CHAPTER-05

METHOD OF VERIFYING THE PHASE CENTER
Method of Verifying the Phase Center:

5.0 Concept of Phase Center :-

The so’s which means called stage center: the point of convergence of curve of intersectant line through the indistinguishable and phasic plane of radio wire radiation field and the plane including gathering device center. Essentially, the spheriform point of convergence of the roundabout wave radiation field in the far zone is corresponding stage center.

We understand that with the movements of repeat, the dynamic district of log-infrequent dipole gathering device is advancing. Consequently the zone of its stage center is developing. For broadband gathering contraptions, this paper about the log-discontinuous dipole receiving wire, we consider it to be Yagi-Uda radio wire whose radiation qualities are relative with the radio wire on the paper on the properties’ reason of log-incidental radio wire. As we most likely am mindful, radiative essentialness of log-periodic gathering mechanical assembly is done by the dynamic range. Specifically the dipole’s length receiving wire is essentially identical to full wavelength. This paper relies on upon the use of electromagnetic entertainment business programming to remove the stage point of convergence of log-intermittent dipole receiving wire. The stage center can be grabbed direct from CST propagation programming on the constrained’s reason difference time space procedure. For FEKO programming on the Moment’s reason Method and HFSS programming on the restricted’s reason segment procedure, the results of stage center can’t be reenacted particularly. Despite the way that the stage center can be particularly obtained by entertainment programming, it is crucial to completely show exploratory data and pay thought on the steadfastness on the outlining. This proposition has been examining the schedules for affirming stage center from CST generation programming.
Remembering the finished objective to affirm the result’s precision about stage center with CST reenactment programming, the approach is taken in this paper: checking the region of stage center from CST and the partition within utilizing in order to break focuses set HFSS. Checking data is dismembered with the thought and use of probability and numerical estimations remembering the final objective to affirm the result’s rightness. Especially isolated into three stages: the first step, finding the essential fragment whether Phi or Theta on the H plane or E plane in HFSS programming. After that choosing the achieve in the 3dB column width; moreover, here inputting in order to find out the stage center in CST half of 3dB shaft width. The third walk in HFSS moving coordination the stage’s zone focus and looking at to one side and right. Inside 3dB wave width, the territory of stage center relating little instabilities close by changes of the edge is
basically right on the H plane or E plane. Differentiating and stage center from CST, the purpose of upgrading the result's precision is proficient. Fig 1 gives HFSS diversion model which reveals to move the headings to the position of stage core interest. A couple reasons of speculations ought to be cleared up amid the time spent affirmation. In the first place, why should center the essential part in particular planes? An impressive measure of business reenactment programming tasks are nonappearance of amalgamation. It is hard to comprehend exact worth. Thusly using the models of change, it is uncommon to hold the genuine fragment in the examination. By then why is the 3dB shaft width asked for to settle the issue? The rule reason is to ensure round wave radiation toward far zone. In doing thusly, the result's error is diminished past what numerous would consider conceivable.

5.1 Applying variance ideas by verify the result of phase center :-

The examination of progress was extraordinarily joined in exploratory estimations. Use parameter estimation and theory testing to deal with the immeasurable volumes of data. The degree which each and every imperative part affect the trial's eventual outcome is grabbed. In this work, the repeat of 2.0GHz. The eventual outcomes of are known as test spotlights on the crucial fragments of the essential H plane inside 3dB column width. The test targets are affected by two components, independently: under the 3dB column width degrees (0 changes every two degrees) and the center detachment from top of the nourishment line of the radio wire to the reason for headings. The state of components is known as the level. In the reenactment, the bar width and the qualities of x centre from foundation of the heading to vertex of the sustenance line of the radio wire change. Likewise, only a test's delayed consequence is grabbed in the blend of two variables. It is two-Fig the examination of change without excess. In this work, a numerical model about the application with the examination of progress is set up:

On the suspicion that these two variables, for instance, An and B may impact the trial's result, and A have 63 level, seven level are have a spot with B. the
general set $X_{ij}$ (each rEPhi results) are regularly self-sufficient, and obey $N(ij,2)$ The estimation of these recognition are $X_{ij}(i = 1,2,...,63;j=1,2,...,7)$ During gazing upward the suspicion, if the typical estimations of the general game plan of the two variables under every level are proportionate, the theory come through. The comparisons of two-segment examination of progress without are given bene. Due to space confinements, Table 1 gives the information piece of this paper. As indicated by the information acquired FA 6.00,FB 62.00 are ascertained with the above equations. At the point when critical level is

$$S_d = n\sum_{i=1}^{n}(X_i - \bar{X})^2 \quad (1), \quad S_g = m\sum_{j=1}^{m}(X_j - \bar{X})^2 \quad (2), \quad S_x = \sum_{i=1}^{n}\sum_{j=1}^{m}(X_{ij} - X_i - X_j + \bar{X})^2$$

$$F_a = \frac{\frac{S_x}{\frac{m-1}{S_x}}}{\frac{m}{(m-1)(n-1)}} \geq F_{as} \{m-1,(m-1)(n-1)\} \quad (4), \quad F_a = \frac{\frac{S_g}{\frac{n-1}{S_g}}}{\frac{n}{(m-1)(n-1)}} \geq F_{ns} \{n-1,(m-1)(n-1)\}$$

From now on, two variables have a colossal impact on the guideline section in the affirmation about stage center of the model. With a particular finished objective to check the stage’s precision center from CST, the criticalness of contrast in this paper is used. The condition about the little change of unpredictable variables unveiled that contrasting with the position of stage center is insignificant impacted. The point differentiating and the reenactment results is accomplished. The change’s comparison is key.

$$DX = EX - (EX)^2$$

exhibits the data learned in repeat 2.0GHz. From the source estimation of the heading to top of receiving wire nourishment line $x= - 7.7mm$ the distinction $DX = - 69.8033 js$ insignificant. Differentiating and stage center from CST, we can see that the refinement is 10mm between the two results. Regardless, the past detachment is similar to the resonating’s length half-wave dipole 75mm. Since segment B is basically influenced on rEPhi, the wonders above can be illuminated in the check. In conform, the territory of stage point of convergence of log-discontinuous dipole receiving wire is set aside a few
minutes spent affirmation and examination, and the clarifications behind these failures are all around cleared up.

5.2. Boundary Conditions

Remembering the deciding objective to have the ability to get the constants $C_k$ from a point of confinement condition must be joined on each of the diverse $k$ parts. The limit condition to be satisfied is that the vector potential is discontinuous over the parts’ terminations. Considering the physical operation of the show and the dispersal of potential on the $k$th part, it can be seen that beside immovably isolated coupled segments the responsibilities to $A_z(k, z)$ by the coupled segments will associate with consistent. Thusly the vector potential may be thought to be included a steady part notwithstanding an evolving part. The consistent part may be removed from the right-hand side of (7) by exhibiting the vector potential difference, described for the $k$th segment

$W_{zk}(z) = A_z(k, z) - A_z(l_k)$ \(\text{(8)}\)

Hence the limit condition requires the variable piece of the vector potential to vanish at the component closes, i.e.

$W_{zk}(l_k) = 0$ \(\text{(9)}\)

It will further be discovered helpful to characterize a consistent $U_k$, corresponding to the steady piece of the vector potential, as

$$U_k = \frac{-j\omega}{\beta} A_z(l_k) = \frac{-j\zeta_0}{4\pi} \sum_{k=1}^{\infty} \int_{-l_k}^{l_k} L_{\epsilon, t}(z') K(t_k, z') \frac{d}{dz'}$$ \(\text{(10)}\)

Where

$$K(l_k, z') = \frac{\exp(-j\beta R'_{ki})}{R'_{ki}} \left\{ \right.$$ \(\text{(11)}\)

Substituting (10) into (7) and solving boundary to eliminate $C_k$ gives
\[
\sum_{k=1}^{N} \int_{-l}^{l} z I(z') K D(z, z') dz' = j 4 \pi \zeta \cos \beta_{l} \\
X \{U_k [\cos \beta z - \cos \beta l] + \frac{1}{2} V_{Ok} \sin \beta (l - z)\} \text{ For } k = 1, 2, 3, \ldots, N
\]

which represents the set of vector potential difference equations characterizing the log-periodic array, where the difference kernel is given by

\[
K D(z, z') = \frac{\exp(-j \beta R_{ki})}{R_{ki}} - \frac{\exp(-j \beta R'_{ki})}{R'_{ki}}
\]

5.3 Functional Variation of Vector Potential Components:

The difference kernel (13) may be separated as follows,

\[
K D(z, z') = K D R(z, z') + K D I(z, z')
\]

where

\[
K D R(z, z') = K R(z, z') - K R(lk, z')
\]

\[
K D I(z, z') = -K I(z, z') + K I(lk, z')
\]

It will be seen that the right-hand side of (12) is communicated as far as two source capacities V Ok and U k, the previous of which is a potential distinction limited at z k = 0 and the recent a field of steady abundance disseminated over the whole length of every component. Subsequently the present circulation in every component may be viewed as the whole of two segments of which one is produced specifically by V Ok, as though the receiving wire were disengaged, and the other is affected by U k as in a getting radio wire put in a uniform field. The main term in the present's piece kept up by U k is and the main term in that kept up straightforwardly by V Ok is sin Whence it is sensible
to accept that the present may be composed as the aggregate of two sections, contingent upon the source capacities; i

\[ I_z(z') = I_v(z') + I_u(z') \]  \hspace{1cm} (15)

which adding functional dependence becomes,

\[ I_z(z') = A_i \sin \beta (l_i - |z'|) + B_i (\cos \beta z' - \cos \beta l_i) \]  \hspace{1cm} (16)

where \( A_i \) and \( B_i \) are in general complex amplitude constants. The vector potential difference on the lefthand side of (12) may now be written as

\[ W_{v_k}(z) + W_{u_k}(z) = \sum_{l=1}^{N} \int_{-l_i}^{l_i} \left[ I_{v_l}(z') + I_{u_l}(z') \right] \times [KDR(z,z') + jKDI(z,z')] dz' \]  \hspace{1cm} (17)

The two gatherings \( W_{v_k}(z) \) and \( W_{u_k}(z) \) are the vector potential contrast circulations on a component \( k \) in the exhibit because of the dissemination from every single other component \( /= 1, 2, 3, \ldots N \), with a source appropriation corresponding to \( \sin \beta/(l_i - |z'|) \) and separately on them. With a specific end goal to particular the integrals of (17) further it is important to examine the vector's reliance potential contrast segments \( W_{v_k}(z) \) a
Table 5.1

<table>
<thead>
<tr>
<th>Potential component</th>
<th>$d_{k1}$ range</th>
<th>Functional variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re($W_{yk1}$)</td>
<td>$0 \leq \frac{d_{k1}}{\lambda} \leq 0.25$</td>
<td>$\sin \beta(l_k -</td>
</tr>
<tr>
<td></td>
<td>$\frac{n}{4} &lt; \frac{d_{k1}}{\lambda} \leq \frac{n + 2}{4}$</td>
<td>$(-1)^{(n+1)/2} (\cos \beta z - \cos \beta l_k)$</td>
</tr>
<tr>
<td>Im($W_{yk1}$)</td>
<td>$0 \leq \frac{d_{k1}}{\lambda} \leq 0.5$</td>
<td>$-(\cos \beta z - \cos \beta l_k)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{n}{2} &lt; \frac{d_{k1}}{\lambda} \leq \frac{n + 1}{2}$</td>
<td>$(-1)^{(n+3)/2} (\cos \beta z - \cos \beta l_k)$</td>
</tr>
<tr>
<td>Re($W_{uk1}$)</td>
<td>$0 \leq \frac{d_{k1}}{\lambda} \leq 0.25$</td>
<td>$(\cos \beta z - \cos \beta l_k)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{n}{4} &lt; \frac{d_{k1}}{\lambda} \leq \frac{n + 2}{4}$</td>
<td>$(-1)^{(n+1)/2} (\cos \beta z - \cos \beta l_k)$</td>
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<td></td>
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<td>$(-1)^{(n+3)/2} (\cos \beta z - \cos \beta l_k)$</td>
</tr>
</tbody>
</table>

5.4 ADMITTANCE MATRIX APPROACH TO LOGPERIODIC ANTENNAS:

The log occasional reception apparatus, depicted in Fig 5.4, comprises of parallel direct dipole components orchestrated one next to the other in a plane and framing a coplanar cluster. Streams at the dipoles’ bases are utilized to focus the log’s example occasional receiving wire. To ascertain the dipole base streams, permission lattice way to deal with LPDA, which utilizes the
shared couplings between the dipoles, is used. Permission framework way to
deal with LPDA will be definite in this part. Dipole base streams will be utilized
for the reflector's outline reception apparatus which will be planned in the
extent of this proposal.

Fig 5.2 Log Periodic Dipole Array

5.5 CHARACTERIZATION OF A TWO-PORT MICROWAVE NETWORK:

Characterize a two-port microwave network.

\[
\begin{align*}
V_1 & \rightarrow Z_{c} \rightarrow I_2 \\
I_1 & \left[ \begin{array}{cc}
A & B \\
C & D \\
\end{array} \right] \rightarrow V_2
\end{align*}
\]

Fig 5.3, A two Port Network

Definitions of some parameters in Fig 5.5 are as follows:

Zc- The transmission line of characteristic impedance
\( \gamma \) is the propagation constant,

\[ \gamma = a + jb \]

where

\( \alpha \): is attenuation constant (Np/m) and

\( \beta \): is the phase constant (rad/m),

\( L \): is the length of the transmission line.

For Fig 5.3, ABDC matrix of the transmission line in terms of currents and voltages can be written as follows:

\[
\begin{bmatrix}
V_1 \\
I_1 \\
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix} \begin{bmatrix}
V_2 \\
I_2 \\
\end{bmatrix}
\]

(5.1)

Where

\[ A = \cosh \gamma l = D \]

\[ B = Z_c \sinh \gamma l \]

and

\[ C = \frac{1}{Z_c} \sinh \gamma l \]

For lossless transmission lines, \( \alpha = 0 \Rightarrow \cosh \alpha = 1 \) and \( \sin \alpha = 0 \) since

\[ \cosh \gamma l = \cosh (\alpha + j\beta) l = \cosh \alpha \cosh \beta l + \sinh \alpha \sinh \beta l \]

\[ = \cosh \beta l \]

And

\[ \sin \gamma l = \sin (\alpha + j\beta) l = \sin \alpha \cosh \beta l + \sinh \alpha \sin \beta l \]

\[ = \sin \beta l \]

Thus, the new forms of A, B, C and D are:

\[ A = \cos \beta l = D, \quad B = Z_c j \sin \beta l \quad \text{and} \quad C = \frac{1}{Z_c} j \sin \beta l \]

(5.2)
5.6 Admittance Matrix Approach :-

In this area, an induction grid way to deal with discover the streams at the dipole’s bases components of a log intermittent dipole radio wire will be given. In the first place, streams for the reception apparatus made out of two dipoles will be researched and after that the method for the receiving wire with two dipoles will be stretched out to a strategy for a radio wire made out of three and four dipoles separately. At long last, a system to locate the present at the dipoles’ bases will be outlined for a log intermittent receiving wire made out of N dipoles. Comparative system is displayed in Stutzman and Thiele [7].

5.6.1 Transmission Matrix Representation :-

For Fig 5.4, voltages and currents can be related as follows:

Substituting equation (5.2) in equation (5.1) gives:

\[ I_1 = I_2 \cos \beta l + jY C V_2 \sin \beta l \]
\[ V_1 = -V_2 \cos \beta l + jY^{-1}_C I_2 \sin \beta l \]

Reviewing the above equations in matrix form

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} =
\begin{bmatrix}
\cos \beta l & jY^{-1}_C \sin \beta l \\
Y C \sin \beta l & \cos \beta l
\end{bmatrix}
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\]
For the log occasional radio wire, transmission lines between the dipole components are jumbled and joined with one another. This befuddling of the lines is spoken to in Fig 5.5 as takes after.

![Diagram of transmission lines with crossover](image)

**Fig:- 5.5 Transmission Line Segment with cross over of lines**

Considering the crisscrossing of the lines, Equations 5.3.a and 5.3.b takes the following form:

\[ I_1 = I_2 \cos \beta l - jY_c V_2 \sin \beta l \]  \hspace{1cm} (5.5-a)

\[ V_1 = -V_2 \cos \beta l + jY_c^{-1} l_2 \sin \beta l \]  \hspace{1cm} (5.5-b)

**Let c = \cos \beta l and S = \sin \beta l**  \hspace{1cm} (5.6)

### 5.6.2.1 Two Antennas related by a Transmission Line:

In this segment, the present connection between the two radio wires joined by a transmission line will be explored.
In Fig 5.6, the excitation current is spoken to by $I_s$, the self inductions of the two reception apparatuses are spoken to by $Y_{11}$ and $Y_{22}$ and the shared permissions of the receiving wires are spoken to by $Y_{12}$ and $Y_{21}$. $Y_L$ is the heap permission. $Y_N$ is an induction grid for the transmission line between the two reception apparatuses and characterized as.

$$ [Y_N] = \frac{Y_c}{jS} \begin{bmatrix} C & 1 \\ 1 & 0 \end{bmatrix}, \quad Y_c = Z_c^{-1} $$ (5.7)

In Fig 5.6. applying Kirchhoff’s current law at node A and B:

$$ I_s = I_{A1} + I_1 \quad \text{(5.8)} $$

$$ I_{A2} + I_2 + I_L = 0 \Rightarrow I_{A2} + I_2 + Y_L V_2 = 0 \quad \text{(5.9)} $$

Rewriting eq. (5.2b) by using Eq. (5.6) we get:

$$ V_1 + V_2 C = jY_c C \cdot S I_2 \Rightarrow I_2 = \frac{Y_c}{jS} (V_1 + V_2 C) \quad \text{(5.10)} $$

$$ I_1 = \frac{Y_c}{jS} (V_1 + V_2 C) C - jY_c V_2 S $$

$$ \Rightarrow I_1 = \frac{Y_c}{jS} C V_1 + \left( \frac{Y_c}{jS} C^2 j Y_c S \right) V_2 $$

$$ \Rightarrow I_1 = \frac{Y_c}{jS} C V_1 + \frac{Y_c}{jS} \left( C^2 j Y_c S \frac{jS}{Y_c} \right) V_2 $$

Since $\cos^2 \beta l + \sin^2 \beta l = 1$ ($C^2 + S^2 = 1$), then:
I_1 = \frac{Y_c}{jS} (C.V_1 + V_2) \quad (5.11)

Rewriting eq. (5.10) and (5.11) in matrix

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \frac{Y_c}{jS} \begin{bmatrix}
C & 1 \\
1 & C
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = [Y_A] \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \quad (5.12)
\]

Rewriting eq. 5.8 and 5.9 in matrix form:

\[
\begin{bmatrix}
I_S \\
0
\end{bmatrix} = \begin{bmatrix}
I_{A1} \\
I_{A2}
\end{bmatrix} + \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} + \begin{bmatrix}
0 \\
Y_L V_2
\end{bmatrix}
\]

Where

\[
\begin{bmatrix}
I_{A1} \\
I_{A2}
\end{bmatrix} = [Y_A] \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \quad and \quad [Y_A] = [Z_A]^{-1} \quad (5.12, 5.13 \text{ and } 5.14)
\]

In Equation 5.14, \([Z_A]\) is a matrix which is composed of the self impedances of the dipole elements and the mutual impedances between the dipole elements.

For example for an antenna composed of two dipoles,

\[
[Z_A] = \begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\]

Where;

\(Z_{11}\) is the self impedance of the first dipole.

\(Z_{22}\) is the self impedance of the second dipole.

\(Z_{12}=Z_{21}\) is the mutual impedance between two dipoles.

Replacing equation 5.12 and equation 5.14 into equation 5.13:

\[
\begin{bmatrix}
I_S \\
0
\end{bmatrix} = [Y_A] \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} + [Y_N] \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & Y_L
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \quad \Rightarrow
\]
\[
\begin{bmatrix}
I_S \\
0
\end{bmatrix} = (I)A \begin{bmatrix}
V_1 \\
0
\end{bmatrix} + [Y_N] + \begin{bmatrix}
0 & 0 \\
0 & Y_L
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \tag{5.15}
\]

For a given excitation current, \( I \) (for example \( I = \text{1 ampere} \)) \( \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \) can be solved.

Finally the currents at bases of the dipoles can be found using equation (5.14).

5.6.1.3 Three Antennas Connected By Two Transmission Lines :-

Let us consider three antennas associated with one another by two transmission lines. The current relations on the dipoles can be related through the similar procedure which is described above.

**Fig 5.7 Three antennas connected by two transmission Lines**

In Fig 5.7, \( I_S \) represents the excitation current, \( Y_{11}, Y_{22} \) and \( Y_{33} \) represent the self admittances of the antennas and the mutual admittances of the antennas are represented by \( Y_{12}, Y_{21}, Y_{13}, Y_{31}, Y_{23}, Y_{32} \). \( Y_{N}^{(1)} \) and \( Y_{N}^{(2)} \) are admittance matrices for the transmission lines between the dipoles and \( Y_L \) is the load admittance.

In Fig 5.7, applying Kirchoff’s current law at node A, B and C:

\[
I_S = I_{A_1} + I_{1}^{(1)} \tag{5.16}
\]
\[
0 = I_{A2} + I_2^{(1)} + I_1^{(2)} \tag{5.17}
\]
\[
0 = I_{A3} + I_2^{(2)} + Y_LV_3 \tag{5.18}
\]

where the superscripts on I’s show the line number.

Rewriting the Equations:- 5.16, Equations:- 5.17 and Equation:- 5.18 in matrix form:

\[
\begin{bmatrix}
I_0 \\
0
\end{bmatrix} =
\begin{bmatrix}
I_{A1} & I_1^{(1)} \\
I_{A2} & I_1^{(2)} \\
I_{A3} & 0
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
Y_LV_3
\end{bmatrix} \tag{5.19}
\]

Where

\[
\begin{bmatrix}
I_1^{(1)} \\
I_2^{(1)} \\
0
\end{bmatrix} =
\begin{bmatrix}
Y_N^{(1)} & V_1 \\
V_2 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
I_1^{(2)} \\
I_2^{(2)}
\end{bmatrix} =
\begin{bmatrix}
Y_N^{(2)} & V_2 \\
V_3 & 0
\end{bmatrix}
\]

Let \( Y_N^{(1)} = \begin{bmatrix} a^{(1)} & b^{(1)} & V_1 \\ b^{(1)} & a^{(1)} & V_2 \end{bmatrix} \) and \( Y_N^{(2)} = \begin{bmatrix} a^{(2)} & b^{(2)} & V_2 \\ b^{(2)} & a^{(2)} & V_3 \end{bmatrix} \)

Then;

\[
\begin{align*}
Y_N^{(1)} \cdot 
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} &=
\begin{bmatrix}
a^{(1)} & b^{(1)} & V_1 \\ b^{(1)} & a^{(1)} & V_2
\end{bmatrix} \\
Y_N^{(2)} \cdot 
\begin{bmatrix}
V_2 \\
V_3
\end{bmatrix} &=
\begin{bmatrix}
a^{(2)} & b^{(2)} & V_2 \\ b^{(2)} & a^{(2)} & V_3
\end{bmatrix}
\end{align*}
\]

\[
\begin{bmatrix}
I_1^{(1)} \\
I_2^{(1)} \\
0
\end{bmatrix} +
\begin{bmatrix}
0 \\
I_1^{(2)} \\
I_2^{(2)}
\end{bmatrix} =
\begin{bmatrix}
a^{(1)}V_1 + b^{(1)}V_2 \\
b^{(1)}V_1 + a^{(1)}V_2 + b^{(2)}V_3 \\
0
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
b^{(2)}V_2 + a^{(2)}V_3
\end{bmatrix} =
\begin{bmatrix}
a^{(1)}V_1 + b^{(1)}V_2 \\
b^{(2)}V_1 + a^{(1)}V_2 + b^{(2)}V_3 \\
0
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
b^{(2)}V_2 + a^{(2)}V_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
a^{(1)} \\
b^{(1)} \\
0
\end{bmatrix} +
\begin{bmatrix}
a^{(2)} & b^{(2)} \\
b^{(2)} & a^{(2)}
\end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} =
\begin{bmatrix}
a^{(1)}V_1 + b^{(1)}V_2 \\
b^{(2)}V_1 + a^{(1)}V_2 + b^{(2)}V_3 \\
0
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
b^{(2)}V_2 + a^{(2)}V_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
a^{(1)} \\
b^{(1)} \\
0
\end{bmatrix} +
\begin{bmatrix}
a^{(2)} & b^{(2)} \\
b^{(2)} & a^{(2)}
\end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{5.20}
\]

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Combining equations (5.19) and (5.20)

\[
\begin{bmatrix}
I_s \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
I_{A1} \\
I_{A2} \\
I_{A3}
\end{bmatrix} +
\begin{bmatrix}
0 & b^{(1)} & 0 \\
0 & a^{(1)+a^{(2)}} & b^{(2)} \\
0 & b^{(2)} & a^{(2)}
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & Y_L
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\] (5.21)

Where:

\[
\begin{bmatrix}
I_{A1} \\
I_{A2} \\
I_{A3}
\end{bmatrix} = Y_A \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\] (5.22)

Rewriting equation (5.21) by combining it with equation (5.22)

\[
\begin{bmatrix}
I_s \\
0 \\
0
\end{bmatrix} = Y_A \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} +
\begin{bmatrix}
0 & b^{(1)} & 0 \\
0 & a^{(1)+a^{(2)}} & b^{(2)} \\
0 & b^{(2)} & a^{(2)}
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & Y_L
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\] = (Y_A + Y_N) \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & Y_L
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\] (5.23)

Where

\[
Y_N =
\begin{bmatrix}
a^{(1)} & b^{(1)} & 0 \\
b^{(1)} & a^{(1)} & 0 \\
0 & 0 & 0
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 \\
0 & a^{(2)} & b^{(2)} \\
0 & b^{(2)} & a^{(2)}
\end{bmatrix} =
\begin{bmatrix}
[Y_N^{(1)}] & 0 \\
0 & 0 & 0
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

In Equation 5.23, Y_A represent the inverse of the ZA, impedance matrix which is formed by the self and mutual impedances of the dipole elements, as explained earlier.

For a given I_s, \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} can be solved. Finally dipoles current can be calculated using equation 5.22.

5.6.1.4 Four Antennas connected by Three Transmission Line:-
Give us a chance to consider four radio wires which are joined by three transmission lines. The present relations at the dipoles' bases can be connected through the comparative technique which is portrayed previously.

![Diagram of four antennas connected by three transmission lines]

**Fig 5.8 Four antennas connected by three transmission lines**

**Fig 5.9 Detailed view of A1, A2, A3 and A4**

In fig 5.8 applying Kirchhoff's current law at node A, B, C and D.

\[
I_s = I_{A1} + I_1^{(1)} + I_1^{(1)} \quad (5.24)
\]

\[
0 = I_{A2} + I_2^{(1)} + I_1^{(2)} \quad (5.25)
\]

\[
0 = I_{A3} + I_2^{(2)} + I_1^{(3)} \quad (5.26)
\]

\[
0 = I_{A4} + I_2^{(3)} + Y_1 Y_4 \quad (5.27)
\]
Rewriting the Equation 5.24, Eq. 5.25, Eq. 5.26 and Eq. 5.27 in matrix form:

\[
\begin{bmatrix} I_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} I_{A_1} \\ I_{A_2} \\ I_{A_3} \\ I_{A_4} \end{bmatrix} + \begin{bmatrix} I_1^{(1)} \\ I_2^{(1)} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_2^{(2)} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_4^{(3)} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ Y_L V_4 \end{bmatrix}
\]

After some manipulations:

\[
\begin{bmatrix} I_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = Y_A + \begin{bmatrix} [Y_N^{(1)}] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & [Y_N^{(3)}] & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

(5.28)

Where \([Y_N^{(1)}]\) is a 2X2 matrix.

For a given \(I_s\), \(\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}\) can be solved. Finally the currents at the bases of the dipoles can be found using

\[
\begin{bmatrix} I_{A_1} \\ I_{A_2} \\ I_{A_3} \\ I_{A_4} \end{bmatrix} = Y_A \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}
\]

(5.29)

### 5.6.1.5 N antennas connected by N-1 Transmission Lines:

-
For a log-intermittent dipole reception apparatus comprising of N dipole components, the technique to discover the streams at the dipole’s bases components is as per the following:

1) YA lattice which is the reverse of the ZA ought to be shaped. In the event that there are N dipole components on the log-intermittent dipole reception apparatus, ZA is a N-by-N framework. The corner to corner passages of ZA are the self impedances of the dipole components and the off-inclining components are the common impedances between the dipole components.

2) For sample, \( ZA(2,4)=ZA(4,2) \) is the shared impedance between the second and the fourth dipole components. For the transmission lines between the dipole components, \( \left[ (i) \right] \) N Y 2-by-2 line lattices ought to be framed.

3) Load permission, YL, ought to be determined and ought to be set into the keep going section of the N-by-N grid as found in Equation 2.28.

4) As one can find in Equation 2.28, \( \left[ (i) \right] \) N Y 2-by-2 line networks ought to be set into the inclining and sub-askew parts of the N-by-N framework. At the point when every one of the 2-by-2 lattices are put in the N-by-N framework, the last passage of the first YN grid is summed up with the first section of the second YN network and this strategy is rehearsed up to the last YN framework as should be obvious the mathematical statement 5.21. At the end the last passage of the last YN grid is summed with the heap permission YL.

5) Following Equation 2.28, the voltage values on the dipole components ought to be found for a given excitation current, Is.

6) By duplicating these voltage framework and YA grid, one can get the present qualities at the dipoles’ base Equation 5.29.

5.7 Mutual Impedance between Linear Elements :-
The info impedance of radio wires can be ascertained when they emanate into an unbounded medium. Fig 5.10 demonstrates the receiving wire’s impedance which is joined with a transmission line. This impedance is known as driving –Point-impedance. This driving Point-receiving wire must be coordinated keeping in mind the end goal to coordinate any reception apparatus. This driving Point-impedance is known as the self impedance of the radio wire on the off chance that he receiving wire is confined means there are no close-by items. The genuine piece of this self impedance is called self-reactance (radiation resistance), and the nonexistent part is a self reactance.

![Diagram of transmission line with an antenna and an equivalent impedance](image)

**Fig 5.10 Transmission line with an antenna and with an equivalent impedance**

The vicinity of adjacent article, which may be direct reception apparatus, modify the present circulation .the emanated field and the info impedance. The present conveyance on one is influenced by the field transmitted by the other one. These shared influences are required to plan the reception.
apparatuses. The driving Point-impedance is subject to the shared impedance between the radio wires and other component.

5.7 Analysis of Dipole Antenna

In Fig 5.11, the current distribution on the dipoles are assumed to be center-fed, can be written as

\[
I(x = 0, y = 0, z') = \begin{cases} 
  a_z \cdot I_0 \cdot \sin[k(l - z')] & , 0 \leq z' \leq a \\
  a_z \cdot I_0 \cdot \sin[k(l + z')] & , \phi = \phi_0, \leq 0 
\end{cases}
\]

From fig 5.11, Magnetic field radiated by the dipole is given by:

\[
\mathbf{H} = a_o \mathbf{H_0} = a_o \frac{1}{4\pi} \oint \mathbf{\nabla \times A} = a_o \frac{1}{4\pi} \frac{dA_z}{dp}
\]

\[
\mathbf{H} = -a_o \frac{I_0}{4\pi} \frac{1}{y} \left[ e^{-jkR_1} + e^{-jkR_2} - 2\cos(\phi) e^{-jkR} \right]
\]

Where
The electric field can be calculated using Maxwell’s equations:

\[
E = \frac{1}{j \omega \varepsilon} \nabla \times A
\]

After some analytical details, two components of electrical field can be found. These are the \(-y\) and the \(-z\) components of the electrical field and are equal to:

\[
E_z = -j \frac{l_0}{4\pi} \left[ (e^{-jkR_1}/R_1) + (e^{-jkR_2}/R_2) - 2\cos(kl)(e^{-jkR/r}) \right]
\]

\[
E_y = -j \frac{l_0}{4\pi y} \left[ (z-1)(e^{-jkR_1}/R_1) + (z+1)(e^{-jkR_2}/R_2) - 2\cos(kl)(e^{-jkR/r}) \right]
\]

(5.32, 5.33)

5.8 Analysis of self and Mutual Impedance of linear elements

Two linear antennas of a system can be represented by T-equivalent form of a two port (Four terminals) network as show in Fig 5.12.
Fig: 5.12 Two Port Network and its T equivalent

The voltage current relations for Fig 5.12 are as follows:

\[ V_1 = Z_{11} I_1 + Z_{12} I_2 \]
\[ V_2 = Z_{21} I_1 + Z_{22} I_2 \] \hspace{1cm} (5.34)

Where \( Z_{11} \) and \( Z_{22} \) are self impedances of the antennas.

\[ Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_2=0} \]
\[ Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_1=0} \]

is the impedance at port 1 and is known as mutual impedance due to the current at port 2 (with port 1 is open circuit).

\[ Z_{12} = Z_{21} \]

is the impedance at port 2 and is known as mutual impedance due to the current at port 1 (With port 2 is open circuit).

For reciprocal Networks

Antenna's driving Point-impedance can be written as:
From eq. (5.35) it is clear that the driving Point-impedance of the reception apparatus relies on the self impedance of the receiving wire itself and in addition the common impedance between the two radio wires and the present proportions.

\[
Z_{1d} = \frac{v_1}{i_1} = Z_{11} + Z_{12} \left( \frac{I_{z_1}}{I_{i_1}} \right)
\]

\[
Z_{2d} = \frac{v_2}{i_2} = Z_{22} + Z_{21} \left( \frac{I_{z_2}}{I_{i_2}} \right)
\]  

(5.35)

Fig 5.13 Position of two dipole antennas used in self and mutual impedances

Shared coupling between the receiving wires can be analyzethiond by method for more mind boggling systems, yet here disentangled model utilizing affected emf strategy will be utilized. The accompanying comparison can be composed for Fig 5.13.

\[
V_{21} = -\frac{1}{l_{z_2-1}} \int_{-l_{z_2-1}}^{l_{z_2-1}} E_{z_21}(z') I_2(z') dz'
\]  

(5.36)
Where

V21 is the induced open-circuit voltage in antenna 2, referred to its current at the input terminals, due to the radiation from antenna 1.

The input current of the antenna 2 is I2i,

EZ21(Z') is the E-field component of antenna 1, which is parallel to antenna 2.

The current distribution along antenna 2 is I2(z').

The mutual impedance between two antennas can be defined as follow:

\[
Z_{21} = \frac{V_{21}}{I_{1i}} = - \frac{1}{I_{1i} I_{2i-1}} \int_{-\infty}^{\infty} E_{z21}(z') I_2(z') \, dz' 
\]

(5.37)

Equation (5.36) can also be written in the following form:

\[
Z_{21} = \frac{V_{21}}{I_{1i}} = j \frac{I_{1m} I_{2m}}{4 \pi I_{1i} I_{2i-1}} \int_{-\infty}^{\infty} \sin[k(l - z')] \left\{ \frac{e^{j k R_1}}{k R_1} + \frac{e^{j k R_2}}{k R_2} - 2 \cos(kh) \frac{e^{j k R_0}}{k R_0} \right\} \, dz' 
\]

(5.38)

\[
k R_1 = \frac{1 + 1 - \sqrt{1 + 2}}{\sqrt{1 + 2}} \\
k R_2 = \frac{1 + 1 + \sqrt{1 + 2}}{\sqrt{1 + 2}} \\
k R_0 = \frac{1 + \sqrt{1 + 2}}{\sqrt{1 + 2} + 1} = \frac{120 \pi}{2}
\]

Where \( \eta \) is the intrinsic impedance of free space.

For a very thin dipole with length 2l, the current at input terminals, \( i/l \), can be related to the maximum current, \( m/l \) as follows:

\[
i_l = I_m \sin(kl) \quad (5.39)
\]
Following equation can be obtained by replacing equation 5.39 into equation (5.38):

\[
Z_{21} = \frac{j.30}{(\sin(\kappa h))(\sin(\kappa l))} \int_I \left( e^{j\kappa R_1} + e^{j\kappa R_2} - 2\cos(\kappa h) e^{j\kappa R_0} \sin(\kappa (l - z)) \right) dz
\]

(5.40)

The half length of the dipole when h( or l) is approximately equal to-

\[
k \left( \frac{\lambda}{2} \right) \sin(\kappa h) = \sin \left( \frac{2\pi}{\lambda} k \frac{\lambda}{2} \right) = \sin(k \pi)
\]

Where k=1,2,3,4…n, so one can expect very high values of \(Z_{21}\). When \(l = 0.1395m = 0.93 \lambda/2\), then the self impedance will be \(R + jX = 4428 + j3509\). In fig 5.14 self impedance and reactance of the dipole antenna can be seen. In fig 5.14 dipoles are equal to length \(l\) and self impedance takes very high values.
5.9 Self Impedances of Linear Antennas

Thickness of the dipole is imperative so as to focus self impedance. Dipole
Thickness is considered by approximating the self impedance of two dipoles
having the same length and range of the dipole separated from one another.
Self impedance can be computed from the same equation as the common
coupling between two components. The supposition of sinusoidal current over
every dipole is still substantial.

Fig 5.15 Geometry used in the calculation of Self Impedance

In the above Fig, current is assumed to be concentrated at the center. Distance $s$ can be calculated as:

$$s = \cos \theta \left( \sqrt{2a^2 + z^2} \right) = \sqrt{2a^2 + z^2} \left( \frac{2}{a^2 + \frac{a^2}{2} + \frac{z^2}{2}} \cos \theta \right)$$

(5.41)

For very thin dipoles, $z \gg a$, so $s \approx \frac{2a^2 + z^2}{\sqrt{2a^2 + z^2}}$

Comparing with equation 5.38, we can replace spacing $s$ by $2a$. By doing so the self impedance of the dipole can be approximated by finding the mutual coupling between two dipoles having same length which are $2a$ apart from each other where $a$ is the radius of the dipole.
5.10 Mutual Coupling for Dual Polarized Log Periodic Antennas :-

In a double energized log occasional reception apparatus, for a given recurrence the two half wave dipole components are opposite to one another. Shared coupling between these opposite components can be examined.

Two receiving wires joined by a transmission line are given in fig 5.6.2.1. For this the relationship in the middle of current and voltage are researched and the outcome is summed up to N radio wires associated by N-1 transmission lines. As a straight Point-the accompanying Fig can be considered when we are making utilization of two opposite radio wires.

![Diagram](image.png)

**Fig:- 5.16 Two antennas connected by a transmission line in dual polarized LPA**

In Fig 5.16(a),"italic" images demonstrate the dipole's impact which is opposite to it and in fig 5.16 (b) Y11 is the common induction of the dipole reception apparatus which are opposite to one another and which are at the same level. The common induction of the first and second receiving wires
which are opposite to one another is $Y_{12}$. It is a bit much that $V_1$ ought to be equivalent to $V_1'$.

**Fig. 5.17** Detailed view of dipole antennas which are perpendicular to each other

In fig 5.16(a) it is obvious that $Y_{11} > Y_{11}'$, $Y_{12} > Y_{12}'$ as a mutual coupling between orthogonal elements are small compared to one between elements in the same plane. $Y_{12}'$ is expected to be very small compared with $Y_{11}'$. Therefore $Y_{12}'$ is neglected among $Y_{11}, Y_{12}, Y_{11}'$.

**Fig 5.18 approximations on Fig 5.16 (a)**

In area 2.3.1, it is expressed that, the slanting passages of $ZA$, which is the reverse of $YA$ lattice, are the self impedances of the dipole components. Considering the impact of orthogonal components and making a few approximations expressed over, the askew sections of $YA$ lattice will be
Y11 + Y11’ rather than Y11 just. Along these lines, the following occupation is to discover Y11’ and contrast it and Y11.

5.11 Calculation of MCOA:-

Keeping in mind the end goal to Fig shared coupling between two orthogonal antennas, it is vital to compute electric field which is parallel to reception apparatus 2 because of receiving wire 1. Electric field ought to be taken along the y pivot.

E_y from the Fig 5.19 can be calculated when z=x=0, then

$$E_y = \frac{j}{4\pi y} \left[ (-l)(e^{-jkR_1/R_1}) + (l)(e^{-jkR_2/R_2}) \right]$$

Fig.-5.19 Two Dipole antennas orthogonal to each other

$$R_1 = R_2 = \sqrt{\rho^2 + l^2}$$

Since z=0, E_y is turned to be 0, assuming the wire radius is very small. Since E_y is zero, Z_{21} = 0.
If we don’t assume the radius of the second dipole is zero, i.e., the dipole is not very thin, corresponding Fig is as follows:

Fig 5.20 Two dipoles orthogonal to each other with no assumption of zero wire radius of the second antenna

In Fig: 5.20 \( x^2 + z^2 = a^2 \)

Where

\( X = a \sin \theta \)
\[ Z = a \cos \theta (5.42) \]

Equation 5.31 is turned out to be:

\[ \rho = \sqrt{x^2 + yz^2} = \sqrt{a^2 \sin 2\theta + y^2} \]

\[ R_1 = \sqrt{\rho^2 + (a \cos \theta - h)^2} \]

\[ R_2 = \sqrt{\rho^2 + (a \cos \theta + h)^2} \]

\[ r = \sqrt{\rho^2 + a \cos \theta^2} \quad (5.43) \]

Substituting Equation 5.42 and Equation 5.43 into Equation 5.33, one gets:

\[ E_y = \frac{j b}{4 \pi y} \left( \frac{a \cos \theta - 1}{\sqrt{\rho^2 + a \cos \theta - h}} \right) e^{-jk \frac{R_1 \rho}{R_2}} - \frac{a \cos \theta + 1}{\sqrt{\rho^2 + a \cos \theta + h}} e^{-jk \frac{R_1 \rho}{R_2}} - 2a \cos \theta \cos (kl) \]

\[ Z_{21} = \frac{V_{21}}{I_{1i}} = - \frac{1}{I_{1i} I_{2i}} \int_{-1}^{+1} E_y(y) I_2(y) dy \quad (5.44) \]

Assuming \( I_{1i} \) and \( I_{2i} \) are 1 and \( J_2(y) = I_2(y) / 2\pi a \)

\[ Z_{21} = - \int_{0}^{2\pi + 1} E_y(y) (I_2(y) / 2\pi a) dy \, d\theta \quad (5.45) \]
5.12 Far field patterns of Log Periodic antenna

In fig 5.22, the current distribution on the dipoles, which are center-fed can be written as:

\[
I(x=0,y=0,z') = \begin{cases} 
   a_0 I_0 \sin \left( \frac{k}{2} - z' \right) & , 0 \leq z' \leq \frac{1}{2} \\
   a_0 I_0 \sin \left( \frac{k}{2} + z' \right) & , -1 \leq z' \leq 0 
\end{cases}
\]

Then the E-field of the single dipole is:

\[
E = \frac{I_0 e^{jn} \sin \left( \frac{k}{2} - z' \right)}{2\pi \sin (k/2)} \cos(k/2) \cos \theta - \cos(k/2) \sin \theta \quad \text{[2]} \quad (5.46)
\]
In Equation 5.46, using Fig 5.22, \( r \, r R \sin n \, n \approx\), which is the far field approximation for phase terms, in the numerator and \( r \, r \, n \approx\), which is the far field approximation for amplitude terms, in the denominator. Substituting these equality and approximation in Equation 5.46

\[
E_{\theta n} = \frac{e^{jkr}}{r} \frac{j}{2\pi \sin(k(l_n/2))} \left( \frac{\cos(k(l_n/2) \cos \theta) - \cos(k(l_n/2))}{\sin \theta} \right). \tag{5.47}
\]

where

\[
\begin{align*}
F_{n}(\theta) &= \left[ \frac{\cos(k(l_n/2) \cos \theta) - \cos(k(l_n/2))}{\sin \theta} \right]. \tag{[2]} \end{align*}
\]

**Fig 5.23 Zoomed view of Fig 5.21 where dipoles are in xz plane**

Equation 5.47 is the \( n \) Eq of one dipole. Total E plane pattern which is produced by \( N \) dipole:

\[
E_{0} = \frac{e^{jkr}}{r} \frac{j}{2\pi \sin(k(l_n/2))} \sum_{n=1}^{N} I_{n} \, e^{j\theta} \, f_{n}(\theta) \tag{5.48}
\]

Equation 5.48 is the E-plane pattern of an antenna consisting of \( N \) dipole. In Fig 5.21, E-plane is the xz plane where \( \phi = 0 \). E-plane of an antenna is the plane where electric field lines occur.

### 5.13 GENERALIZATION FOR THE FAR-FIELDS OF THE ANTENNA :-
Fig 5.24 Orientation of a log-periodic antenna on the Cartesian coordinate system

Referring Fig:- 5.24:

\[ F_{\text{ANT}}(\theta, \phi) = \frac{j}{2\pi r \sin(k(l_n/2))} e^{jkr} \sum_{n=1}^{N} I_n e^{jkrn} \hat{r}_n(\theta, \phi) \]

where \[ \hat{r}_n(\theta, \phi) = \begin{bmatrix} \cos(k(l_n/2)\cos\theta) - \cos(k(l_n/2)) \sin\theta \n \sin(k(l_n/2))\sin\theta \end{bmatrix} \]

\[ r^* n = R_n ax + \sin\theta \sin\phi ay + \cos\theta az \]

Finally

\[ F_{\text{ANT}}(\theta, \phi) = \frac{j}{2\pi r \sin(k(l_n/2))} e^{jkr} \sum_{n=1}^{N} I_n e^{jkrn} \sin\theta \cos\phi \hat{r}_n(\theta, \phi) \]

(5.49)

Using Equation 5.49, E and H plane patterns can be found. As explained before, E-plane, where the electric field lines occur, is the xz plane where \( \phi = 0 \). H-plane is the xy plane where \( \theta = 90^0 \).
In calculating the H-plane of the antenna, far field approximations also can be made by using Fig 5.49. \( r_n \approx r - R_n \cos \phi \), which is the far field approximation for phase terms, in the numerator and \( r_n \approx r \), which is the far field approximation for amplitude terms.

H plane pattern is,

\[
E_\theta = \frac{e^{jkr}}{R} \frac{j}{2\pi \sin(k(l_n/2))} \sum_{n=1}^{N} I_{on} e^{jkn \cos \phi} F_j(m(\theta, \phi)) \bigg|_{\theta=90} \tag{5.50}
\]

Where

\[
\sum_{n=1}^{N} I_{on} e^{jkn \cos \phi} F_j(m(\theta, \phi)) \bigg|_{\theta=90} = [1 - \cos(k(l_n/2))]
\]

Fig 5.25 Zoomed view of Fig 5.21 where dipoles are in XZ plane