CHAPTER-4

ELECTRON ACCELERATION BY A CHIRPED LASER PULSE IN VACUUM UNDER THE INFLUENCE OF MAGNETIC FIELD

4.1 INTRODUCTION

The development of theoretical concepts and experimental models for the investigations of charged particle dynamics under interactions with laser fields are the key area of research during last few decades [57, 105]. The extensive studies of laser electron interaction target to the betterment of electron accelerations for high energy gains. Laser pulse polarization plays an important role in transferring energy to electrons and hence enhancing electron acceleration [96]. The obtained electron energies can further be increased and retained for larger distances by optimal use of magnetic fields [73]. Thus the polarization as well as magnetic field remains the key characteristics for obtaining high electron energy gains [55, 66]. The variations in frequency of laser pulse effectively improve the energy gain by the accelerated electrons from laser pulse [46]. Use of chirped laser pulse enforces the maintaining of resonance during laser electron interactions for longer duration [117, 118]. Salamin and Jisrawi used a quadratic frequency chirp to achieve high electron energy during laser electron interaction in vacuum [119]. The suitable application of magnetic field can further enhance electron acceleration gradient [108].

Afhami and Eslami [116] studied numerically the effect of nonlinear chirped Gaussian laser pulse parameters on the electron acceleration. They proposed that the nonlinear chirped pulse has a much smaller divergence than that of linear chirped pulse. The main goal of this work is to study the combined effect of chirping by a LP laser pulse and pinching by an azimuthal magnetic field on accelerated electron in vacuum. For this purpose, we organize this chapter as follows: dynamics of electron due to a LP chirped laser pulse is described in section 4.3. In section 4.4, we show the numerical results and discuss the final electron energy and electron trajectory for different chirp factors. The conclusions are presented in section 4.5.

4.2 LINEAR FREQUENCY CHIRP

The frequency chirped laser pulse enforces an effective electron laser interaction. The betatron resonance between the electron and the electric field of the laser pulse is maintained for a longer duration with a LP chirped laser pulse. The most common chirped form is the linear chirp in which the instantaneous frequency changes linearly during the chirped pulse. The linear chirped laser pulse provides an improved platform for electron laser interactions than that with unchirped laser pulse.

A linear frequency chirp $\omega(\xi)$ is expressed as:

$$\omega(\xi) = \omega_0 (1 + \alpha\xi) \, , \tag{4.1}$$

where α is the frequency chirp factor, ω_0 is the initial frequency of laser, $\xi = z - ct$ is the retarded coordinate, and c is the velocity of light in vacuum. Eq. (4.1) represents an arbitrary frequency chirp for linear and negative chirp.

4.3 FIELD DISTRIBUTION FOR LINEARLY POLARIZED CHIRPED LASER PULSE

We consider a LP chirped laser pulse propagating along the z-direction with transverse component of electric field ($\vec{E} = \hat{x}E_x$) as:

$$E_x = \frac{E_0}{f} \sin(\phi) \exp\left(-\frac{(\xi - \xi_0)^2}{\sigma^2} - \frac{r^2}{r_0^2 f^2} \right), \tag{4.2}$$

where $\phi = k(\xi)\xi + \tan^{-1}(z / Z_R) - zr^2 /(Z_R r_0^2 f^2) + \phi_0$, $f^2 = 1 + (z / Z_R)^2$, $k(\xi) = \omega(\xi)/c$ is the wave number, $Z_R = kr_0^2 / 2$ is the Rayleigh length, ϕ_0 is the initial phase, σ is the laser pulse length, $r^2 = x^2 + y^2$, r_0 is minimum laser spot size, and ξ_0 is the initial position of the pulse peak. The longitudinal component of electric field is neglected because of very small amplitude in comparison with transverse component. The magnetic field components of laser pulse can easily be obtain from Maxwell's equations and expressed as:

$$\vec{B} = -ic \frac{\vec{\nabla} \times \vec{E}}{\omega(\xi)} \, , \tag{4.3}$$

The laser pulse imparts energy to the electron that carries current in the z-direction and experiences an azimuthal magnetic field. Here we consider the effect of azimuthal magnetic field on electron acceleration. In general, for LP laser with transverse component of electric field in x-direction propagating through vacuum, the azimuthal magnetic field has two components and is expressed as $\vec{B}_\theta = (\hat{x}y - \hat{y}x)(B_0 / r_0)\exp(-r^2 / r_0^2)$. For $y = 0$, the B_θ becomes perpendicular to laser electric field, due to which the cyclotron frequency increases with increasing value of x. Thus electron undergoes betatron oscillations in the presence of azimuthal magnetic field. For $x = 0$, the B_θ becomes parallel to laser electric field. This correspond to an ordinary mode propagation which becomes unaffected by the presence of static magnetic field. Thus for a LP laser pulse with a transverse x component, the azimuthal magnetic field appears with y component only and is expressed as:

$$\vec{B}_\theta = -\hat{y}B_0 \frac{x}{r_0} \exp\left(-\frac{x^2}{r_0^2}\right), \tag{4.4}$$

where the constant B_0 represents the maximum amplitude of magnetic field. Figure 4.1 shows a schematic of vacuum acceleration of electron by a LP chirped laser pulse with an azimuthal magnetic field.

4.4 ELECTRON DYNAMICS AND RELATIVISTIC ANALYSIS

The equations expressing electron momentum and energy are:

$$\frac{dp_x}{dt} = -eE_x + e\beta_z B_y + e\beta_z B_\theta, \tag{4.5}$$

$$\frac{dp_y}{dt} = 0, \tag{4.6}$$

$$\frac{dp_z}{dt} = -e\beta_x B_y - e\beta_x B_\theta, \tag{4.7}$$

$$\frac{d\gamma}{dt} = -e\beta_x E_x, \tag{4.8}$$

where p_x, p_y and p_z are x, y and z components of the momentum $\vec{p} = \gamma m_0 \vec{v}$ respectively; β_x, β_y and β_z are x, y and z components of the normalized velocity $\vec{\beta} = \vec{v}/c$ respectively; $\gamma^2 = 1 + (p_x^2 + p_y^2 + p_z^2)/(m_0 c)^2$ is the Lorentz factor, $-e$ is the charge and m_0 is the rest mass of the electron respectively. The following are the normalized parameters:

$$a_0 \to \frac{eE_0}{m_0 \omega_0 c}, \quad \tau \to \omega_0 t, \quad \sigma' \to \frac{\omega_0 \sigma}{c}, \quad r_0' \to \frac{\omega_0 r_0}{c}, \quad z_0' \to \frac{\omega_0 z_0}{c}, \quad x' \to \frac{\omega_0 x}{c}, \quad y' \to \frac{\omega_0 y}{c},$$

$$z' \to \frac{\omega_0 z}{c},\ \beta_x \to \frac{v_x}{c},\ \beta_y \to \frac{v_y}{c},\ \beta_z \to \frac{v_z}{c},\ p_0' \to \frac{p_0}{m_0 c},\ p_x' \to \frac{p_x}{m_0 c},\ p_y' \to \frac{p_y}{m_0 c},\ p_z' \to \frac{p_z}{m_0 c}$$

$$,\ k_0' \to \frac{ck_0}{\omega_0},\ \alpha' \to \frac{\alpha c}{\omega_0}, \text{ and } b_0 \to \frac{eB_0}{m_0 \omega_0 c}.$$

where a_0 is the normalized value of laser intensity parameter; τ is the normalized time; σ' is the normalized laser pulse length; z_0' is the normalized initial position of the pulse peak; x', y' and z' are the normalized x, y and z coordinates; p_0' is the normalized initial momentum of electron; p_x', p_y' and p_z' are the x, y and z components of the normalized momentum; k_0' is the normalized value of initial wave number; α' is the normalized chirp parameter and b_0 is the normalized value of magnetic field. Equations (4.5) to (4.8) are the coupled ordinary differential equations. We solve these equations numerically for the electrons trajectory and energy.

We consider that the electron is initially injected at a small angle δ with respect to the direction parallel to propagation of laser pulse with $\vec{p}_0 = \hat{x} p_0 \sin\delta + \hat{z} p_0 \cos\delta$ where p_0 is the initial momentum of the electron [55].

4.5 RESULTS AND DISCUSSION

We have chosen the following dimensionless parameters for numerical analysis: $a_0 = 2.5$ (corresponding to laser intensity $I \sim 8.5 \times 10^{18}\ W/cm^2$), $a_0 = 5$ (corresponding to

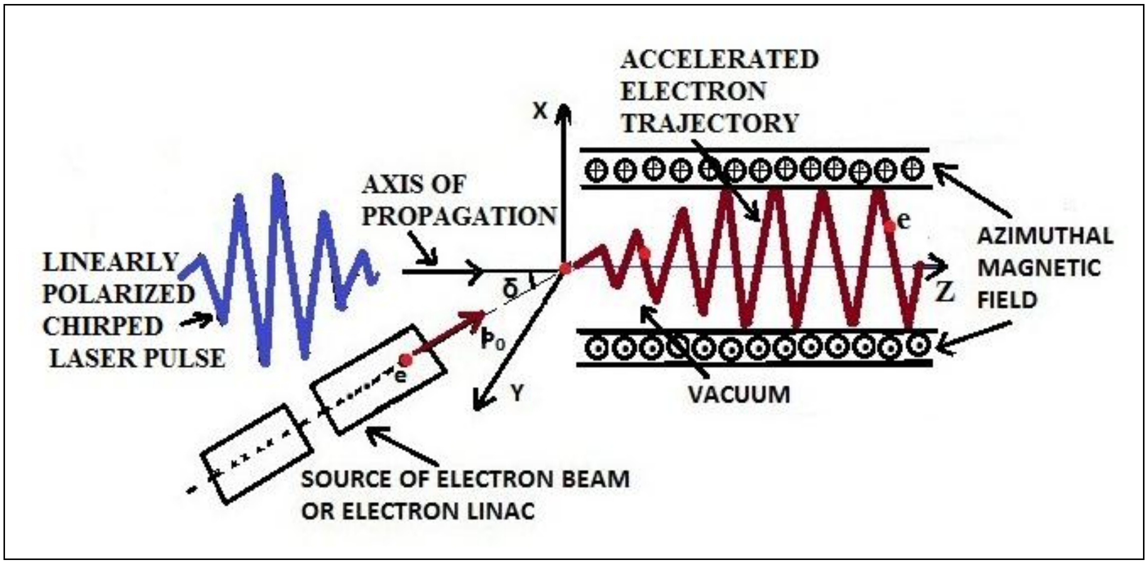

Figure 4.1. A schematic showing the vacuum acceleration of electron by a LP chirped laser pulse in the presence of an azimuthal magnetic field.

laser intensity $I \sim 3.46\times10^{19}\,W/cm^2$) and $a_0 = 10$ (corresponding to laser intensity $I \sim 1.37\times10^{20}\,W/cm^2$) with wave length $\lambda_0 \sim 1\mu m$; $r_0' = 900$ (corresponding to laser spot sizes $r_0 \approx 151\mu m$); and $p_0' = 1$, 2 and 2.5; $\sigma' = 70$ (corresponding to laser pulse duration of $200fs$); $\delta = 10^{\circ}$; $\phi_0 = 0$; $b_0 = 0.00225$ (corresponding to a magnetic field of $240kG$), $b_0 = 0.0041$ (corresponding to a magnetic field of $438kG$) and $b_0 = 0.009$ (corresponding to a magnetic field of $962kG$); and $z_0' = -300$.

Figure 4.2, shows the trajectory of electron in three dimensional plane without and with azimuthal magnetic field with and without chirping for $a_0 = 2.5$, $p_0' = 1$ and $b_0 = 0.009$. The electron gets accelerated during interaction with laser pulse even without external azimuthal magnetic field. But within a small distance it gets decelerated, goes out of phase and losses its energy as depicted from fig. 4.2(a). In the presence of azimuthal magnetic field a confined trajectory for longer distance appears as in fig. 4.2(b). The trajectory appears more confined with linearly chirped laser pulse than that with un-chirped pulse which depicts that the presence of

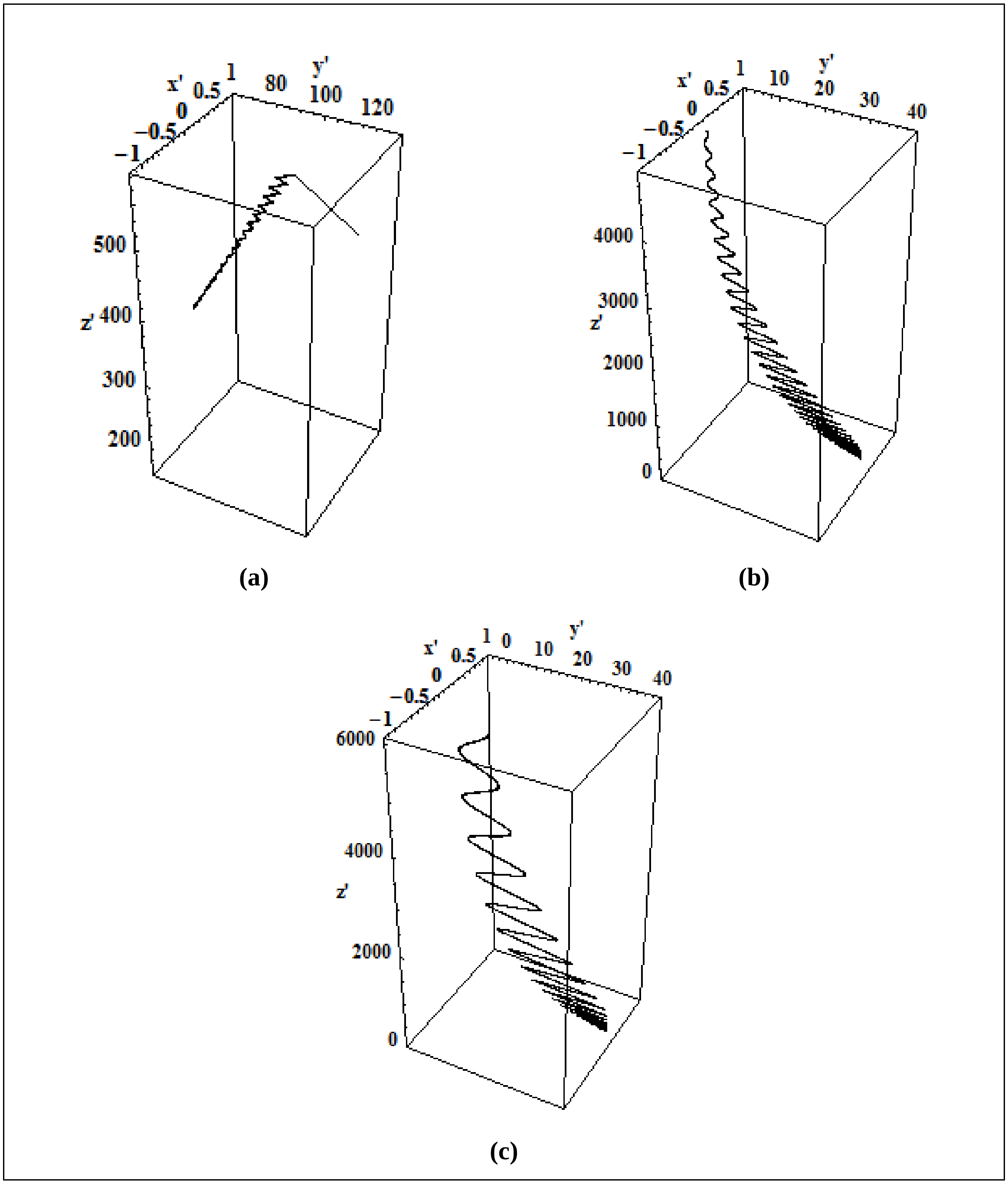

Figure 4.2. Trajectory of electron in 3D plane without and with chirping for LP laser pulse with $a_0 = 2.5$ **and** $p_0' = 1$**. (a)** $\alpha' = 0$**,** $b_0 = 0$ **(b)** $\alpha' = 0$**,** $b_0 = 0.009$ **and (c)** $\alpha' = 0.0005$ **and** $b_0 = 0.009$**. The other parameters are** $r_0 = 900$**,** $\sigma' = 70$**,** $\phi_0 = 0$**,** $\delta = 10^\circ$ **and** $z_0' = -300$**.**

chirping increases the duration of laser electron interaction and hence provide the better transfer of energy. The electron oscillates in the direction of propagation of laser during the interaction with laser pulse. The azimuthal magnetic field enhances the strength of $\vec{v} \times \vec{B}$ force and enforces an accelerated motion for larger distance by deflecting the escaping electron along the $+z$ direction which indicates the retaining of high energy at larger distances.

Figure 4.3, shows the variation of electron energy gain γ with normalized magnetic field b_0. In fig. 4.3(a) and 4.3(b), electron energy gain has been analysed at different values of normalized initial momentum with and without chirping for $a_0 = 2.5$ and 5 respectively. The electron energy gain appears higher with chirped laser pulse than that with unchirped laser pulse in the presence of azimuthal magnetic field. For $a_0 = 2.5$ and $p_0' = 2.5$ the electron energy gain is $\gamma = 651$ in the presence of chirp factor $\alpha' = 0.00085$, which is 31.5% higher than $\gamma = 495$ in the absence of chirping. The obtained energy gain with chirped pulse for $a_0 = 5$ and $p_0' = 2.5$ is about 34% higher than that in the absence of chirping. Fig. 4.3(c), shows the variation of energy gain γ with b_0 for different values of laser pulse intensity parameters $a_0 = 10$, 20, 30, 40 and 50 at $p_0' = 2.5$ and selective values of chirp factor α'. The selective values of chirp factor corresponding to the laser pulse intensities are obtained by optimization for maximum energy gain. High energy gain appears at the small values of magnetic field. A pre-accelerated electron trapped with laser pulse in the presence of low and optimum magnetic field. After attaining maximum gain at resonance the electron energy decreases and almost saturates for the larger values of magnetic field. The electron energy increases with laser pulse intensity. For $a_0 = 20$ the higher energy gain $\gamma = 2675$ is achieved at $b_0 = 0.00325$ with $\alpha' = 0.00075$ which is higher than $\gamma = 2476$ for $a_0 = 10$ at $b_0 = 0.00275$ with $\alpha' = 0.0008$ at $p_0' = 2.5$. Comparatively higher energy gain is obtained with higher values of laser pulse intensity parameter $a_0 = 30$, 40 and 50. Thus electron energy gain increases with laser intensity.

Figure 4.4, shows the variation of energy gain γ by the electron with accelerating distance z normalized with Rayleigh length Z_R. In fig. 4.4(a) and 4.4(b), we see the variation of

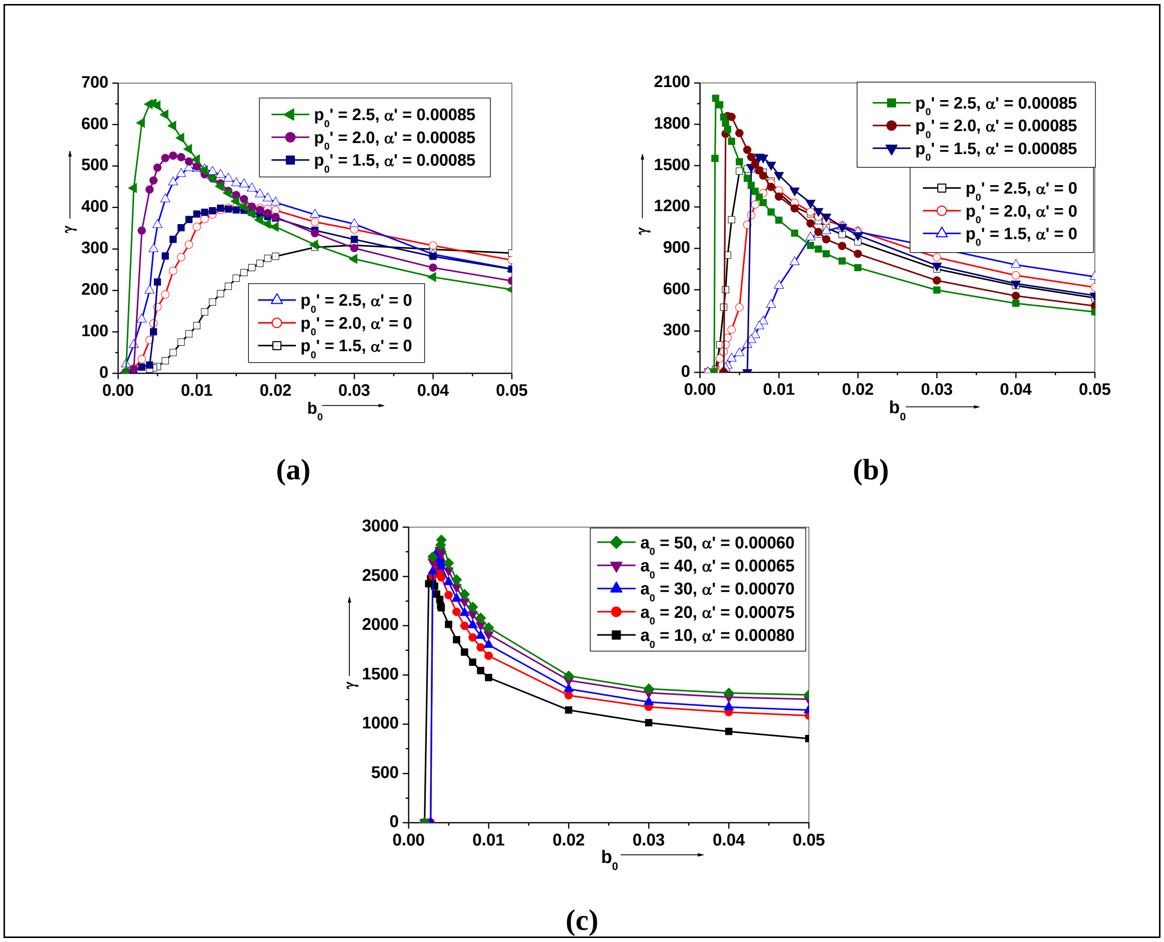

Figure 4.3. Energy gain γ with normalized magnetic field b_0 for different values of normalized initial momentum and intensity parameters for LP laser pulse. (a) $p_0'=1.5$, 2 and 2.5 at $a_0=2.5$ with $\alpha'=0$ and $\alpha'=0.00085$ (b) $p_0'=1.5$, 2 and 2.5 at $a_0=5$ with $\alpha'=0$ and $\alpha'=0.00085$ and (c) $a_0=10$, 20, 30, 40 and 50 at $p_0'=2.5$. Rest of the parameters are same as taken in fig. 4.2.

γ with z/Z_R for different values of normalized momentum $p_0'=1.5$, 2 and 2.5 with optimum values of magnetic field for $a_0=2.5$ and 5. The electron energy gain increases during the interaction with laser pulse. After achieving the maximum value, the energy gain saturates for larger distances. Fig. 4.4(c), shows the variation of energy gain γ with z/Z_R for the optimum values of magnetic field b_0 and chirp factor α' with $p_0'=2.5$ for $a_0=10$, 20, 30, 40 and 50

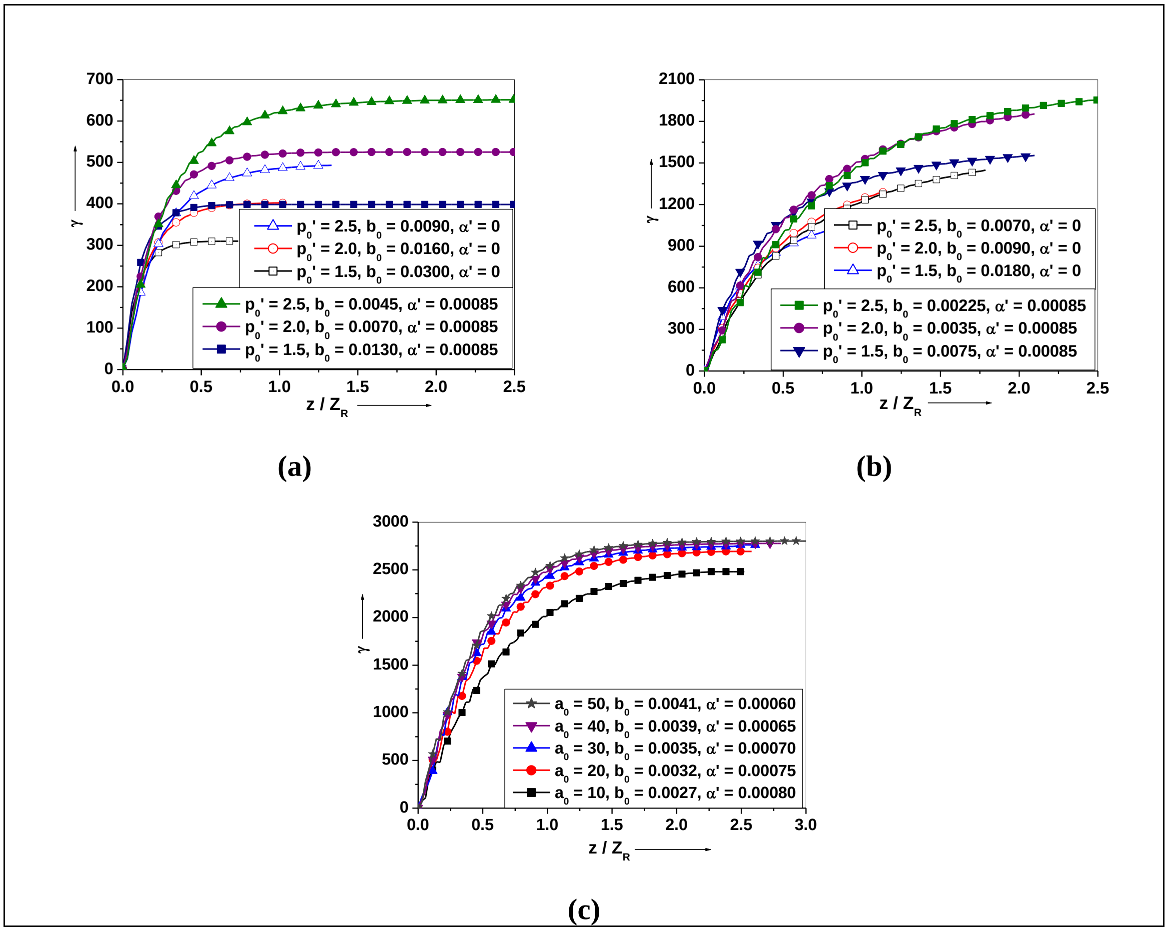

Figure. 4.4. Electron energy gain γ with accelerating distance z normalized with Rayleigh length Z_R for distinct values of normalized magnetic field for LP laser pulse. (a) $p_0'=1.5$, 2 and 2.5 at $a_0=2.5$ with $\alpha'=0$ and $\alpha'=0.00085$, (b) $p_0'=1.5$, 2 and 2.5 at $a_0=5$ with $\alpha'=0$ and $\alpha'=0.00085$, and (c) $a_0=10$, 20, 30, 40, and 50 at $p_0'=2.5$. Rest of the parameters are same as taken in fig. 4.2.

respectively. With the optimum value of magnetic field, initial momentum and suitable chirp factor, the electron gain higher energy with distance due to resonance. As appearing an energy gain of $\gamma = 2476$ (corresponding to $1.26\,GeV$) is achieved with laser intensity $a_0 = 10$ (corresponding to $1.37\times10^{20} W/cm^2$) and chirping factor $\alpha' = 0.0008$ in the presence of azimuthal magnetic field $b_0 = 0.0027$ (corresponding to $288\,kG$). We have observed the accelerating distance of about three times the Rayleigh length as depicted from fig. 4.4(c). The higher energy gain is achieved with higher values of laser pulse intensities and optimum values of magnetic field, initial momentum, and chirp factor.

In the present study the high energy gain of about $1.47GeV$ is achieved with a chirped laser pulse of peak intensity $I \sim 3.4\times10^{21} W/cm^2$ with magnetic field. This intensity is about half of that was used to obtain only $100\,MeV$ energy with a LP laser pulse [57]. The magnetic field used in our scheme is feasible and can be generated experimentally [94, 115]. An electron acceleration gradient of GeV/m was achieved [108] by using circularly polarized laser pulse with axial magnetic field of about $600\,kG$. We employ a comparatively smaller magnetic field of about $438\,kG$ with LP chirped laser pulse to achieve the acceleration gradient of the order of GeV/m. Higher energy gain can be achieved with higher intensity laser pulse with azimuthal magnetic field. The observed value of angle of emittance remains small with higher intensity pulses. Further, the radiative losses are not much [66] due to the pre accelerated electron.

4.5 CONCLUSION

In recent years, there is much attention in accelerating the electrons by chirped laser fields in vacuum. We have additionally introduced the influence of azimuthal magnetic field on electron acceleration with a LP chirped laser pulse. Our results highlight the importance of chirping of laser pulse in the presence of azimuthal field for enhanced electron acceleration. We have presented that an electron with approximately $2.5MeV$ of initial energy can gain energy of about $1.47GeV$ from a LP chirped laser pulse in the presence of relatively small azimuthal magnetic field. Thus with suitable selection of parameters like laser pulse intensity, initial electron momentum, static magnetic field and chirp factor, the higher energy of the order of GeV can be achieved with an electron of few MeV of initial energy. The presented model led to a substantial increase in electron acceleration energy.