CHAPTER-8

ELECTRON INJECTION FOR ENHANCED ENERGY GAIN BY A RADIALLY POLARIZED LASER PULSE IN VACUUM IN THE PRESENCE OF MAGNETIC WIGGLER

8.1 INTRODUCTION

Particle acceleration by using laser in vacuum and plasmas is an emerging area of research during last few decades [16, 35]. Many theoretical and experimental models were proposed and developed to study the electron energy gain during laser electron interaction [13, 60, 105]. The plasma based schemes for electron acceleration by lasers are capable to generate very high energy gain but constraint to suffer with plasma related instabilities. Such instabilities are absent in vacuum based on electron acceleration by laser. The variations of laser parameters such as pulse polarizations, beam width, initial phase and frequency amplifications were analysed for sequential improvements in electron energy gains [97, 102, 118, 121]. The rest electrons can be accelerated to high energy of the order of GeV by using the RP laser pulse in vacuum [61]. The RP laser pulse present more advantage over linearly and circularly polarized laser pulses [41, 76] for electron [66, 86] and proton [103] acceleration in vacuum. It is due to its unique focusing property with a strong longitudinal component and small spot size. Gupta *et al*. [66] studied the effect of externally applied axial magnetic field on electron energy gain by RP laser pulse and observed that a pre accelerated (initial energy of $1.1 MeV$) electron can be accelerated to $1.5 GeV$ of energy with $1 MG$ of magnetic field by using RP laser pulse of peak intensity $10^{19} W/cm^2$. Varin *et al*. [106] analysed the direct electron acceleration by RP laser pulse. A pre-accelerated electron when injected in laser path gets accelerated toward the low-intensity regions of the laser beam by the transverse electric field component. The electron trajectories are controlled by $\vec{v}\times\vec{B}$ force. The parallel component of electric field in the direction of propagation provides an extra push to strengthen the longitudinal motion of the electron. The external magnetic field further enhances $\vec{v}\times\vec{B}$ force which enforces the electron for efficient oscillation in the direction parallel to propagation of the laser pulse. An electron can gain and retain sufficient energy if a wiggler magnetic field of optimum magnitude and period is applied externally [48]. Mirzanejhad *et al*. [63] studied the effect of a helical magnetic wiggler and axial guide field on electron bunch acceleration by using an inverse free electron laser (IFEL). The wiggler cause the trajectory of electron to oscillate in transverse direction, thereby support in projecting a component of the laser pulse electric field in the direction of the electron motion. They observed an acceleration gradient of about $0.2GeV/m$.

In this chapter we have presented the electron acceleration with a RP laser pulse and a magnetic wiggler in vacuum. An electron while interacting with RP laser pulse is influenced by a force due to longitudinal component of electric field of laser. The laser magnetic field component strengthens the $\vec{v}\times\vec{B}$ force. As a result electron with a few MeV of initial energy is accelerated around the direction parallel to propagation of the laser. The electron gains and retains high energy till the saturation of betatron resonance and afterwards it gets decelerated, de-phased with laser field and loses its gained energy. The presence of external wiggler magnetic field encircles the trajectory of accelerated electron which improves the strength of $\vec{v}\times\vec{B}$ force and enforces the retaining of betatron resonance for longer duration. It restricts the deceleration of electron and supports the retaining of gained energy for longer duration. The experimental availability of magnetic field of the order of MG [65, 113, 115] favours our model to use a wiggler magnetic field of such strength for electron acceleration with a RP laser pulse in vacuum. The maximum transfer of energy from laser field to the electron is observed at a particular angle of injection of electron with respect to the direction parallel to propagation of laser pulse. To the best of our knowledge electron injection for the enhancement of electron energy was never analysed by using RP laser pulse with magnetic wiggler in vacuum. This work is organized as follows: Section 8.2, describes the electromagnetic fields and electron dynamics used to study the electron acceleration. We solve numerically the coupled differential equations by using a relativistic simulation code to find the trajectory and energy of electron in vacuum. The numerical results for energy and injection of electron are discussed in section 8.3. Finally, we draw conclusions in the section 8.4.

8.2 WIGGLER MAGNETIC FIELD

For a cartesian coordinate system with circularly polarized laser propagation along z-axis, the wiggler magnetic field [63] is oriented in transverse direction and is related as $\vec{B}_w = B_{0w}(\hat{x}\cos(k_w z) + \hat{y}\sin(k_w z))$. For a radially polarized laser propagation along z-axis, the externally applied wiggler magnetic field is in azimuthal direction and is given by:

$$\vec{B}_w = \hat{\theta} B_{0w} \sin(\theta - k_w z), \tag{8.1}$$

where k_w is the wiggler wave number, B_{0w} is the amplitude, and θ is the azimuthal angle.

8.3 ELECTRON DYNAMICS AND RELATIVISTICS ANALYSIS

A RP laser beam with paraxial approximation and propagating parallel to z-axis with electric field ($\vec{E} = \hat{r}E_r + \hat{z}E_z$) components are expressed as [97]:

$$E_r = E_0 \frac{r}{r_0 f^2} \cos(\phi) \exp\left[-\left\{ \frac{[t-(z-z_L)/c]^2}{\tau^2} \right\} - \frac{r^2}{r_0^2 f^2} \right], \tag{8.2}$$

$$E_z = E_0 \frac{2}{k_0 r_0 f^2} \left[\left(1 - \frac{r^2}{r_0^2 f^2} \right) \sin(\phi) - \frac{z r^2}{Z_R r_0^2 f^2} \cos(\phi) \right] \exp\left[-\left\{ \frac{[t-(z-z_L)/c]^2}{\tau^2} \right\} - \frac{r^2}{r_0^2 f^2} \right], \tag{8.3}$$

where $\phi = \omega_0 t - k_0 z + 2\tan^{-1}(z/Z_R) - zr^2/(Z_R r_0^2 f^2) + \phi_0$, $f^2 = 1 + (z/Z_R)^2$, $k_0 = \omega_0 / c$, $Z_R = k_0 r_0^2 / 2$ is the Rayleigh length, r is the radial coordinate, τ is the pulse duration, r_0 is minimum laser spot size, ω_0 is the laser frequency, z_L is the initial position of pulse peak and c is the velocity of light in vacuum. The magnetic field components related to the laser pulse can be obtained through Maxwell's equations and expressed as:

$$\vec{B}_L = c \frac{\vec{k}_0 \times \vec{E}}{\omega_0}, \tag{8.4}$$

The total magnetic field is $\vec{B} = \vec{B}_L + \vec{B}_w$.

Figure 8.1, shows a schematic of vacuum acceleration of electron by a RP laser pulse with wiggler magnetic field.

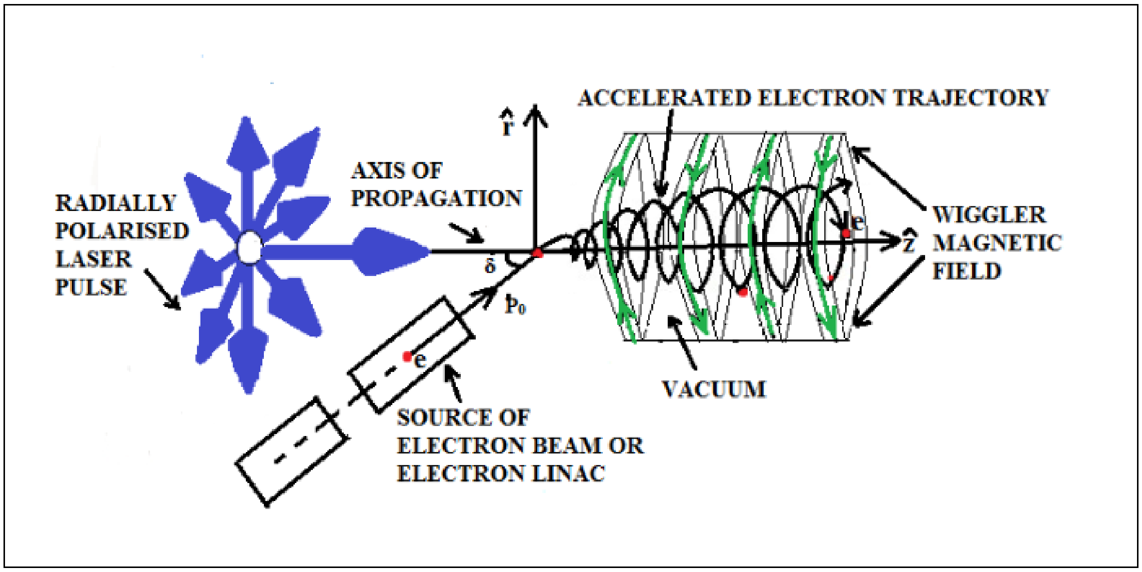


Figure 8.1. A schematic showing the vacuum electron acceleration by a Radially polarized (RP) laser pulse with wiggler magnetic field.

The equations governing momentum and energy of electron are the following:

$$\frac{dp_r}{dt} = -eE_r + e\beta_z (B_{L\theta} + B_{w\theta}), \tag{8.5}$$

$$\frac{dp_z}{dt} = -eE_z + e\beta_r (B_{L\theta} + B_{w\theta}), \tag{8.6}$$

$$\frac{d\gamma}{dt} = -e(\beta_r E_r + \beta_z E_z), \tag{8.7}$$

where $\gamma^2 = 1 + (p_r^2 + p_z^2)/m_0^2 c^2$ is the Lorentz factor, p_r and p_z are the radial and longitudinal components of electron momentum $\vec{p} = \gamma m_0 \vec{v}$ respectively, β_r and β_z are the radial and longitudinal components of the normalized velocity $\vec{\beta} = \vec{v}/c$, and $-e$ and m_0 are the electron's charge and rest mass respectively.

The equations (8.5)-(8.7) form a set of coupled ordinary differential equations. These equations have been solved numerically with a computer simulation code for electron trajectory and energy. In thic chapter the time, length, velocity, momentum, and energy are normalized by $1/\omega_0$, c/ω_0, c, $m_0 c$, and $m_0 c^2$ respectively. The normalized laser intensity and normalized wiggler magnetic field parameters can be expressed as, $a_0 = eE_0 / m_0\omega_0 c$ and $b_{w0} = eB_{w0} / m_0\omega_0 c$ respectively. We consider that the electron is initially injected [55, 118, 121] at a small angle δ to the direction parallel to propagation of laser pulse with

$$\vec{p}_0 = \hat{r} p_0 \sin\delta + \hat{z} p_0 \cos\delta, \tag{8.8}$$

where p_0 is the initial momentum of the electron.

8.4 RESULTS AND DISCUSSION

For numerical simulations, we set parameters, $a_0 = 25$ (corresponding to laser intensity $I \sim 8.5\times10^{20}\ W/cm^2$), $a_0 = 100$ (corresponding to laser intensity $I \sim 1.36\times10^{22}\ W/cm^2$), wave length $\lambda_0 \sim 1\mu m$; $r_0' = 700$ (corresponding to laser spot sizes $r_0 \sim 120\mu m$), laser pulse length $\sigma = 70$; initial position of pulse peak $z'_L = -100$; initial electron position $z_i = 0$ and $r_i = 0$; initial phase $\phi_0 = 0$, normalized wiggler wave length $\lambda_w / \lambda_L = 4500$, 6000 and 8500; $b_{w0} = 9\times10^{-5}$ (corresponding to wiggler magnetic field $\sim 9.62 kG$) and 10×10^{-5} (corresponding to wiggler magnetic field $\sim 10.69 kG$); $\theta = 30^\circ$ and 40°; $\delta = 0^\circ$ and 10°; and $p'_0 = 1$.

Figure 8.2, shows the variation of electron energy gain with normalized longitudinal distance z' at different values of laser intensity parameter $a_0 = 25$, 50, and 100 without and with wiggler magnetic field. We consider a unit value for function $\sin(\theta - k_w z)$ and analyze the variation of energy gain by electron for different values of b_{w0}. The electron energy gain increases with z' and approaches its maximum value at resonance. It is due to the betatron oscillations which set up between the accelerated

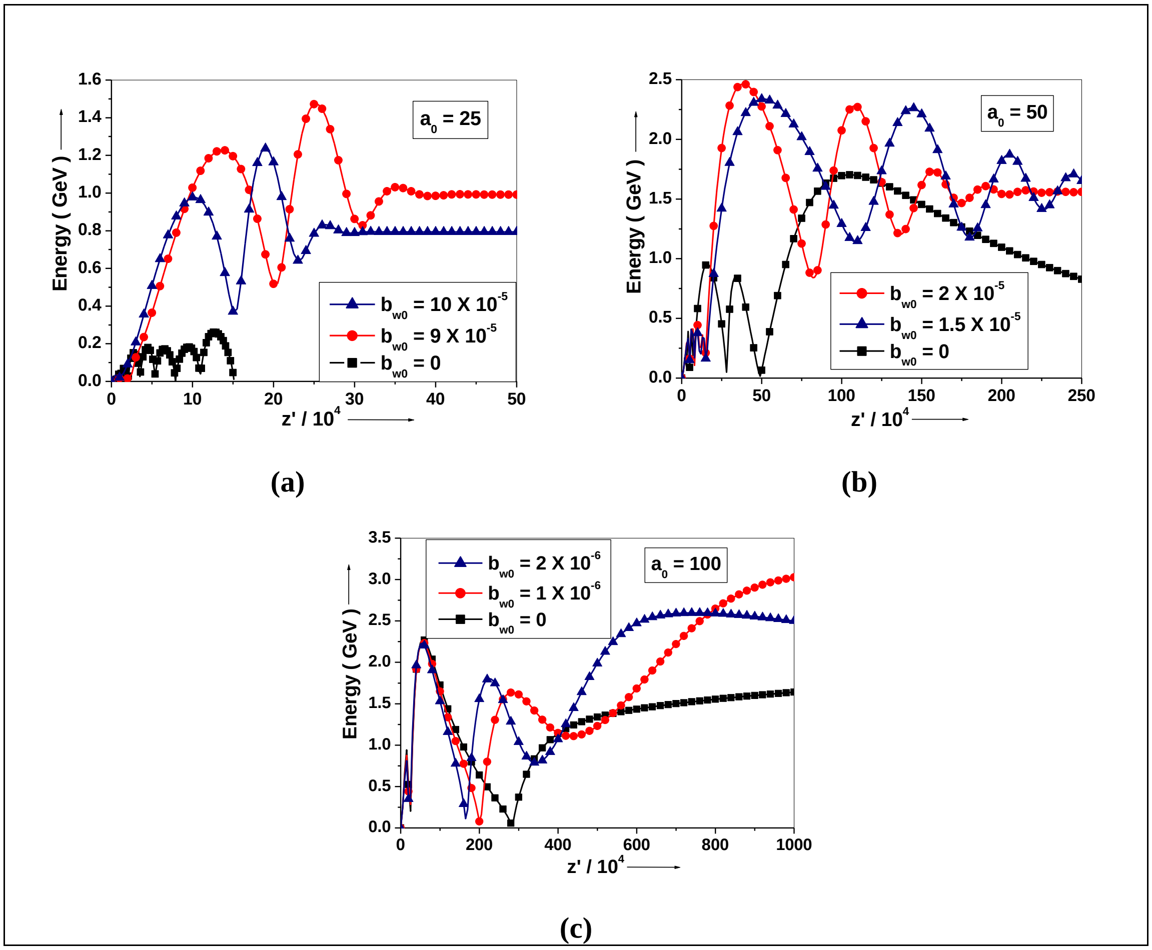


Figure 8.2. Electron energy variations with normalized distance z' in the absence and presence of normalized wiggler magnetic field b_{w0} at distinct values of laser intensity parameter a_0. (a) $a_0 = 25$ and $b_{w0} = 0$, 9×10^{-5}, and 10×10^{-5} (b) $a_0 = 50$ and $b_{w0} = 0$, 1.5×10^{-5}, and 2×10^{-5}, and (c) $a_0 = 100$, and $b_{w0} = 0$, 1×10^{-6}, and 2×10^{-6}. The other parameters are $p'_0 = 1$, $r_0' = 700$, $\sigma = 70$, $z'_0 = -100$, $z_i = 0$, $r_i = 0$, $\delta = 10^\circ$, and $\phi_0 = 0$.

electron and the electric field of the laser pulse. Hence, the electron retains considerable energy. The net electron energy gain by the electron during their acceleration appears sensitive to the laser intensity and magnetic wiggler. It is also seen that the electron can gain very high energy with wiggler magnetic field as wiggler magnetic field increases the time of interaction between the laser pulse and the electron. Singh and Kumar [97] used RP laser pulse with a different set of laser parameters and obtained the highest electron energy of $140MeV$ and $0.5GeV$ for $a_0 = 25$ and $a_0 = 50$ respectively. However, we have employed a RP laser pulse with magnetic wiggler and obtained the highest electron energy of about $1.5GeV$ and $2.5GeV$ for $a_0 = 25$ and $a_0 = 50$ respectively as depicted in fig. 8.2(a) and 8.2(b). For $a_0 = 50$ with normalized wiggler magnetic field $b_{w0} = 2\times10^{-5}$, the energy gain is about $2.5GeV$, which is higher than the energy of $2.25GeV$, attainted with $b_{w0} = 1.5\times10^{-5}$. Thus the value $b_{w0} = 2\times10^{-5}$ is an optimum value of wiggler magnetic field b_{w0} for the maximum energy gain with $a_0 = 50$. If we make small change in the value of b_{w0}, a significant change in the electron energy gain is seen. One should realize the sensitiveness of the wiggler magnetic field while optimizing the value of chirp parameter for maximum electron energy gain. For $a_0 = 100$ at optimum wiggler magnetic field, above $3GeV$ electron energy is achieved whereas without magnetic field it is about $2.25GeV$ with same intensity as depicted in fig. 8.2(c). It is interesting to notice that the maximum energy gain in the presence of a wiggler magnetic field with a RP laser pulse is about 4 times greater than that in the absence of wiggler magnetic field at $a_0 = 25$.

Figure 8.3, shows the variation plot of electron energy gain with normalized wiggler wave length λ_w / λ_L, normalized wiggler magnetic field b_{w0}, and azimuthal angle θ for $a_0 = 25$. One can see that the parameters λ_w / λ_L, b_{w0} and θ are optimized at values for which maximum energy gain is obtained. The optimum value of wiggler wave length and magnetic field plays a vital role in maintaining the resonance for longer duration. For $a_0 = 25$, with $b_{w0} = 9\times10^{-5}$ and $\theta = 30^\circ$ the maximum energy gain appears with $\lambda_w / \lambda_L = 4500$ as shown in fig. 8.2(a). Fig. 8.2(b) and 8.2(c), shows the variation plot of electron energy gain with normalized wiggler wave length λ_w / λ_L, and azimuthal angle θ. Thus, for $a_0 = 25$, the optimum values of normalized wiggler wave length and magnetic field are $\lambda_w / \lambda_L = 8500$ and $b_{w0} = 10\times10^{-5}$ respectively at $\theta = 40^\circ$ for which maximum energy gain of about $2.3GeV$ is obtained.

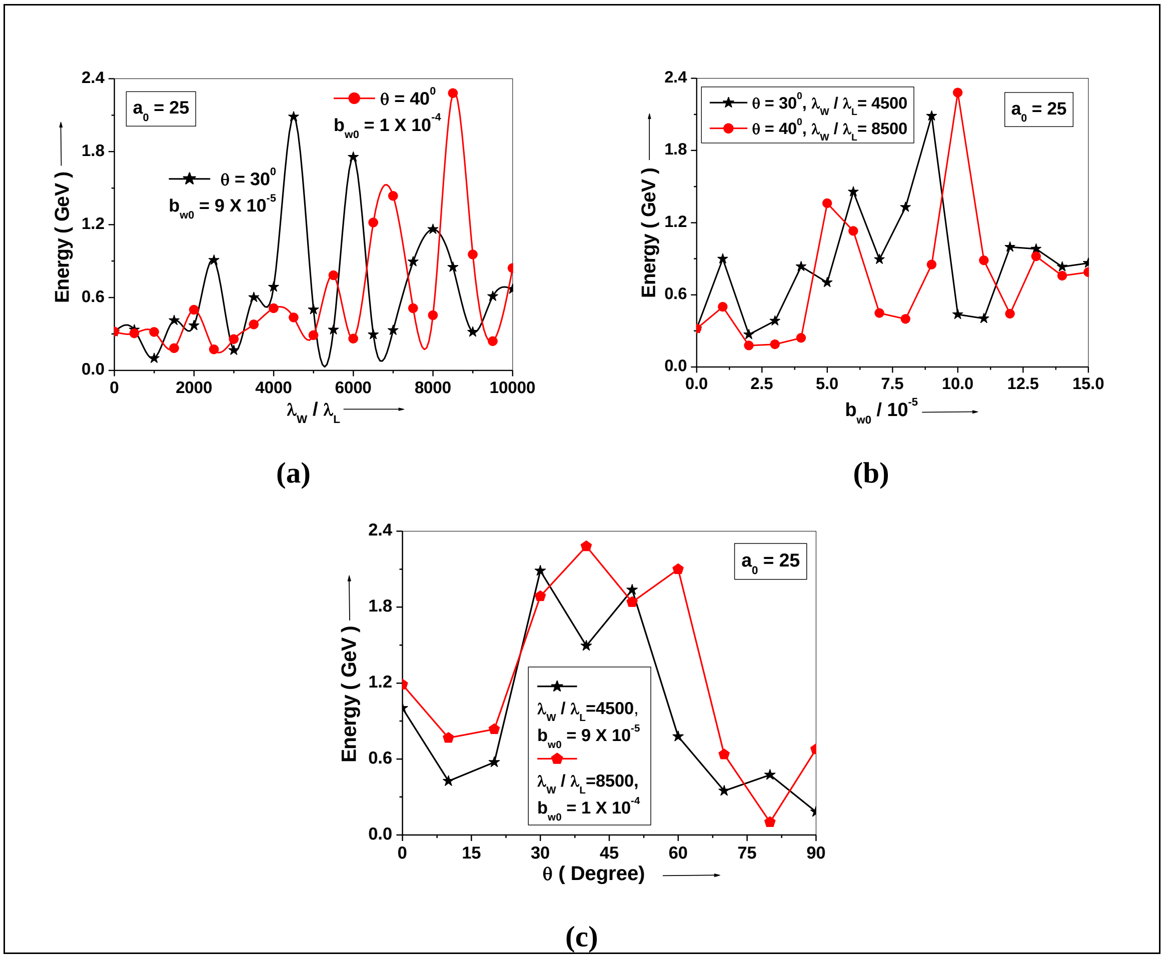
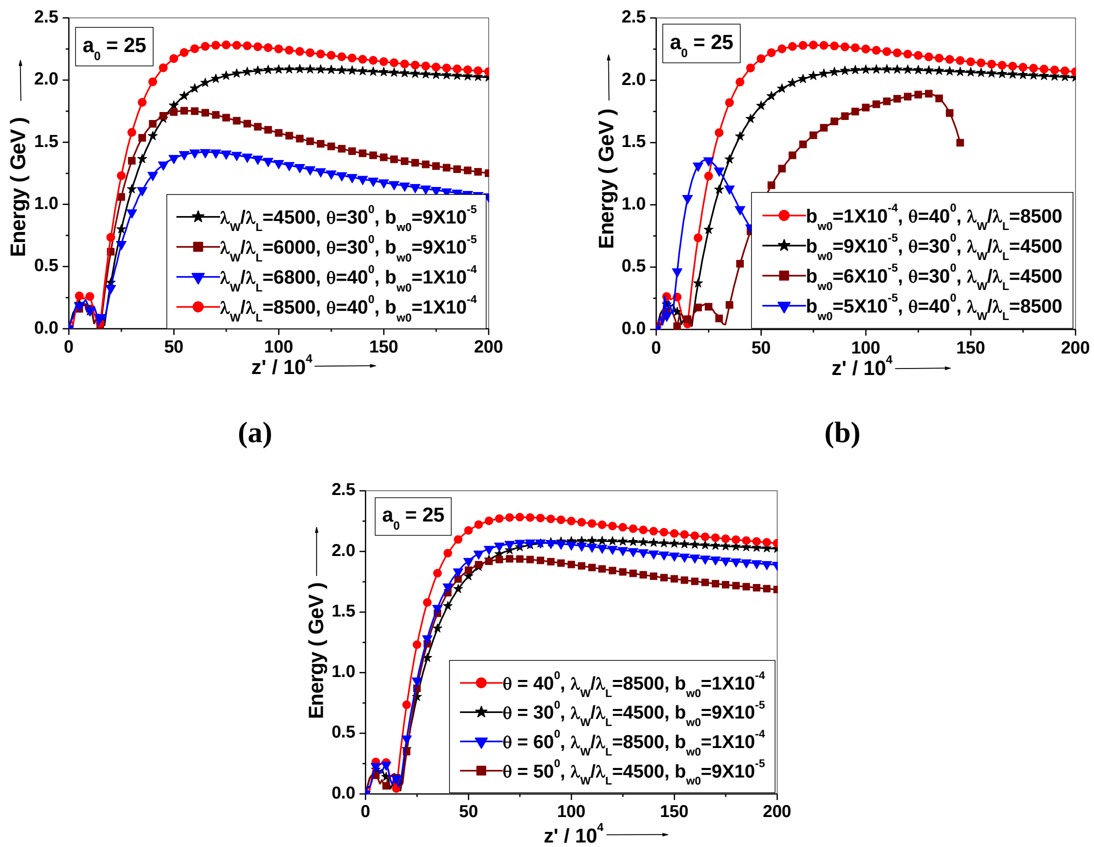


Figure 8.3. Electron energy for $a_0 = 25$ as a function of (a) Normalized wiggler wavelength λ_W / λ_L with $\theta = 40^\circ$, $b_{w0} = 10\times10^{-5}$ and $\theta = 30^\circ$, $b_{w0} = 9\times10^{-5}$, (b) Normalized wiggler magnetic field b_{w0} with $\theta = 40^\circ$, $\lambda_W / \lambda_L = 8500$ and $\theta = 30^\circ$, $\lambda_W / \lambda_L = 4500$, and (c) Azimuthal angle θ with $\lambda_W / \lambda_L = 8500$, $b_{w0} = 10\times10^{-5}$ and $\lambda_W / \lambda_L = 4500$, $b_{w0} = 9\times10^{-5}$. Rest of the parameters are same as taken in fig. 8.2.

Figure 8.4, shows the variation of energy gain by electron with normalized longitudinal distance z' for different values of λ_W / λ_L, b_{w0} and θ at $a_0 = 25$. The optimum values of λ_W / λ_L, b_{w0} and θ for maximum energy gain with $a_0 = 25$ are same as taken in fig. 8.3. We have observed that if we change the values of λ_W / λ_L, b_{w0} and θ then the electron energy gain varies. So we can optimize the value of these parameters



(a) (b)

(c)

Figure 8.4. Electron energy for $a_0 = 25$ **as a function of normalized distance** z' **for (a) Normalized wiggler wavelength** $\lambda_W/\lambda_L = 8500$**,** 6800 **with** $\theta = 40^\circ$**,** $b_{w0} = 10\times10^{-5}$ **and** $\lambda_W/\lambda_L = 4500$**,** 6000 **with** $\theta = 30^\circ$**,** $b_{w0} = 9\times10^{-5}$ **(b) Normalized wiggler magnetic field** $b_{w0} = 1\times10^{-4}$**,** $b_{w0} = 5\times10^{-5}$ **with** $\theta = 40^\circ$**,** $\lambda_W/\lambda_L = 8500$ **and** $b_{w0} = 9\times10^{-5}$**,** $b_{w0} = 6\times10^{-5}$ $\theta = 30^\circ$**,** $\lambda_W/\lambda_L = 4500$ **and (c) Azimuthal angle** $\theta = 40^0$**,** 60^0 **with** $\lambda_W/\lambda_L = 8500$**,** $b_{w0} = 10\times10^{-5}$ **and** $\theta = 30^0$**,** 50^0 **with** $\lambda_W/\lambda_L = 4500$**,** $b_{w0} = 9\times10^{-5}$**. Rest parameters are same as taken in fig. 8.2.**

and obtain the maximum energy gain

We have considered that the electron is injected at a small angle $\delta = 10^\circ$ with respect to the direction parallel to propagation of laser pulse. The angle of the electron

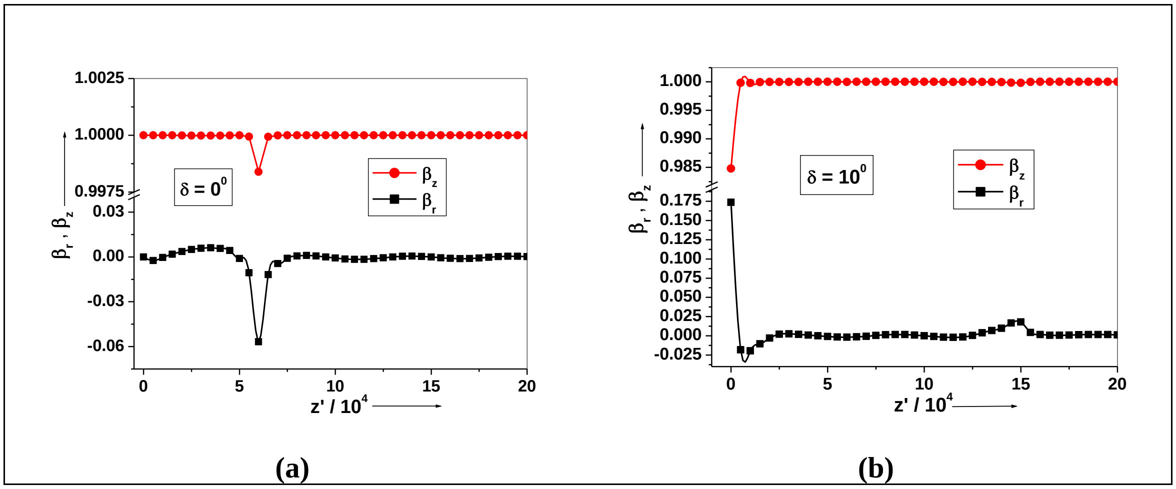


Figure 8.5. Normalized parallel velocity β_z and perpendicular velocity β_r as a function of normalized propagation distance z' for $a_0 = 25$ with $b_{w0} = 10\times10^{-5}$, $\lambda_W / \lambda_L = 8500$, $\theta = 40^\circ$ and angle of electron injection as (a) $\delta = 0^\circ$ and (b) $\delta = 10^\circ$ Rest parameters are same as taken in fig. 8.2.

injection determines the initial velocity of electron during interaction with laser pulse. As per equation (8.8) the initial velocity of a pre-accelerated electron when injected at a small angle δ possesses two components, normalized transverse component $\beta_r = \beta_0 \sin\delta$ and normalized longitudinal component $\beta_z = \beta_0 \cos\delta$.

Figure 8.5, shows the variation of normalized velocity component β as a function of normalized distance z'. The transverse component β_r of velocity is non-zero when electron is sideway injected at a small angle $\delta = 10^\circ$ as depicted in fig. 8.5(b), whereas this component is initially zero for axial injection at $\delta = 0^\circ$ as appears in fig 8.5(a).

Figure 8.6, shows the variation of electron energy gain with normalized longitudinal distance z' at two different values of angle of electron injection $\delta = 0^\circ$ and $\delta = 10^\circ$ with respect to the direction parallel to propagation of laser pulse for laser intensity parameter $a_0 = 25$.

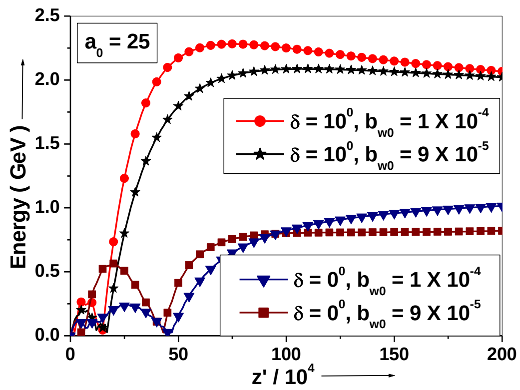


Figure 8.6. Electron energy variations for $a_0 = 25$ with normalized distance z' for two different values of angle of electron injection $\delta = 0^\circ$ and $\delta = 10^\circ$ with $\lambda_W / \lambda_L = 8500$, $\theta = 40^\circ$, $b_{w0} = 10 \times 10^{-5}$ and $\lambda_W / \lambda_L = 4500$, $\theta = 30^\circ$, $b_{w0} = 9 \times 10^{-5}$. Rest parameters are same as taken in fig. 8.2.

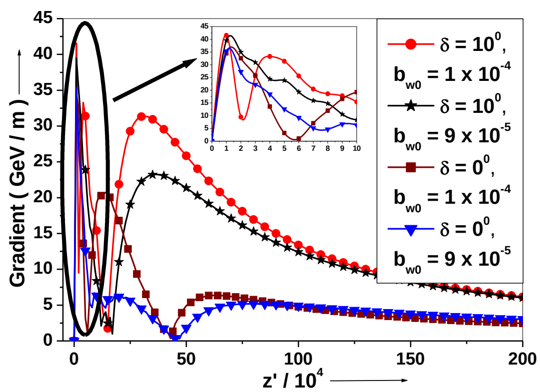


Figure 8.7. Acceleration gradient variations with normalized distance z' for $a_0 = 25$ with two different values of angle of electron injection $\delta = 0^\circ$ and $\delta = 10^\circ$ at $\lambda_W / \lambda_L = 8500$, $\theta = 40^\circ$, $b_{w0} = 10 \times 10^{-5}$ and $\lambda_W / \lambda_L = 4500$, $\theta = 30^\circ$, $b_{w0} = 9 \times 10^{-5}$. Rest parameters are same as taken in fig. 8.2.

The electron energy gain is about 1.3 times higher with a sideway injection of electron than that with axial injection as depicted from fig. 8.6. It is because of the non-zero transverse component of velocity which provides initially an additional kick to the electron for high energy gain. Hence the electron energy gain with sideway injection is higher than that with axial injection.

Figure 8.7, shows the variation plots of acceleration gradient with normalized distance z'. The acceleration gradient [108] can be obtained by using relation $d\gamma / dz = -e\vec{\beta}.\vec{E} / \beta_z$ and is plotted with normalized distance z' for two values of δ at $a_0 = 25$ with $p_0' = 1$ and optimum value of b_{w0}, λ_W / λ_L, and θ for axial and sideway injection of electron as derived from fig. 8.6. We have observed an acceleration gradient of about $42 GeV/m$ with $a_0 = 25$(corresponding to laser intensity $I \sim 8.5 \times 10^{20}\ W/cm^2$) in sideway injection. Higher acceleration gradient can be obtained with higher values of laser intensity parameter.

The radiative loss during deceleration can be obtained by the following expression (Lienard result) of the radiated power [38]:

$$P(t) = \frac{2e^2\gamma^6}{3c}\left[\left(\frac{d(\vec{v})}{dt}\right)^2 - \left(\vec{v} \times \frac{d\vec{v}}{dt}\right)^2\right]. \qquad (8.8)$$

This expression shows that the radiated power depends on the quiver velocity of electron. As per numerical values of parameters taken, the radiation loss $(\Delta W \sim P \times \Delta t)$ appears to be very large, where Δt is the interaction duration. The gained energy loses via radiations. However, a pre-accelerated electron can retain sufficient energy and the radiation loss remains small. We have employed a pre-accelerated electron with normalized initial momentum $p_0' = 1$. Thus the radiation loss can be minimized for a pre-accelerated electron [66].

8.5 CONCLUSIONS

We have observed electron acceleration to GeV energy by a RP laser pulse in vacuum in the presence of wiggler magnetic field. In the absence of wiggler magnetic field the accelerated electron after attaining maximum energy gets out of phase with the field and loses its gained energy quickly. The presence of a wiggler magnetic field not only increases the electron energy gain but also support in retaining the gained energy for longer distances. This is due to the fact that the wiggler magnetic field increases the duration of electron laser interactions and hence maintains the betatron resonance for longer distances. We have observed about 4 times higher energy gain in the presence of wiggler magnetic field with a RP laser pulse than that in the absence of wiggler magnetic field. An effective enhancement in electron energy gain is observed with a sideway injection of electron at a small angle of about 10° with respect to the direction parallel to propagation of laser pulse than that with axial injection. The maximum energy gain with a sideway injection is about 1.3 times higher than that with axial injection.