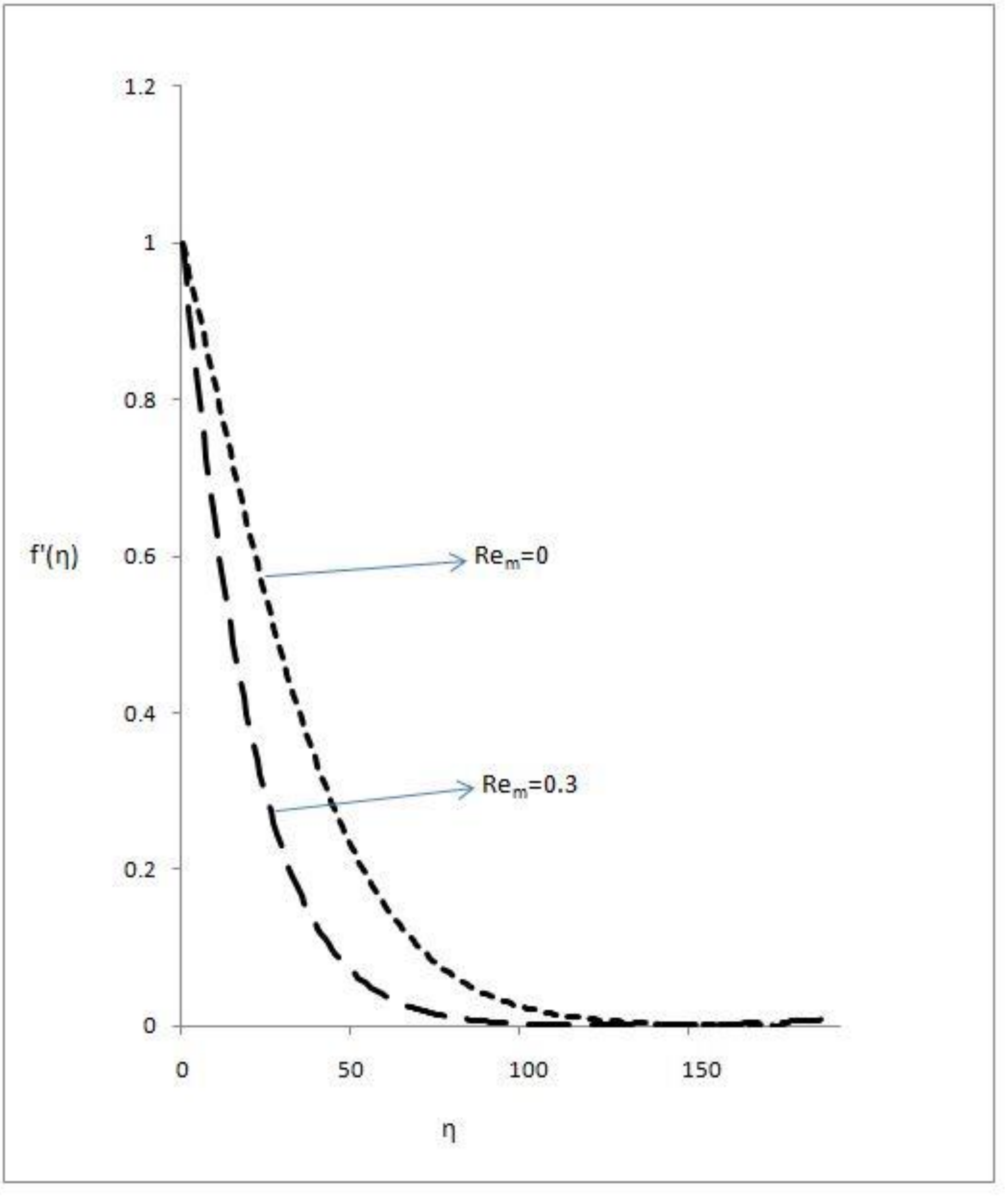

Figure 2.5 Velocity profile against η for different values of Re_m with $Pr = 0.02$, $n = 0.3$ & $Ec = 0.0$

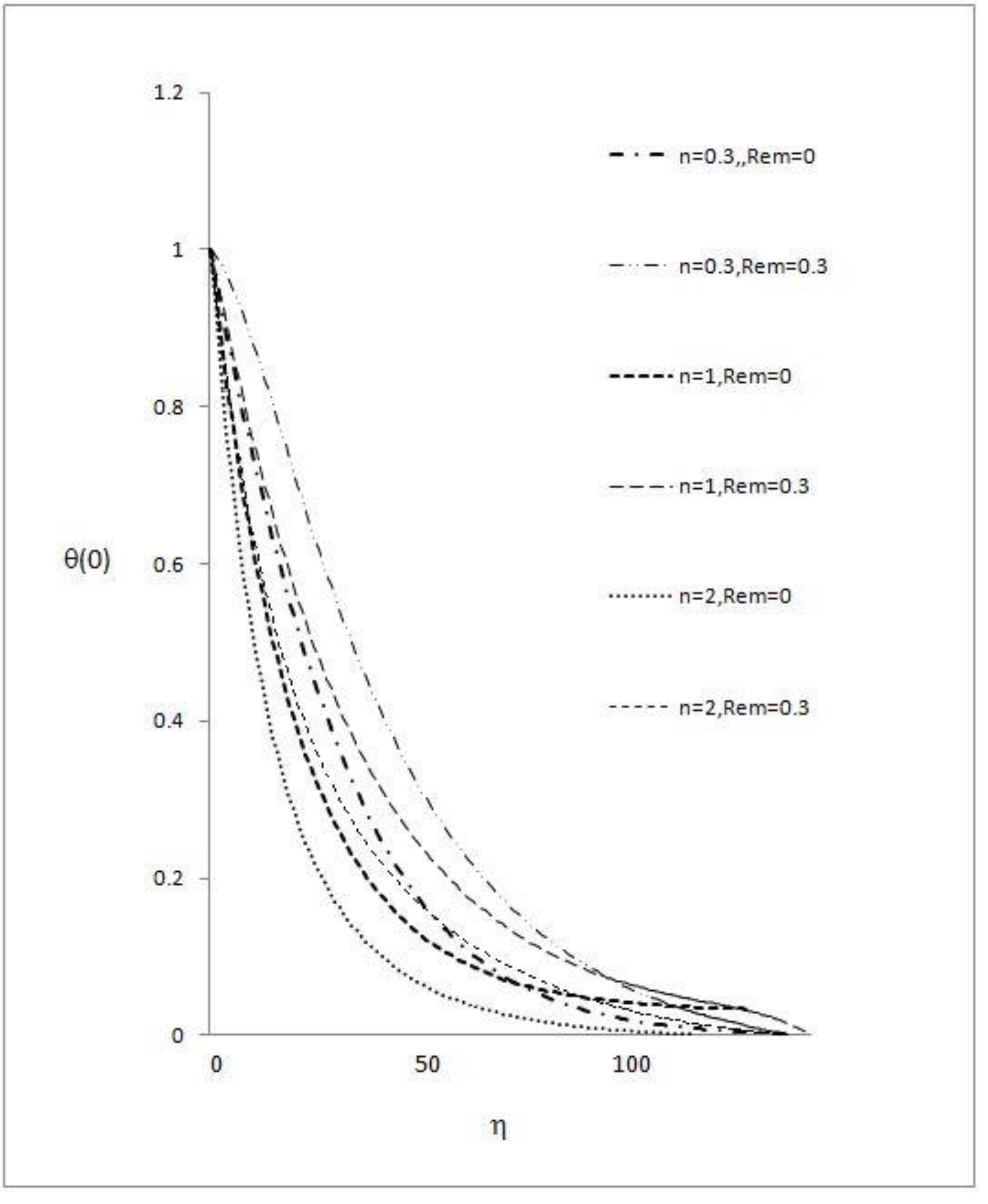

Figure 2.6 Temperature profile against η for different values of n and Re_m with Pr = 0.02 and θ_r = 2.0

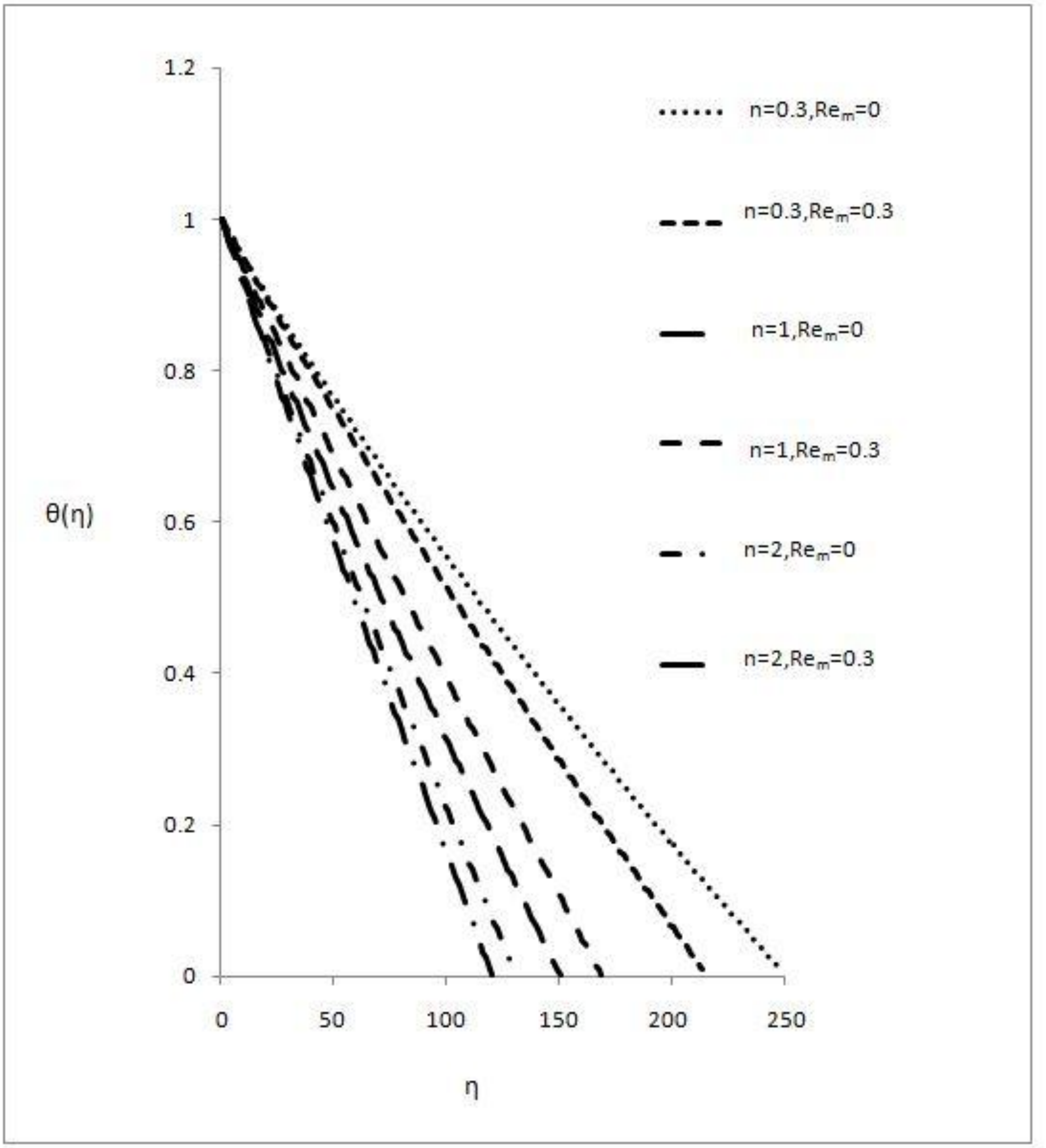

Figure 2.7 Temperature profile against η for different values of n and Re_m with Pr = 0.02 , Ec=0.0 & θ_r=2.0

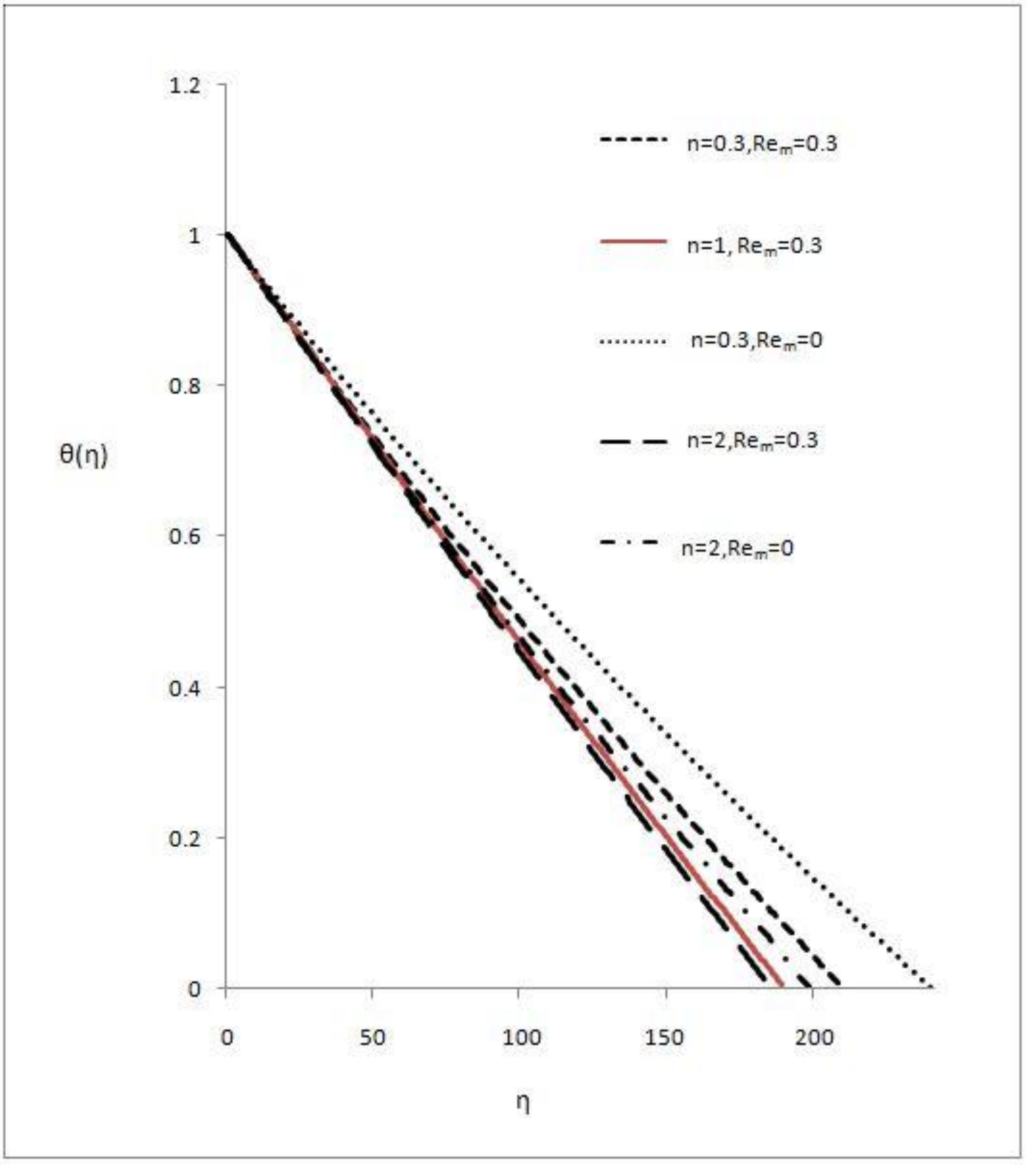

Figure 2.8 Temperature profile against η for different values of n and Re_m with $Pr = 0.02$, $Ec= 0.5$ and $\theta_r = 2.0$

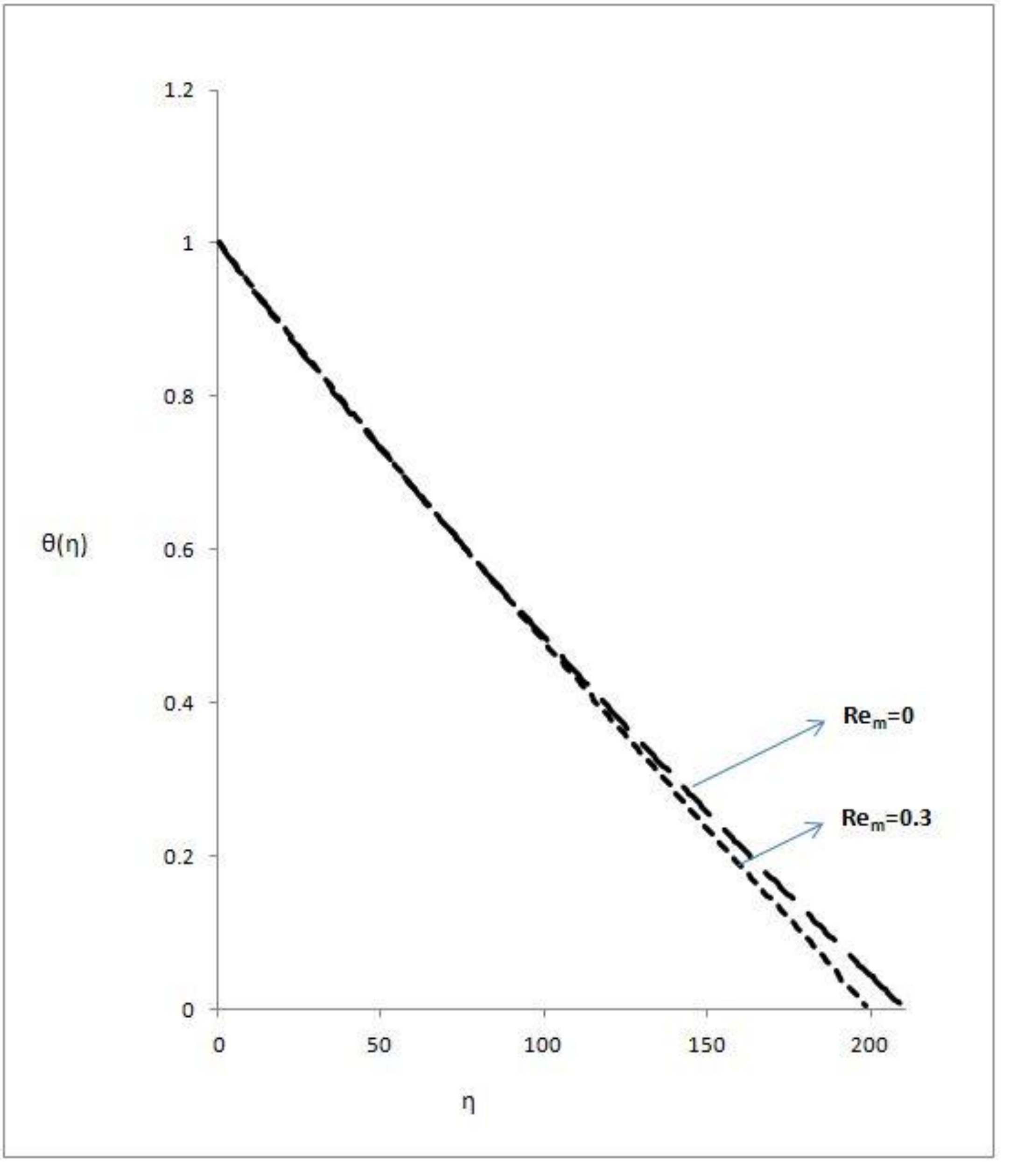

Figure 2.9 Temperature profile against η for different values of Re_m with Pr = 0.02, n = 0.3 & Ec = 0

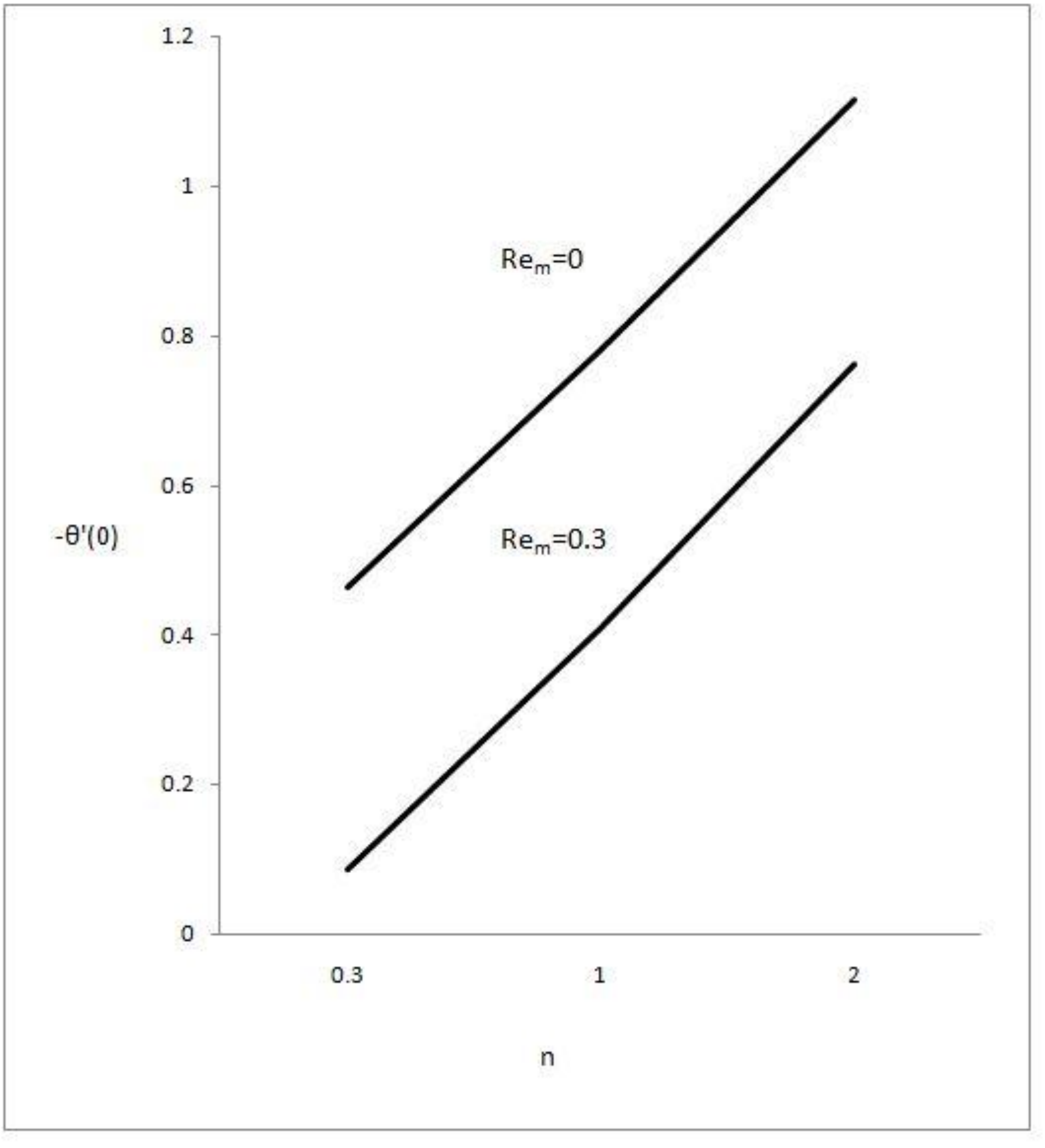

Figure 2.10 Heat transfer rate

2.4 CONCLUSION

The effects of power law index of the surface temperature variation (exponent), magnetic parameter and variable parameter on a hydro magnetic flow and heat transfer on a continuously moving surface have been studied numerically using the Runge-Kutta Gill method together with the shooting technique. From the previous results and discussion, we conclude the following

- The velocity decreases with the increase of power law index of the surface temperature variation (exponent) and the magnetic parameter

- The temperature decreases with the increase of the power law index of the surface temperature variation (exponent) and the magnetic parameter.

- The heat transfer rate increases rapidly with the increase of power law index of the surface temperature variation (exponent) whereas when the magnetic parameter increases the heat transfer rate decreases.