CHAPTER-2

NUMERICAL SIMULATION OF BOUNDARY LAYER FLOW
OF A NANOFLUID PAST AN EXPONENTIAL STRETCHING
SHEET

Part of this chapter 2 is published in the Journal of Bulletin of Mathematics and
2.1. INTRODUCTION

The hypothetical studies of fluids flowing over a incessant stretching sheet has strained attention of engineers and scientists for the preceding hardly any decades owing to the piece of information that this kinds of fluids corresponds to fairly a small number of scientifically noteworthy fluids. A range of applications of these fluids are there in the industrialized as well as cooling and drying of papers and textiles, extrusion of polymer fluids, synthetic and normal gels, cooling of everlasting metallic plate in cooling soak and spinning of fibres etc.

The heat transfer rate from the sheet into the fluid is very noteworthy as in such applications, it induces a straight crash on the superiority of the merchandise. On the other hand, the widespread conformist heat transfer fluids such as water, ethylene glycol, and engine oil have imperfect heat transfer capabilities owing to their low thermal conductivity, while metals have much advanced thermal conductivities than these fluids. Therefore, dispersing high thermal conductive solid particles in a predictable heat transfer fluid may augment the thermal conductivity of the ensuing fluid.

Behind the revolutionary work by Sakiadis [1961], a huge quantity of literature is obtainable on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surface [2003,2011]. Though flow and heat transfer over exponentially stretching sheet and also investigated the heat transfer in the flow captivating and exponentially changing wall temperature. Bidin and Nazar [2009], Ishak [2011] and Nadeem et al. [2010-2011] numerically examined the flow and heat transfer more than an exponentially stretching surface by means of thermal radiation. Elbashbeshy [2001]
numerically inquire into the flow and heat transfer over an exponentially stretching surface bearing in mind wall mass suction. Sanjayanand and Khan [2005] studied the viscous-elastic boundary layer flow and heat transfer owed to and exponentially stretching sheet. Partha et al. [2005] obtained a similarity solution for mixed convection flow over an exponentially stretching surface by captivating into report the control of viscous dispel on the convective transport. Al-odat et al. [2006] explored the outcome of magnetic field on thermal boundary layer on an exponentially stretching unbroken surface with an exponential temperature distribution. Sajid and Hayat [2008] found the influence of thermal radiation on the boundary layer flow and heat transfer of an incompressible viscous fluid due to an exponentially stretching sheet and they reported series solutions for velocity and temperature using HAM.

Changhoon Lee [2012] premeditated the boundary layer flow of nanofluid over and exponentially stretching surface. He solved it methodically by an HAM method. It is well known that Choi [1995] be the earliest to initiate the phrase ‘nanofluid’ that stand for the fluid in which nano-scale particles are pendant in the base fluid with low thermal conductivity such as water, ethylene glycol, oil etc. The exercise of chemical accumulation is a technique purposeful to augment the heat transfer act of base fluids. Nanofluids have been exposed to augment the thermal conductivity and convective heat transfer appearance of base liquids [2001]. There are plentiful biomedical usage that rivet nanofluids such as magnetic cell division, drug delivery, hyperthermia and disparity augmentation in magnetic resonance imaging.

Thus enthused by the above uttered investigations and applications of exponential stretching sheet, we feel suitable to speak about steady laminar two dimensional boundary layer flows and heat transfer of nanofluids above an exponential stretching sheet.
2.2. MATHEMATICAL FORMULATION

We look upon for steady, incompressible, laminar, two dimensional boundary layer flow of a viscous nanofluid past a flat sheet coinciding with the plane \( y=0 \) and the flow limited to \( y>0 \). The flow is created owing to stretching of the sheet caused by the concurrent application of two identical and differing force along the x-axis. Keeping the origin fixed, the sheet is afterwards stretched with a velocity \( u = u_0(x) = U_0 \exp(x/L) \), where \( U_0 \) is the reference velocity, \( L \) is the reference length and \( x \) is the coordinate calculated all along the stretching surface altering exponentially with the expanse from the slit as shown in Fig 1.

![Fig 1. Physical model and co-ordinate system](image)

It is assumed that at the stretching surface, the temperature \( T \) and the nanoparticles fraction \( C \) take constant values \( T_0 \) and \( C_0 \) respectively. When \( y \) attains infinity, the ambient values of temperature \( T \) and nanoparticles fraction \( C \) are denoted by \( T_\infty \) and \( C_\infty \) respectively. The fluid is
a water based nanofluid containing three types of nanoparticles Cu, Al₂O₃ and TiO₂. It is further assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium.

The basic steady conservation of mass, momentum, thermal energy and nanoparticles equations for nanofluids can be written in Cartesian co-ordinates x and y as, see Kuznetsov and Nield [2009]. The flow and heat transfer characteristics under the boundary layer approximations are governed by the following equations.

\[
\begin{align*}
\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} &= 0 \\
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= u \left( \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \left( \frac{\partial^2 T}{\partial y^2} \right) + \tau \left[ D_B \left( \frac{\partial C}{\partial y} \right) + \frac{D_t}{T_m} \left( \frac{\partial^2 T}{\partial y^2} \right) \right] \\
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D_B \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_t}{T_m} \left( \frac{\partial^2 T}{\partial y^2} \right)
\end{align*}
\]  

(2.2.1) \hspace{1cm} (2.2.2) \hspace{1cm} (2.2.3) \hspace{1cm} (2.2.4)

The boundary conditions are

\[
\begin{align*}
v &= 0, \quad u = u_w(x), \quad T = T_w, \quad C = C_w, \quad \text{at} \quad y = 0 \\
u = v = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \to \infty
\end{align*}
\]

(2.2.5) \hspace{1cm} (2.2.6)

Where \( u \) and \( v \) are the velocity components along \( x \) and \( y \) axis respectively. \( v = \mu / \rho_t \) is the kinematic viscosity, \( \alpha = \kappa / (\rho C_p) \) is the thermal diffusivity, \( D_B \) is the Brownian diffusion coefficient, \( D_t \) is the thermophoresis diffusion coefficient and \( \tau = (\rho a) / (\rho a)_t \) is the ratio between the effective heat capacity of the nanoparticles material and heat capacity of the nano fluid. \( T \) is the temperature inside the boundary layer, \( T_\infty \) is the temperature far away from the sheet. \( u_w(x) = U_o \exp(x/L) \) is the stretching velocity of the sheet. \( T_w = T_\infty + b \exp(x/2L) \) is the
temperature of stretching surface and \( C_w = C_m \cdot e^{\exp(x/2L)} \) is nanoparticles volume fraction at the stretching surface.

We are interested in similarity solution of the above boundary value problem therefore we introduce the following similarity transformations (dimensionless quantities).

\[
\eta = \sqrt{\frac{U_0}{2L}} \exp(x/2L), \quad \psi = \sqrt{\frac{2
u L U_0}{\nu}} \exp(x/2L) f(\eta), \\
\theta(\eta) = \frac{T - T_m}{T_w - T_m}, \quad \phi(\eta) = \frac{C - C_m}{C_w - C_m}, \\
u = U_0 \exp(x/L)f'(\eta), \quad v = -\sqrt{\frac{\nu U_0}{2L}} \exp(x/2L) \{ f(\eta) + \eta f'(\eta) \}
\]

\( f' \) denotes the non-dimensional stream function, the prime denotes differentiation with respect to \( \eta \) and the stream function \( \psi \) is defined in the usual way as \( u = \partial \psi / \partial y, \quad v = -\partial \psi / \partial x \). Making use of transformations \((2.2.7)\) in \((2.2.1)\), we can realize incompressibility condition (i.e. continuity equation) is identically satisfied and the governing eqns \((2.2.2) - (2.2.4)\) takes the form of non-linear ordinary differential equations:

\[
f'' + \frac{1}{f'} - 2f'^2 = 0 \quad (2.2.8) \\
0'' + \nu f 0' + Pr Nb \phi'0 + Pr Nt 0'^2 = 0 \quad (2.2.9) \\
\phi'' + Le (f' \phi' - f \phi') + \frac{Nt}{Nb} \theta'' = 0 \quad (2.2.10)
\]

The boundary conditions are

\[
f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad \text{at} \quad \eta = 0 \\
f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \quad \text{as} \quad \eta \to \infty \quad (2.2.11)
\]
Where \( u, \Theta \) and \( \phi \) are dimensionless velocity, temperature and nanoparticles concentration, respectively. \( \eta \) is the similarity variable, the prime denote differentiation with respect to \( \eta \) and the governing parameters appearing in eqs (8) to (10) are defined by:

\[
\begin{align*}
Pr & = \frac{u}{\alpha} \quad \rightarrow \text{Prandtl number} \\
Le & = \frac{u}{D_\alpha} \quad \rightarrow \text{Lewis number} \\
Nb & = \frac{(\rho c)_r D_\phi (C_u - C_w)}{(\rho c)_r u} \quad \rightarrow \text{Brownian motion parameter} \\
NT & = \frac{(\rho c)_r D_T (T_u - T_w)}{(\rho c)_r T_u} \quad \rightarrow \text{Thermophoresis parameter}
\end{align*}
\]

(2.2.12)

It is important to note that this boundary value problem reduces to the classical problem of flow and heat and mass transfer due to a stretching surface in a viscous fluid when \( Nb \) and \( NT \) are zero in eqs (2.2.9)-(2.2.10) (The boundary value problem for \( \phi \) is then becomes ill-posed and is of no physical significance).

The important physical quantities of interest in this problem are local Skin friction coefficient \( C_f \), the local Nusselt number \( Nu \) and the local Sherwood number \( Sh \) are defined as:

\[
C_f = -\frac{\tau_w}{\rho u^2}, \quad Nu = \frac{\dot{q}_w}{k(T_u - T_w)}, \quad Sh = \frac{\dot{q}_m}{D_\phi (C_u - C_m)}
\]

(2.2.13)

Where wall shear stress \( \tau_w \), wall heat flux \( \dot{q}_w \), mass flux \( \dot{q}_m \) are given by:

\[
\tau_w = \rho \left( \frac{\partial u}{\partial y} \right)_{\eta=0}, \quad \dot{q}_w = -k \left( \frac{\partial T}{\partial y} \right)_{\eta=0}, \quad \dot{q}_m = -D_\phi \left( \frac{\partial \phi}{\partial y} \right)_{\eta=0}
\]

(2.2.14)
Where \( C_r \), \( \text{Nu}_x (\text{Nur}) \), \( \text{Sh}_x (\text{Shr}) \), \( \text{Re}_x \) are the skin friction, local Nusselt number, local Sherwood number and local Reynolds number respectively.

By solving eqs.(2.2.13) using eqs.(2.2.7),(2.2.14). We get

\[
C_r \sqrt{2 \text{Re}_x} = f'(0), \quad \left( \frac{2}{X} \frac{\text{Nu}_x}{\sqrt{\text{Re}_x}} \right) = \phi(0) - \text{Nur}, \quad \left( \frac{2}{X} \frac{\text{Sh}_x}{\sqrt{\text{Re}_x}} \right) = \psi(0) - \text{Shr}
\]

(2.2.15)

Where \( X = x / l \) is dimensionless coordinate along the sheet, \( l \) is the length of the sheet, \( C_r \), \( \text{Nu}_x (\text{Nur}) \), \( \text{Sh}_x (\text{Shr}) \), \( \text{Re}_x \) are the skin friction, local Nusselt number, local Sherwood number and Reynolds number respectively.

2.3. NUMERICAL SOLUTION

An well-organized fourth order Runge-Kutta method along with shooting technique has been employed to study the flow model of the above coupled non-linear ordinary differential equations (2.2.8)-(2.2.10) for different values of governing parameters viz. Prandtl number \( Pr \), Lewis parameter \( L_e \), Brownian motion parameter \( Nb \) and thermophoresis parameter \( Nt \). The non-linear ordinary differential equations are first decomposed into a system of first order differential equations. The coupled ordinary differential equs.(2.2.8)-(2.2.10) are third order in \( f \) and second order in \( \theta \) and \( \phi \) which have been reduced to a system of seven simultaneous equations for seven unknowns. In order to numerically solve this system of equations using Runge-Kutta method, the solution requires seven initial conditions but two initial conditions in \( f \) one initial condition in each of \( \theta \) and \( \phi \) are known. However, the values of \( f, \theta \) and \( \phi \) are known at \( \eta \rightarrow \infty \) These end conditions are utilized to produce unknown initial conditions at \( \eta = 0 \) by shooting technique. The most important step of this scheme is to choose the appropriate finite
value of \( \eta_\infty \). Thus to estimate the value of \( \eta_\infty \), we start with some initial guess value and solve the boundary value problem consisting of Eqs. (2.2.8)-(2.2.10) to obtain \( f'(0), \theta'(0) \) and \( \phi'(0) \).

The solution process is repeated with another larger value of \( \eta_\infty \) until two successive values of \( f''(0), \theta'(0) \) and \( \phi'(0) \) differ only after desired significant digit. The last value of \( \eta_\infty \) is taken as the finite value of the limit \( \eta_\infty \) for the particular set of physical parameters for determining velocity, temperature, and concentration, respectively are \( f(0), \theta(0) \) and \( \phi(0) \). After accomplishment of all the initial conditions we solve this system of simultaneous equations using fourth order Runge-Kutta integration scheme. The value of \( \eta_\infty \) is selected to vary from 5 to 20 depending on the physical parameters governing the flow so that no numerical oscillation would occur. Thus, the coupled boundary value problem of third-order in \( f \), second order in \( \theta \) and \( \phi \) has been reduced to a system of seven simultaneous equations of first-order for seven unknowns is as follows:

The eqs. (2.2.8)-(2.2.10) can be expressed as:

\[
\begin{align*}
\dot{f} & = -ff' + 2f'' \\
\dot{\theta} & = -Pr \left[ f\theta' + Nt\theta'' \right] \\
\dot{\phi} & = -Le\left( \phi' - f\phi' \right) + Pr \left( \frac{Nt}{Nb} \right) \left[ f\phi' + Nt\phi'' \right] 
\end{align*}
\]  
(23.16)

Now we can define new variables by the equations:

\[
\begin{align*}
f_1 &= f, \quad f_2 = f', \quad f_3 = f'', \quad f_4 = \theta, \quad f_5 = \theta', \quad f_6 = \phi, \quad f_7 = \phi'
\end{align*}
\]  
(2.3.17)
The coupled higher order non-linear differential equations (2.2.8)-(2.2.10) with the boundary conditions (2.2.11) may be transformed to seven equivalent first order differential equations and boundary conditions as given below:

\[ f'_1 = f_2, \]
\[ f'_2 = f_3, \]
\[ f'_3 = -f'_1 f_2 + 2f_2^2, \]
\[ f'_4 = f_5, \]
\[ f'_5 = -Pr\left[f'_2 f_3 + Nb f'_2 f_4 + N t f'_2\right], \]
\[ f'_6 = f_7, \]
\[ f'_7 = -Le(f'_1 f_3 - f'_2 f_6) + Pr\left(\frac{N t}{Nb}\right)[f'_3 f_4 + Nb f'_2 f_3 + N t f'_2] \]  \( (2.3.18) \)

A prime denotes the differentiation with respect of \( \eta \) and the boundary conditions are:

\[ f_1(0) = 0, \quad f_2(0) = 1, \quad f_3(0) = 1, \quad f_4(0) = 1, \quad f_5(\infty) = 0, \quad f_6(\infty) = 0, \quad f_7(\infty) = 0 \]  \( (2.3.19) \)

In the present analysis, the boundary value problem is first converted into an initial value problem (IVP). Then the IVP is solved by suitably guessing the missing initial value using the shooting method for several sets of parameters. The step size \( h=0.1 \) is used for the computational purpose. The error tolerance of \( 10^{-6} \) is also being used. The results obtained are displayed through tables and graphs, and the main features of the problems are discussed and analyzed.
2.4. RESULTS AND DISCUSSION

The numerical solutions are obtained for velocity, temperature and concentration profiles for different values of governing parameters. The obtained results are displayed through graphs from, Figs.2-13(b) for velocity, temperature and concentration profiles respectively.

Figs 2, 3 and 4. shows the effects of Nt and Nb for the selected values of Pr and Le numbers. As expected, the boundary layer profiles for the temperature profile $\theta(\eta)$ are essentially the same form as in the case of a regular fluid. It is observed that the temperature profile increases as the parameters (fig 2,Nt,Nb=0.1,0.3,0.5) ,(fig3,Nb=0.1,0.2,0.3) and (fig 4,Nt=0.1,0.3,0.5) increases, which results in thickening of thermal boundary layer of the fluid. An increase in Nb corresponds to the effective motion of nanoparticles within the flow. The intensity of this chaotic motion increases the kinetic energy of the nanoparticles and as a consequence the nanofluid's temperature rises.

In nanofluids the Brownian motion takes place due to the size of nanoparticles which is of nanometer scale and at this level, the particle motion and its effect on the fluid have a pivotal role over heat transfer. From the definition of thermophoresis parameter Nt, it is obvious that larger values of Nt correspond to the larger temperature difference and shear gradient. Thus increase in Nt leads to the larger temperature inside the boundary layer as depicted in Fig 4.

Figs. 5 and 6 shows the effects of Pr and Le numbers on the temperature profiles for the selected values of Nb and Nt. It is observed that the temperature decrease, as the parameters in (fig5, Pr,Le=1,10) and (fig6, Pr=10,15,20) increases, which results in thinning of thermal boundary layer thickness of the fluid.

Figs. 7, 8 and 9 shows the effects of Nb ,Le and Nt parameters on concentration profiles for the selected values of other parameters. It is observed that the concentration profile
decrease as the parameters (fig 7, Nb = 0.1, 0.3, 0.5), (fig 8, Le = 10, 20, 30) increases, while concentration profile increases as the parameter (fig 9, Nt = 0.1, 0.2, 0.3) increases. In figs 7 and 8 which results in thinning of concentration boundary layer thickness of the fluid. Whereas in fig 9, it is noticed, the thickening of concentration boundary layer thickness of the nano fluid.

From fig 2. and fig 7, we can say that the temperature profiles converge quickly than the concentration profiles. The thickness of the boundary layer for the concentration profiles $\Phi(\eta)$ is found to be lesser than the thermal boundary layer thickness when Le > 1. It decreases with the increase in Nb and this decrease diminishes when Nb > 5.

**Figs. 10(a) and 10(b)** shows, the variation in dimensionless heat transfer rates (i.e. Nusselt number) vs Nt for Pr = 1 and Pr = 10 respectively. These figures illustrate the effects of Pr and Nb on the dimensionless heat transfer rates for the same combination of Le. It is noticed that the local Nusselt number decreases with the increase in Nb and Nt, but increase with increase in Pr, with higher Prandtl number has a relatively lower thermal conductivity, which results in reduction of the thermal boundary layer thickness.

**Figs. 11(a) and 11(b)** shows for both the cases of LSS and ESS, the variation in dimensionless heat transfer rates vs Nt for Le = 5 and Le = 25 respectively. These figures show the effects of Le and Nb on the dimensionless heat transfer rates for the same combination of Prandtl numbers. It is noticed that the local Nusselt number decreases with the increase in Nb and Nt but decrease with increase in Le. The alteration in the dimensionless heat transfer rates is found to be higher for smaller values of Nb and this change decreases with the increase of Nt.

**Figs. 12(a) and 12(b)** depicts the variation in dimensionless mass transfer rates (i.e. local Sherwood number) vs Nt for Pr = 1 and Pr = 10 respectively. These figures show the effects of Pr and Nb on the dimensionless mass transfer rates for the same combination of Le.
Further it is noticed from figs 12(a) and 12(b), that the local Sherwood number increases with the increase in Nb and Nt, but increases with increase in Pr.

**Figs. 13(a) and 13(b)** shows the disparity in dimensionless concentration rate $-\phi'(0)$ vs Nt for Le=5 and Le=25 respectively. These figures show the effects of Le and Nb on dimensionless heat transfer rates for the same combination of Prandtl numbers. It is noticed that in fig 13(a) the local Sherwood number increases with the increase in Nb and Nt for Le=5 and in fig 13(b) local Sherwood number decreases with the increase in Nb and Nt for Le=25, but however local Sherwood number increase with increase in Le.

Finally, a comparison with published work available in the literature has been performed in order to check the accuracy of the present results.

From table 1, it shows a test of accuracy of the solution, the values of local Nusselt number $-\theta'(0)$ for different values of Prandtl number are compared with solutions reported by Magyari and Keller[1999], El-Aziz[2009], Bidin and Nazar[2009], Anur Isha[2011] and Swati Mukhopadhyay[2012]. The table shows the numerical solution obtained by the fourth order Runge-Kutta method along with shooting technique is in very good agreement. Therefore, we are confident that results obtained by us are very much accurate to analyze the flow problem.

Tables 2 and 3 shows the variation of the local Nusselt number and local Sherwood number respectively for different values of Nb, Nt for Pr=10, Le=10. It is noticed that local Sherwood number is a decreasing function, while it is an increasing function for (Nb=0.1 to Nb=0.5 keeping Nt =0.1,0.2,0.3,0.4,0.5) and initially decreasing function for (Nt=0.1 to Nt=0.5 for Nb=0.1,0.2) later appears to be an increasing function for (Nt=0.1 to 0.5 for Nb=0.3,0.4,0.5).
Finally, a comparison with published papers available in the literature has been done in order to check the accuracy of the present results.

**From table 1,** it shows a test of accuracy of the solution, the values of local Nusselt number $-\theta(0)$ for different values of Prandtl number are compared with solutions reported by Magyari and Keller [1999], El-Aziz [2009], Bidin and Nazar [2009], Anur Ishak [2011] and Swati Mukhopadhyay [2012]. The table shows the numerical solution obtained by the present fourth order Runge-Kutta method along with Shooting technique are in very good agreement. Therefore, we are confident that our results are highly accurate to analyze the flow problem.

**Tables 2 and 3** shows the variation of the reduced Nusselt number (Nur) and reduced Sherwood number (Shr) respectively for different values of Nb, Nt for Pr=10, Le=10. It is observed that Nur is a decreasing function, while Shr is an increasing function for (Nb=0.1 to Nb=0.5 keeping Nt =0.1,0.2,0.3,0.4,0.5) and initially decreasing function for (Nt=0.1 to Nt=0.5 for Nb=0.1,0.2) afterwards increasing function for (Nt=0.1 to 0.5 for Nb=0.3,0.4,0.5).

**2.5. CONCLUSIONS.**

In this paper, effects of Prandtl number (Pr), Lewis number (Le), Brownian motion parameter (Nb), thermophoresis parameter (Nt) on temperature profiles, concentration profiles, local Nusselt number and local Sherwood number, on the boundary layer flow and heat transfer of nanofluids past an exponential stretching sheet is investigated. The variation of the reduced Nusselt number and reduced Sherwood numbers with Nb and Nt for various values of Pr and Le is presented in tabular and graphical forms. The numerical results obtained are in excellent agreement with the previously published data in limiting
condition and for some particular cases of the present study. The following conclusions have been drawn from the present study:

1] The effects of Pr, Le are inversely proportional to temperature where as the reverse effect is seen in case of Nt, Nb.

2] The effects of Nt is directly proportional to concentration (mass fraction) where as the reverse effect is seen in case of Nb and Le.

3] In the case of LSS, it is found that the reduced Nusselt number is decreasing function, while the reduced Sherwood number is increasing function for each of the dimensionless parameters Pr, Le, Nb and Nt considered.

4] The reduced Sherwood number is an increasing function of higher Pr and a decreasing function of lower Pr, while reduced Nusselt number is decreasing function for lower Pr and increasing function for higher Pr for each Le, Nb and Nt parameters.

5] The reduced Nusselt number is a decreasing function of higher Le and a increasing function of lower Le, while reduced Sherwood number is increasing function of higher Le and decreasing function of lower Le for each Pr, Nb, and Nt parameters.
Table 1.
Comparison of results for the local Nusselt number $-\theta(0)$ for $Nt = Nb = Le = 0$.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.954782</td>
<td>0.954785</td>
<td>0.9548</td>
<td>0.9548</td>
<td>0.9547</td>
<td>0.951556</td>
</tr>
<tr>
<td>2.0</td>
<td>----------</td>
<td>------</td>
<td>1.4714</td>
<td>1.4715</td>
<td>1.4714</td>
<td>1.465304</td>
</tr>
<tr>
<td>3.0</td>
<td>1.869075</td>
<td>1.869074</td>
<td>1.8691</td>
<td>1.8691</td>
<td>1.8691</td>
<td>1.859997</td>
</tr>
<tr>
<td>5.0</td>
<td>2.500135</td>
<td>2.500132</td>
<td>---------</td>
<td>2.5001</td>
<td>2.5001</td>
<td>2.485222</td>
</tr>
<tr>
<td>10.0</td>
<td>3.660379</td>
<td>3.660372</td>
<td>---------</td>
<td>3.6604</td>
<td>3.6603</td>
<td>3.630831</td>
</tr>
</tbody>
</table>

Table 2
Variation of Nur with Nb and Nt for $Pr=10, Le=10$.

<table>
<thead>
<tr>
<th>ESS</th>
<th>Nb=0.1</th>
<th>Nb=0.2</th>
<th>Nb=0.3</th>
<th>Nb=0.4</th>
<th>Nb=0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nt</td>
<td>Nur</td>
<td>Nur</td>
<td>Nur</td>
<td>Nur</td>
<td>Nur</td>
</tr>
<tr>
<td>0.1</td>
<td>1.996804</td>
<td>1.398096</td>
<td>1.010997</td>
<td>0.762334</td>
<td>0.600227</td>
</tr>
<tr>
<td>0.2</td>
<td>1.680884</td>
<td>1.214909</td>
<td>0.906994</td>
<td>0.703189</td>
<td>0.565804</td>
</tr>
<tr>
<td>0.3</td>
<td>1.463489</td>
<td>1.087530</td>
<td>0.833110</td>
<td>0.659919</td>
<td>0.539755</td>
</tr>
<tr>
<td>0.4</td>
<td>1.309018</td>
<td>0.995461</td>
<td>0.778371</td>
<td>0.626898</td>
<td>0.519242</td>
</tr>
<tr>
<td>0.5</td>
<td>1.195238</td>
<td>0.926248</td>
<td>0.736165</td>
<td>0.605640</td>
<td>0.502535</td>
</tr>
</tbody>
</table>
Table 3

Variation of Shr with Nb and Nt for Pr=10, Le=10.

<table>
<thead>
<tr>
<th>ESS</th>
<th>Nb=0.1</th>
<th>Nb=0.2</th>
<th>Nb=0.3</th>
<th>Nb=0.4</th>
<th>Nb=0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nt</td>
<td>Shr</td>
<td>Shr</td>
<td>Shr</td>
<td>Shr</td>
<td>Shr</td>
</tr>
<tr>
<td>0.1</td>
<td>2.977544</td>
<td>3.536209</td>
<td>3.661603</td>
<td>3.696171</td>
<td>3.703874</td>
</tr>
<tr>
<td>0.2</td>
<td>2.608756</td>
<td>3.493767</td>
<td>3.676965</td>
<td>3.727476</td>
<td>3.738362</td>
</tr>
<tr>
<td>0.3</td>
<td>2.533735</td>
<td>3.475994</td>
<td>3.690462</td>
<td>3.750683</td>
<td>3.764038</td>
</tr>
<tr>
<td>0.4</td>
<td>2.438263</td>
<td>3.460115</td>
<td>3.697129</td>
<td>3.765798</td>
<td>3.782225</td>
</tr>
<tr>
<td>0.5</td>
<td>2.351303</td>
<td>3.438623</td>
<td>3.696437</td>
<td>3.775423</td>
<td>3.792153</td>
</tr>
</tbody>
</table>
Fig 2. Effects of \( N_t \) and \( N_b \) on temperature profiles

\[ \theta(\eta) \]

\( \eta \)

\( \Pr = 10, \text{Le} = 10 \)

\( N_t, N_b = 0.1, 0.3, 0.5 \)
Fig 3. Effects of Nb on temperature profiles

$\text{Pr}=10, \text{Le}=10, \text{Nt}=0.1$

$\text{Nb}=0.1, 0.3, 0.5$
Fig 4. Effects of \( Nt \) on temperature profiles

\[ Pr=10, Le=10, Nb=0.1 \]

\( \theta(\eta) \)

\( Nt = 0.1, 0.3, 0.5 \)

\( \eta \)
Fig 5. Effects of Pr and Le on temperature profiles

Fig 6. Effects of Pr number on temperature profiles
Fig 7. Effects of Nb on concentration profiles
Fig 8. Effects of Le number on concentration profiles

Pr=10, Nt=0.1, Nb=0.1

Le=10, 20, 30
Fig 9. Effects of Nt on concentration profiles

$\phi(\eta)$

$\eta$

Pr=10, Le=10, Nb=0.1

Nt = 0.1, 0.2, 0.3
Fig 10(a) Effects of $N_b$ on dimensionless heat transfer rate.
Fig 10(b) Effects of Nb on dimensionless heat transfer rates

Pr=10, Le=10

-θ'(0)

Nb=0.3

Nb=0.4

Nb=0.5

Nt
Fig 11(a) Effect of Nt on dimensionless heat transfer rates

Pr=10, Le=5
Fig 11(b) Effect of Nb on dimensionless heat transfer rate

For $Pr=10$, $Le=25$:

- $Nb=0.5$
- $Nb=0.4$
- $Nb=0.3$
Fig 12(a) Effect of Nb on dimensionless concentration rate

Pr=1, Le=10
Fig 12(b) Effect of Nb on dimensionless concentration rate

Pr=10, Le=10

Nb=0.5
Nb=0.4
Nb=0.3
Fig 13(a) Effect of Nb on dimensionless concentration rate

Pr=10, Le=5
Fig 13(b) Effect of Nb on dimensionless concentration rate

Pr=10, Le=25

-\phi'(0)

Nt

Nb=0.3
Nb=0.4
Nb=0.5