INTRODUCTION

The thesis is divided into two parts, each having two chapters. Part I, having chapters I and II, deals with certain aspects of dynamical systems, while part II having chapters III and IV deals with some aspects of non-relativistic quantum systems.

In chapter I we review the method of investigating the integrability and nonintegrability of classical dynamical systems using the action-angle variables. By a comparative study of the two-dimensional harmonic oscillator - an example of integrable system and the standard Henon-Heiles system - an example of nonintegrable system - we illustrate the emergence of stochasticity in phase space as a result of nonintegrability in the later system. The phase spaces of both the cases are 2-tori. The preservation of these KAM tori in the case of the two dimensional harmonic oscillator and their destruction for the Henon-Heiles system are shown by way of illustrations. The later is directly linked to the emergence of chaos in the system.

For going over to the quantum systems corresponding to the classical motion in the stochastic region we use the classical Birkhoff-Gustavson Normal Form (BGNF) expansion to construct approximate integrals of motion in the region around the elliptic fixed points. A quantisation scheme for the BGNF is the Quantum Normal Form (QNF) as prescribed by
Eckhardt. Using the QNF the energy spectrum of a linear harmonic oscillator perturbed by an $X^6$ term is investigated. It is found that the series generated by the QNF is term by term equivalent to the series obtained from the Rayleigh–Schroedinger Perturbation Theory. Finally we discuss the stochasticity phenomenon in the quantum context.

In chapter II we study numerically a one dimensional map $X_{n+1} = X_n \exp[r(1-X_n)]$. Like the logistic map it describes a population with exponential growth at low densities and represents the effect of epidemic diseases at high densities. It is more complicated than the quadratic map, but has the advantage that a positive population stays positive. We study the emergence of period doubling bifurcations as the parameter is varied, ultimately leading to chaos.

Another mapping which demonstrates the destruction of KAM torus – the Cremona map of Siegel and Henon is also discussed. In this map we see the formation of hyperbolic islands where the tori are destroyed and emergence of saddle points, which is responsible for further instabilities as can be seen from the behaviour of the subsequent iterates. Computer programs written in Turbo Pascal for the latter example and Microsoft Quickbasic for the former example are also given.
In chapter III we discuss the use of the SU(1,1) Lie-Algebra as the Spectrum Generating Algebra (SGA) for the Schrödinger eigenvalue problem. We obtain the most general realisation of the SU(1,1) algebra in terms of second order derivative operators in a single variable and discuss the type of potentials for which the eigenvalues of the Schrödinger Hamiltonian can be obtained through the approach of SU(1,1).

Chapter IV deals with some calculational aspects in non-relativistic quantum mechanics. Epstein's off-diagonal extensions of the Feynman-Hellmann Theorem (FHT) and consequent rederivation of the non-degenerate Rayleigh-Schrödinger Perturbation Theory is further pursued to include the degenerate case and also the Lennard-Jones-Wigner perturbation theory. The extension of the FHT is shown to lead to non-diagonal generalisation of the Quantum Virial Theorem. The extended FHT is applied

1. to calculate the matrix elements of the kinetic energy and potential energy of a linear harmonic oscillator,

2. to obtain an expression for the matrix elements of force in a stationary state and

3. to obtain the average K.E. of a quantum system in the semi-classical approximation.