Opt Rev (2015) 22:539–543
DOI 10.1007/s10043-015-0113-6

REGULAR PAPER

Electron acceleration by a chirped laser pulse in vacuum under the influence of magnetic field

Harjit Singh Ghotra[1] · Niti Kant[1]

Received: 4 March 2015 / Accepted: 15 June 2015 / Published online: 27 June 2015

Abstract A linearly polarized (LP) chirped laser pulse is employed for electron acceleration to GeV energy under the influence of azimuthal magnetic field in vacuum. LP laser pulse supports the trapping of pre-accelerated electron during laser–electron interaction in vacuum. The electron gains energy from LP laser pulse and gets accelerated in the direction of propagation of laser pulse. Additionally, the chirping increases the electron laser interaction for longer duration while the azimuthal magnetic field having pinching effect keeps the motion of electron parallel to the direction of propagation of laser pulse leads to enhance the electron acceleration. The combined effect of chirping of laser pulse and pinching of azimuthal magnetic field not only enhances the electron energy gain but also supports in retaining of gained energy by the electron for longer distances. The accelerating distance is observed to be of three times the Rayleigh length where the Rayleigh length is about 6.78 μm. We observe electron energy gain of about 1.47 GeV in the presence of azimuthal magnetic field of about 438 kG with an intense LP laser pulse of peak intensity of about 3.4×10^{21} W/cm^2. Higher electron energy gain may be obtained with highly intense laser pulse.

✉ Niti Kant
nitikant@yahoo.com

[1] Department of Physics, Lovely Professional University, G. T. Road, Phagwara 144411, Punjab, India

1 Introduction

The development of theoretical and experimental models for the investigations of charged particle dynamics under interactions with laser fields are the key area of research during last few decades [1, 2]. The extensive studies of laser–electron interaction target to the betterment of electron accelerations for high energy gains. Laser pulse polarization plays an important role in transferring energy to electrons and hence enhancing electron acceleration [3]. The obtained electron energies can further be increased and retained for larger distances by optimal use of magnetic fields [4]. Thus, the polarization as well as magnetic field remains the key characteristics for obtaining high electron energy gains [5, 6]. The frequency variation of laser pulse effectively improves the energy gain by the accelerated electrons from laser pulse [7]. Use of chirped laser pulse enforces the maintaining of resonance during laser–electron interactions for longer duration [8, 9]. Salamin and Jisrawi used a quadratic frequency chirp to achieve a high electron energy during laser–electron interaction in vacuum [10]. The suitable application of magnetic field further enhances electron acceleration gradient [11, 12].

Afhami and Eslami [13] studied numerically the effect of nonlinear chirped Gaussian laser pulse parameters on the electron acceleration. They proposed that the nonlinear chirped pulse has a much smaller divergence than that of linear chirped pulse. The main goal of this paper is to study the combined effect of chirping by a LP laser pulse and pinching by an azimuthal magnetic field on electron acceleration in vacuum. For this purpose, we organize this paper as follows: dynamics of electron due to a chirped LP laser pulse is described in Sect. 2. In Sect. 3, we will show the numerical results and discuss the final electron energy and electron trajectory for different chirp factors. The conclusions are presented in Sect. 4.

2 Electron dynamics

In this investigation, we consider a chirped LP laser pulse propagating in z-direction with transverse component of electric field ($\vec{E} = \hat{x}E_x$) as:

$$E_x = \frac{E_0}{f}\sin(\varphi)\exp\left(-\frac{(\xi-\xi_0)^2}{\sigma^2} - \frac{r^2}{r_0^2 f^2}\right) \quad (1)$$

where $\varphi = k(\xi)\xi + \tan^{-1}(z/Z_R) - zr^2/(Z_R r_0^2 f^2) + \varphi_0$, $f^2 = 1 + (z/Z_R)^2$, $k(\xi) = \omega(\xi)/c$ is the wave number, $\omega(\xi) = \omega_0(1+\alpha\xi)$ is an arbitrary frequency chirp for linear [8] and negative chirp, α is the frequency chirp factor, $Z_R = kr_0^2/2$ is the Rayleigh length, $\xi = z - ct$ is the retarded coordinate, φ_0 is the initial phase, σ is the laser pulse length, $r^2 = x^2 + y^2$, r_0 is minimum laser spot size, ω_0 is the initial frequency of laser, ξ_0 is the initial position of the pulse peak, and c is the velocity of light in vacuum. The longitudinal component of electric field is neglected because of very small amplitude in comparison with transverse component. The magnetic field components related to the laser pulse can easily be deduced through Maxwell's equations and expressed as:

$$\vec{B} = -ic\frac{\vec{\nabla}\times\vec{E}}{\omega(\xi)} \quad (2)$$

The laser pulse imparts energy to the electron that carries current in the z direction and experiences an azimuthal magnetic field. Here we consider the effect of azimuthal magnetic field on electron acceleration. In general, for LP laser with transverse component of electric field in x direction propagating through vacuum, the azimuthal magnetic field has two components and is expressed as $\vec{B}_\theta = (\hat{x}y - \hat{y}x)(B_0/r_0)\exp(-r^2/r_0^2)$. For $y = 0$, the B_θ becomes perpendicular to laser electric field, due to which the cyclotron frequency increases with increasing value of x. Thus, electron undergoes betatron oscillations in the presence of azimuthal magnetic field. For $x = 0$, the B_θ becomes parallel to laser electric field. This corresponds to an ordinary mode propagation which becomes unaffected by the presence of static magnetic field. Thus, for a LP laser pulse with a transverse x component, the azimuthal magnetic field [4] appears with y component only and is expressed as:

$$\vec{B}_\theta = -\hat{y}B_0\frac{x}{r_0}\exp\left(-\frac{x^2}{r_0^2}\right) \quad (3)$$

where the constant B_0 represents the maximum amplitude of magnetic field. The equations governing electron momentum and energy are:

$$\frac{dp_x}{dt} = -eE_x + e\beta_z B_y + e\beta_z B_\theta \quad (4)$$

$$\frac{dp_y}{dt} = 0 \quad (5)$$

$$\frac{dp_z}{dt} = -e\beta_x B_y - e\beta_x B_\theta \quad (6)$$

$$\frac{d\gamma}{dt} = -e\beta_x E_x \quad (7)$$

where p_x, p_y and p_z are x, y and z components of the momentum $\vec{p} = \gamma m_0\vec{v}$, respectively; β_x, β_y and β_z are x, y and z components of the normalized velocity $\vec{\beta} = \vec{v}/c$, respectively; $\gamma^2 = 1 + (p_x^2 + p_y^2 + p_z^2)/(m_0c)^2$ is the Lorentz factor, $-e$ and m_0 are the electron's charge and rest mass, respectively. The following are the dimensionless variables:

$$a_0 \to \frac{eE_0}{m_0\omega_0 c}, \tau \to \omega_0 t, \sigma' \to \frac{\omega_0\sigma}{c}, r_0' \to \frac{\omega_0 r_0}{c},$$
$$z_0' \to \frac{\omega_0 z_0}{c}, x' \to \frac{\omega_0 x}{c}, y' \to \frac{\omega_0 y}{c}, z' \to \frac{\omega_0 z}{c},$$
$$\beta_x \to \frac{v_x}{c}, \beta_y \to \frac{v_y}{c}, \beta_z \to \frac{v_z}{c}, p_0' \to \frac{p_0}{m_0 c},$$
$$p_x' \to \frac{p_x}{m_0 c}, p_y' \to \frac{p_y}{m_0 c}, p_z' \to \frac{p_z}{m_0 c},$$
$$k_0' \to \frac{ck_0}{\omega_0}, \alpha' \to \frac{\alpha c}{\omega_0}, \text{ and}$$
$$b_0 \to \frac{eB_0}{m_0\omega_0 c}$$

where a_0 is the normalized laser intensity parameter; τ is the normalized time; σ' is the normalized laser pulse length; z_0' is the normalized initial position of the pulse peak; x', y' and z' are the normalized x, y and z coordinates; p_0' is the normalized initial momentum of electron; p_x', p_y' and p_z' are the x, y and z components of the normalized momentum; k_0' is the normalized value of initial wave number; α' is the normalized chirp parameter and b_0 is the normalized value of magnetic field.

Equations (4) to (7) are the coupled ordinary differential equations. We solve these equations numerically for electron trajectory and energy. We assume that the electron is initially injected at a small angle δ to the direction of propagation of laser pulse with $\vec{p}_0 = \hat{x}p_0\sin\delta + \hat{z}p_0\cos\delta$ where p_0 is the initial momentum of the electron [6, 12].

3 Numerical results

We have chosen the following dimensionless parameters for numerical analysis: $a_0 = 2.5$ (corresponding to laser intensity $I \sim 8.5\times10^{18}$ W/cm^2), $a_0 = 5$ (corresponding to laser intensity $I \sim 3.46 \times 10^{19}\ W/cm^2$) and $a_0 = 10$ (corresponding to laser intensity $I \sim 1.37 \times 10^{20}$ W/cm^2) with wave length $\lambda_0 \sim 1$ μm; $r'_0 = 900$ (corresponding to laser spot sizes $r_0 \sim 151$ μm); $Z_R = 6.78$ μm is the Rayleigh length (with $r_0 \sim 151$ μm and $\lambda_0 \sim 1$ μm); and $p'_0 = 1,\ 2$ and 2.5; $\sigma' = 70$ (corresponding to laser pulse duration of 200 fs); $\delta = 10^\circ$; $\varphi_0 = 0$; $b_0 = 0.00225$ (corresponding to a magnetic field of 240 kG), $b_0 = 0.0041$ (corresponding to a magnetic field of 438 kG) and $b_0 = 0.009$ (corresponding to a magnetic field of 962 kG); and $z'_0 = -300$.

Figure 1 shows the electron trajectory in three-dimensional plane in the absence and presence of azimuthal magnetic field with and without chirping for $a_0 = 2.5$, $p'_0 = 1$ and $b_0 = 0.009$. The electron gets accelerated during interaction with laser pulse even in absence of azimuthal magnetic field. But within a small distance it gets decelerated, goes out of phase and loses its energy as depicted from Fig. 1a. In the presence of azimuthal magnetic field, a confined trajectory for longer distance appears as in Fig. 1b. The trajectory appears more confined with linearly chirped laser pulse than that with un-chirped pulse, which depicts that the presence of chirping increases the duration of laser–electron interaction and hence provides the better transfer of energy. The electron oscillates in the direction of propagation of laser during the interaction with laser pulse. The azimuthal magnetic field enhances the strength of $\vec{v} \times \vec{B}$ force and enforces an accelerated motion for larger distance by deflecting the escaping electron along the $+z$ direction which indicates the retaining of high energy at larger distances.

Figure 2 shows the variation of electron energy gain γ as a function of normalized magnetic field b_0. In Fig. 2a, b, electron energy gain has been analyzed at different values of normalized initial momentum with and without chirping for $a_0 = 2.5$ and 5, respectively. The electron energy gain appears higher with chirped laser pulse than that with unchirped laser pulse in the presence of azimuthal magnetic field. For $a_0 = 2.5$ and $p'_0 = 2.5$ the electron energy gain is $\gamma = 651$ in the presence of chirp factor $\alpha = 0.00085$, which is 31.5 % higher than $\gamma = 495$ in the absence of chirping. The obtained energy gain with chirped pulse for $a_0 = 5$ and $p'_0 = 2.5$ is about 34 % higher than that in the absence of chirping. Figure 2c shows the variation of energy gain γ with b_0 for different values of laser pulse intensity parameters $a_0 = 10$, 20, 30, 40, and 50 at $p'_0 = 2.5$ and selective values of chirp factor α'. The selective values of chirp factor corresponding to the laser pulse intensities are obtained by optimization for maximum energy gain. High energy gain appears at the small values of magnetic field. A pre-accelerated electron trapped with laser pulse in the presence of low and optimum magnetic field. After attaining maximum gain at resonance the electron energy decreases and almost saturates for the larger values of magnetic field. The energy gain increases with laser pulse intensity. For $a_0 = 20$ the higher energy gain $\gamma = 2675$ is achieved at $b_0 = 0.00325$ with $\alpha' = 0.00075$ which is higher than for $a_0 = 10$ at with $\alpha' = 0.0008$ at $p'_0 = 2.5$. Comparatively higher energy gain is obtained with higher values of laser pulse intensity parameter, $a_0 = 30$, 40, and 50.

Figure 3 shows the variation of electron energy gain γ as a function of accelerating distance z normalized with Rayleigh length Z_R. In Fig. 3a, b, we see the variation of γ with z/Z_R for different values of normalized momentum $p'_0 = 1.5,\ 2$ and 2.5 with optimum values of magnetic field for $a_0 = 2.5$ and 5. The electron energy gain increases during the interaction with laser pulse. After achieving the maximum value, the energy gain saturates for larger distances. Figure 3c shows the variation of energy gain γ with z/Z_R for the optimum values of magnetic field b_0 and chirp factor α' with $p'_0 = 2.5$ for $a_0 = 10$, 20, 30, 40, and 50,

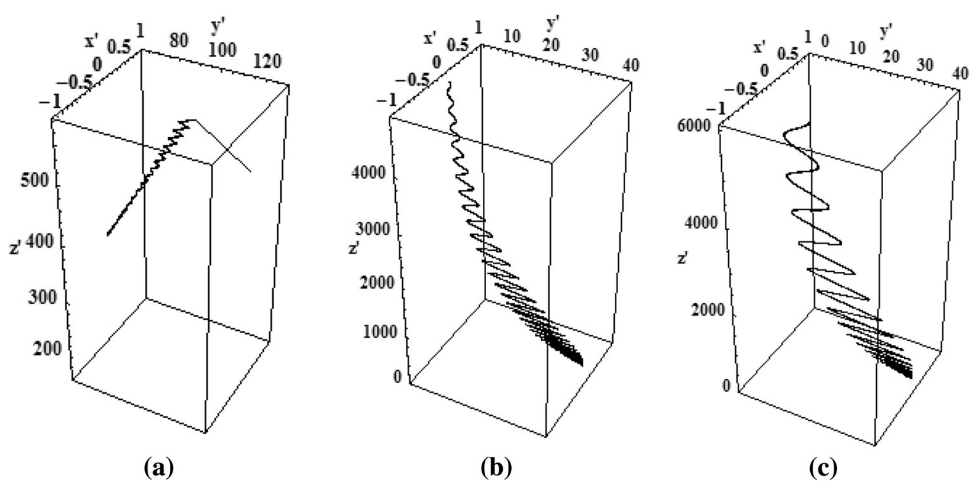

Fig. 1 Electron trajectory in 3D plane in the absence and presence of chirping for LP laser pulse with $a_0 = 2.5$ and $p'_0 = 1$. **a** $\alpha' = 0$, $b_0 = 0$, **b** $\alpha' = 0$, $b_0 = 0.009$ and **c** $\alpha' = 0.0005$ and $b_0 = 0.009$. The other parameters are $r_0 = 900$, $\sigma' = 70$, $\varphi_0 = 0$, $\delta = 10^\circ$ and $z'_0 = -300$

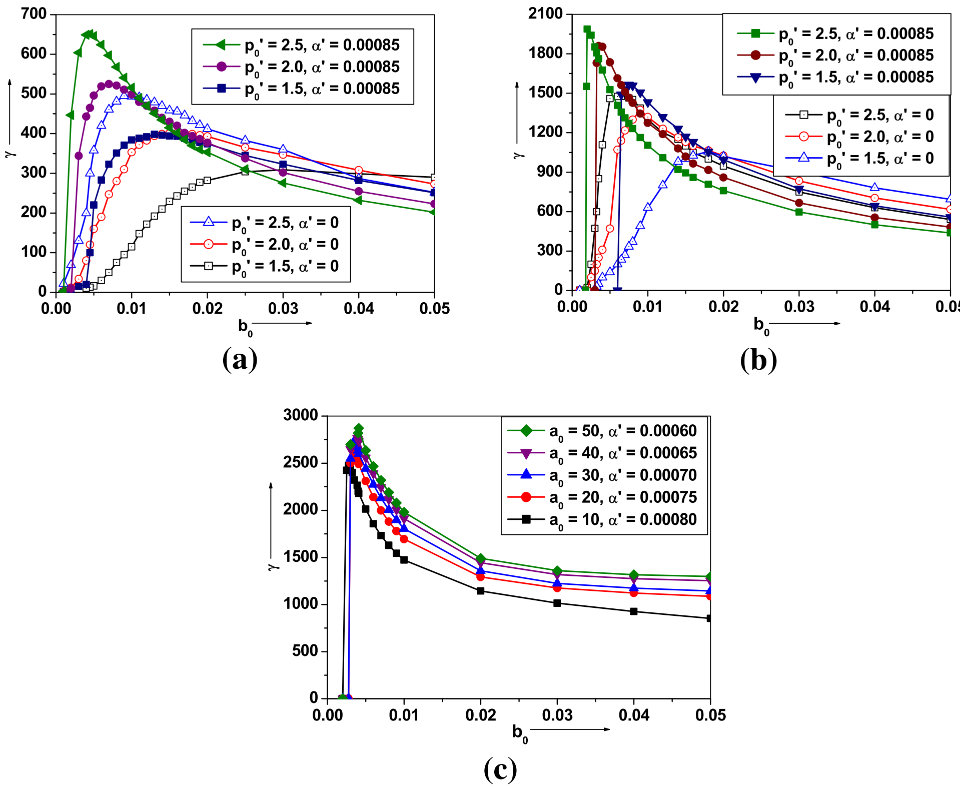

Fig. 2 Energy gain γ as a function of normalized magnetic field b_0 for different values of normalized initial momentum and intensity parameters for LP laser pulses. **a** $p_0' = 1.5$, 2 and 2.5 at $a_0 = 2.5$ with $\alpha' = 0$ and $\alpha' = 0.0085$, **b** $p_0' = 1.5$, 2 and 2.5 at $a_0 = 5$ with $\alpha' = 0$ and $\alpha' = 0.0085$ and **c** $a_0 = 10$, 20, 30, 40 and 50 at $p_0' = 2.5$. The other parameters are same as taken in Fig. 1

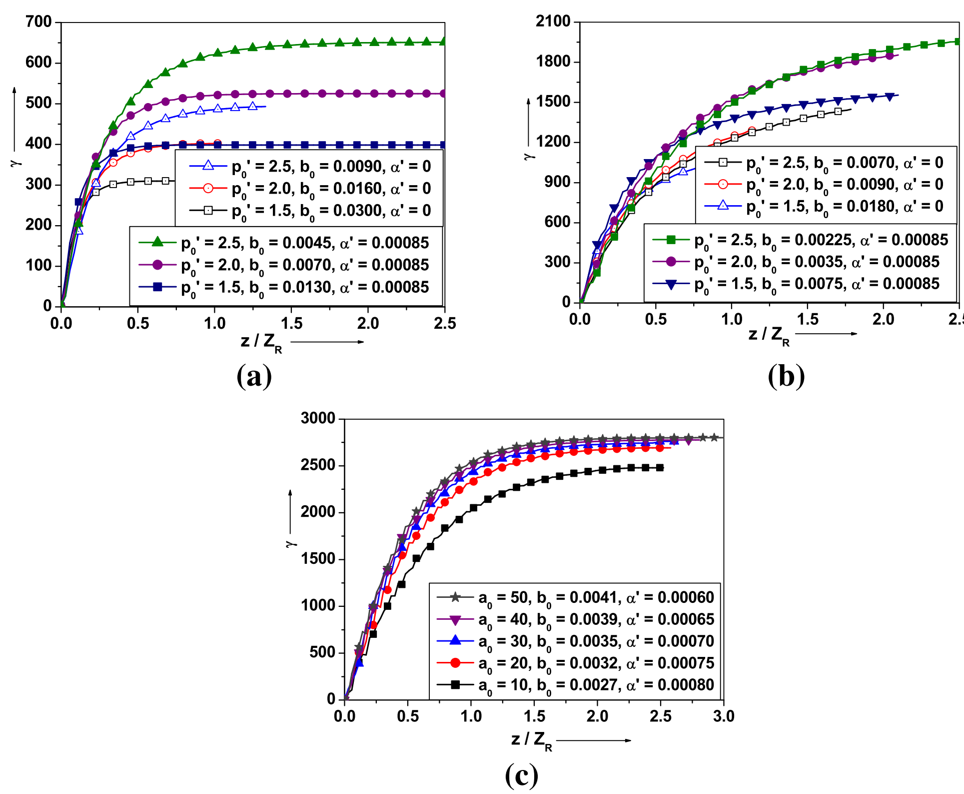

Fig. 3 Electron energy gain γ as a function of accelerating distance z normalized with Rayleigh length Z_R for different values of magnetic field for LP laser pulses. **a** $p_0' = 1.5$, 2 and 2.5 at $a_0 = 2.5$ with $\alpha' = 0$ and $\alpha' = 0.00085$, **b** $p_0' = 1.5$, 2 and 2.5, at $a_0 = 5$ with $\alpha' = 0$ and $\alpha' = 0.00085$, and **c** $a_0 = 10$, 20, 30, 40 and 50 at $p_0' = 2.5$. The other parameters are same as taken in Fig. 1

respectively. With the optimum value of magnetic field, initial momentum and suitable chirp factor, the electron gain higher energy with distance due to resonance. As appearing an energy gain of $\gamma = 2476$ (corresponding to 1.26 GeV) is achieved with laser intensity $a_0 = 10$ (corresponding to 1.37×10^{20} W/cm^2) and chirping factor $\alpha = 0.0008$ in the presence of azimuthal magnetic field $b_0 = 0.0027$ (corresponding to 288 kG). We have observed the accelerating distance of about three times the Rayleigh length as depicted from Fig. 3c. The higher energy gain is achieved with higher values of laser pulse intensities and optimum values of magnetic field, initial momentum, and chirp factor.

In the present study, the high energy gain of about 1.47 GeV is achieved with laser intensity $I \sim 3.4 \times 10^{21}$ W/cm^2 in the presence of magnetic field of 438 kG with chirped laser pulse. This intensity is about half of that was used to obtain only 100 MeV energy with a LP laser pulse [1]. The magnetic field used in our scheme is feasible and can be generated experimentally [14, 15]. An electron acceleration gradient of GeV/m was achieved [11] using circularly polarized laser pulse in the presence of axial magnetic field of about 600 kG. We employ a comparatively smaller magnetic field of about 438 kG with chirped LP laser pulse to achieve the acceleration gradient of the order of GeV/m. Higher energy gain can be achieved with higher intensity laser pulse in the presence of azimuthal magnetic field. The observed values of angle of emittance remain small with higher intensity pulses. Further, the radiative losses are not much [5] due to the pre-accelerated electron.

4 Conclusion

In recent years, there is much attention in accelerating the electrons by chirped laser fields in vacuum. We have additionally introduced the influence of azimuthal magnetic field on electron acceleration with a chirped LP laser pulse. Our results highlight the importance of chirping of laser pulse in the presence of azimuthal field for enhanced electron acceleration. We have presented that an electron with approximately 2.5 MeV of initial energy can gain energy of about 1.47 GeV from a chirped LP laser pulse in the presence of relatively small azimuthal magnetic field. Thus, with suitable selection of parameters like laser pulse intensity, initial electron momentum, static magnetic field and chirp factor, the higher energy of the order of GeV can be achieved with an electron of few MeV of initial energy. The presented model led to a substantial increase in electron acceleration energy.

References

1. Hu, S.X., Starace, A.F.: Laser acceleration of electrons to giga-electron-volt energies using highly charged ions. Phys. Rev. E **73**, 066502 (2006)
2. Wang, X., Zgadzaj, R., Fazel, N., Li, Z., Yi, S.A., Zhang, X., Henderson, W., Chang, Y.Y., Korzekwa, R., Tsai, H.E., Pai, C.H., Quevedo, H., Dyer, G., Gaul, E., Martinez, M., Bernstein, A.C., Borger, T., Spinks, M., Donovan, M., Khudik, V., Shvets, G., Ditmire, T., Downer, M.C.: Quasi-mono-energetic laser-plasma acceleration of electrons to 2 GeV. Nature Commun. **4**, 1988 (2013)
3. Sohbatzadeh, F., Aku, H.: Polarization effect of a chirped Gaussian laser pulse on the electron bunch acceleration. J. Plasma Phys. **77**, 39 (2011)
4. Singh, K.P., Malik, H.K.: Laser Part. Beams **26**, 363 (2008)
5. Gupta, D.N., Kant, N., Kim, D.E., Suk, H.: Electron acceleration to GeV energy by a radially polarized laser. Phys. Lett. A **368**, 402 (2007)
6. Gupta, D.N., Ryu, C.M.: Electron acceleration by a circularly polarized laser pulse in the presence of an obliquely incident magnetic field in vacuum. Phys. Plasmas **12**, 053103 (2005)
7. Khachatryan, A.G., Goor, F.A.V., Boller, K.J.: Interaction of free charged particles with a chirped electromagnetic pulse. Phys. Rev. E **70**, 067601 (2004)
8. Afhami, S., Eslami, E.: Effect of nonlinear chirped Gaussian laser pulse on plasma wake field generation. AIP Adv. **4**, 087142 (2014)
9. Jisrawi, N.M., Galow, B.J., Salamin, Y.I.: Simulation of the relativistic electron dynamics and acceleration in a linearly-chirped laser pulse. Laser Part. Beams **32**, 671 (2014)
10. Salamin, Y.I., Jisrawi, N.M.: Electron laser acceleration in vacuum by a quadratically chirped laser pulse. J. Phys. B: Atomic Mol. Opt. Phys. **47**, 025601 (2014)
11. Galow, B.J., Li, J.X., Salamin, Y.I., Harman, Z., Keitel, C.H.: High-quality multi-GeV electron bunches via cyclotron autoresonance. Phys. Rev. ST Accel. Beams **16**, 081302 (2013)
12. Ghotra, H.S., Kant, N.: Electron acceleration to GeV energy by a chirped laser pulse in vacuum in the presence of azimuthal magnetic field. Appl. Phys. B **120**, 141 (2015). doi:10.1007/s00340-015-6114-0
13. Afhami, S., Eslami, E.: Effect of nonlinear chirped Gaussian laser pulse parameters on the electron acceleration. Opt. Rev. **21**, 475 (2014)
14. Igumenshchev, I.V., Zylstra, A.B., Li, C.K., Nilson, P.M., Goncharov, V.N., Petrasso, R.D.: Self-generated magnetic fields in direct-drive implosive experiments. Phys. Plasmas **21**, 062707 (2014)
15. National High Magnetic Field Laboratory (NHMFL): 60 tesla long pulse magnet. http://www.lanl.gov/orgs/mpa/nhmfl/60TLP.shtml