8.1 Introduction

A great deal of effort has been devoted during the last few years to investigate the stability analysis of double diffusive convection in a fluid-saturated porous layer because of its wide range of applications in various fields such as high quality crystal production, liquid gas storage, oceanography, production of pure medication, solidification of molten alloys, and geothermally heated lakes and magmas etc. The problem of double diffusive convection in porous medium has been extensively studied and the growing volume of work devoted to this area is well documented by Nield and Bejan (2006), Ingham and Pop (2005), Vafai (2000, 2005). The linear stability analysis of double diffusive convection in a porous layer was first undertaken by Nield (1968) for various thermal and solutal boundary conditions. Taunton et al. (1972) extended Nield’s analysis and considered the salt-fingering convection case in a porous layer. Rudraiah et al. (1982) applied linear and nonlinear stability analyses and found that subcritical instabilities are possible in the case of a two-component fluid and there has been much subsequent development of this work (Poulikakos (1986), Taslim and Narusawa (1986), Murray and Chen (1989), Mamou (2002), Mojtabi and Charrier-Mojtabi (2005).

The problem of double diffusive convection has been extensively investigated
for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids. The corresponding problem in the case of a porous medium has also not received much attention until recently. With growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigations of such fluids are desirable. The study of such fluids have applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubrication and colloidal and suspension solutions. In the category of non-Newtonian fluids couple stress fluids have distinct features, such as polar effects. The main feature of couple stresses will be to introduce a size dependent effect that is not present in the classical viscous fluids. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The constitutive equations for couple stress fluids were given by Stokes (1966). The theory proposed by Stokes is the simplest one for the micro-fluids, which allows polar effects such as the presence of couple stress, body couple and non-symmetric tensors. Subsequently, many authors have studied the Rayleigh-Benard problem for couple stress fluids and extensions including the issue of stability/onset (Hiremath and Patil (1993), Sharma and Chandel (2004), Siddeshwar and Pranesh (2004), Malashetty et al. (2006, 2009), Shivakumara (2010), Malashetty et al. (2010), Malashetty and Kollur (2011), Malashetty et al. (2012), Taj et al. (2013), Srivastava and Bera (2013)), but their works differ from the present chapter by considering the effect of heat generation on the system.

The porous material offers its own source of heat in many situations which are of great practical importance. This gives a different way in which a convective flow can be set up through the local heat generation within the porous media. Such a situation can occur through radioactive decay or through, in the present perspective, a relatively
weak exothermic reaction which can take place within the porous material. To be more specific, internal heat is the main source of energy for celestial bodies caused by nuclear fusion and decaying of radioactive materials, which keeps the celestial objects warm and active. It is due to the internal heating of the earth that there exists a thermal gradient between the interior and exterior of the earth’s crust, saturated by multicomponents fluids, which helps convective flow, thereby transferring the thermal energy towards the surface of the earth. Simple ideas of heat generation and transfer inside planets allow us to understand the differences in geological features from one planet to another. Observation also tells us that the earth is active – volcanoes, earthquakes, mountain belts and magnetic fields. These must be due to an internal energy or heat source. Therefore the role of internal heat generation becomes very important in several applications that include geophysics, reactor safety analyses, metal waste form development for spent nuclear fuel, fire and combustion studies and storage of radioactive materials.

There are few studies available on convection with heat generation effects. Chamka (2001) studied similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption. Hill (2005) analyzed double diffusive convection in a porous medium with a concentration based internal heat source. Onset of convection in a porous medium with internal heat generation is investigated by Gasser and Kazimi (2010)]. Capone et al. (2011) analyzed double diffusive penetrative convection simulated via internal heating in an anisotropic porous layer with throughflow. Recently Yadav et al. (2012) investigated effect of internal heat source on the onset of convection in nanofluid layer and also Boundary and internal heat source effects on the onset of Darcy-Brinkman convection in a porous layer saturated by nanofluid. Double diffusive
convection in a saturated anisotropic porous layer with internal heat source is analyzed by Bhaduria (2012). Also Teodosiu et al. (2014) studied CFD modeling of buoyancy driven cavities with internal heat source-application to heated rooms.

Although few literatures on double diffusive convection in a porous medium saturated by ordinary fluid with or without an internal heat source is available, no attention has been devoted to the study of double diffusive convection in a porous layer saturated by a couple stress fluid in the presence of an internal heat source. Therefore in the present work we intend to investigate the onset of double diffusive convection in a couple stress fluid saturated porous layer with an internal heat source employing a modified Darcy model using linear and weak nonlinear stability analyses. The objective of this chapter is to study how the onset criteria for stationary and oscillatory convection are affected by the Darcy-Prandtl number, Lewis number, solute Rayleigh number, normalized porosity, couple stresses and internal heat source. Further, we investigate the effect of internal Rayleigh number and other parameters on heat and mass transfers in a more general porous medium.

8.2 Mathematical Formulation

Consider an infinite horizontal, couple stress fluid saturated porous layer confined between the planes $z=0$ and $z=d$, with the vertically downward gravity force $g$ acting on it. A constant temperatures $\Delta T + T_0$ and $T_0$ with stabilizing concentrations $\Delta S + S_0$ and $S_0$ respectively are maintained between the lower and upper surfaces. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The modified Darcy model, which includes the time derivative, is employed as a momentum equation (see Malashetty et al.
With the Oberbeck–Boussinesq approximation, the basic governing equations are

\[ \nabla \cdot \mathbf{q} = 0, \quad (8.1) \]

\[ \frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \rho \mathbf{g} - \frac{1}{K} \left( \mu - \mu \nabla^2 \right) \mathbf{q}, \quad (8.2) \]

\[ \gamma \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa_r \nabla^2 T + Q(T - T_0), \quad (8.3) \]

\[ \varepsilon \frac{\partial S}{\partial t} + (\mathbf{q} \cdot \nabla) S = \kappa_s \nabla^2 S, \quad (8.4) \]

\[ \rho = \rho_0 \left[ 1 - \beta_f (T - T_0) + \beta_s (S - S_0) \right], \quad (8.5) \]

where the variables and constants have their usual meaning, as given in the nomenclature. Further, \( \gamma = \frac{(\rho c)_m}{(\rho c)_f} \), \( (\rho c)_m = (1 - \varepsilon)(\rho c)_s + \varepsilon(\rho c)_f \), \( c \) is the specific heat of the solid and \( c_p \) is the specific heat of the fluid at constant pressure respectively.

### 8.2.1 Basic State

The basic state of the fluid is assumed to be quiescent and is given by

\[ \mathbf{q}_b = (0,0,0), \quad p = p_b(z), \quad T = T_b(z), \quad S = S_b(z), \quad \rho = \rho_b(z). \quad (8.6) \]

The temperature \( T_b(z) \), solute concentration \( S_b(z) \), pressure \( P_b(z) \) and density \( \rho_b(z) \) satisfy the following equations

\[ \frac{dP_b}{dz} = -\rho_b g, \quad \kappa_r \frac{dT_b}{dz^2} + Q(T_b - T_0) = 0, \quad \frac{d^2 S_b}{dz^2} = 0, \quad (8.7) \]

\[ \rho_b = \rho_0 \left[ 1 - \beta_f (T_b - T_0) + \beta_s (S_b - S_0) \right]. \quad (8.8) \]

Then the conduction state temperature and concentration are given by

\[ T_b(z) = \frac{\Delta T \sin \left( \sqrt{R_i} \left( 1 - \frac{z}{d} \right) \right)}{\sin \sqrt{R_i}} + T_0, \quad S_b = \Delta S \left( 1 - \frac{z}{d} \right) + S_0. \quad (8.9) \]
8.2.2 Perturbed State

On the basic state we superpose infinitesimal perturbations in the form

\[ \mathbf{q} = \mathbf{q}_b + \mathbf{q}', \quad T = T_b(z) + T', \quad S = S_b(z) + S', \quad p = p_b(z) + p', \quad \rho = \rho_b(z) + \rho', \quad (8.10) \]

where primes indicate perturbations. Substituting Eq. (8.10) into Eqs. (8.1)–(8.5) and using Eqs. (8.6)–(8.9), the perturbed equations are given by

\[ \nabla \cdot \mathbf{q}' = 0, \quad (8.11) \]

\[ \frac{\rho_0}{\varepsilon} \frac{\partial \mathbf{q}'}{\partial t} = -\nabla p' + \rho_0 (\beta_T T' - \beta_S S') g - \frac{1}{K} (\mu - \mu_c \nabla^2) \mathbf{q}', \quad (8.12) \]

\[ \gamma \frac{\partial T'}{\partial t} + (\mathbf{q}' \cdot \nabla) T' - \frac{\Delta T}{d} w' = \kappa_r \nabla^2 T' + QT', \quad (8.13) \]

\[ \varepsilon \frac{\partial S'}{\partial t} + (\mathbf{q}' \cdot \nabla) S' - \frac{\Delta S'}{d} w' = \kappa_S \nabla^2 S'. \quad (8.14) \]

By operating curl twice on Eq. (8.12) we eliminate \( p' \) from it and then render the resulting equation and the Eqs. (8.13) and (8.14) dimensionless using the following transformations

\[ (x', y', z') = (x^*, y^*, z^*) d, \quad (u', v', w') = (\kappa_r / d)(u^*, v^*, w^*), \]

\[ t' = t' \left( \frac{\gamma d^2}{\kappa_r} \right), \quad T' = (\Delta T^*) T^*, \quad S' = (\Delta S^*) S^*, \quad (8.15) \]

to obtain non-dimensional equations as (on dropping the asterisks for simplicity),

\[ \left( 1 - C_p \frac{\partial}{\partial t} + 1 - C_p \nabla^2 \right) \nabla^2 w - Ra_r \nabla^2 T + Ra_S \nabla^2 S = 0, \quad (8.16) \]

\[ \left( \frac{\partial}{\partial t} - \nabla^2 - Ri \right) + (\mathbf{V} \cdot \nabla) T + f(z) w = 0, \quad (8.17) \]

\[ \left[ \lambda \frac{\partial}{\partial t} - \frac{1}{Le} \nabla^2 + (\mathbf{V} \cdot \nabla) \right] S - w = 0, \quad (8.18) \]

where

\[ C_p = \mu_c / \mu d^2, \text{ couple stress parameter} \]
\[ Pr_D = \frac{\gamma \varepsilon v d^2}{\kappa_f K}, \text{ Darcy-Prandtl number} \]

\[ Ra_T = \frac{\beta_\gamma g \Delta T d K}{\nu \kappa_f}, \text{ thermal Rayleigh number} \]

\[ Ra_s = \frac{\beta_s g \Delta S d K}{\nu \kappa_f}, \text{ solute Rayleigh number} \]

\[ Ri = \frac{Q d^3}{\kappa_f}, \text{ internal Rayleigh number} \]

\[ \lambda = \varepsilon / \gamma, \text{ normalized porosity} \]

\[ Le = \frac{\kappa_r}{\kappa_s}, \text{ Lewis number} \]

and \( f(z) = \frac{\partial T_b}{\partial z} \). The thermal Rayleigh number characterizes the buoyancy due to the thermal gradient and that due to the solute gradient is indicated by the solute Rayleigh number. It is worth mentioning here that the Darcy-Prandtl number \( Pr_D \) includes the Prandtl number, Darcy number, porosity and the specific heat ratio. The Prandtl number affects the stability of the porous system through this combined dimensionless group. The Darcy-Prandtl number depends on fluid and on the nature of the porous matrix. We allowed the range of values for the Darcy-Prandtl number, i.e. a range of \([1, 50]\) is considered. The ratio between thermal and solute diffusivities is characterized by the Lewis number and can be varied in the range of \([1, 1000]\). The normalized porosity \( \lambda \) is expressed in terms of the porosity of the porous medium \( \varepsilon \) and the solid to fluid heat capacity ratio, \( \gamma \). Since \( 0 < \varepsilon < 1 \), it is clear that \( 0 < \lambda < 1 \).

The boundaries are assumed to be stress free, isothermal and isohaline; the Eqs. (8.16)–(8.18) are to be solved for the boundary conditions

\[ w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \text{ at } z = 0, 1. \quad (8.19) \]
8.3 Linear Stability Analysis

We predict the thresholds of both marginal and oscillatory convections using linear theory. The Eigen value problem defined by Eqs. (8.16)–(8.18) subject to the boundary conditions (8.19) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that the amplitudes of the perturbations are very small, we write

\[
\begin{pmatrix}
  w \\
  T \\
  S
\end{pmatrix} = \begin{pmatrix}
  W(z) \\
  \Theta(z) \\
  \Phi(z)
\end{pmatrix} \exp \left[ i (lx + my) + \sigma t \right],
\]

(8.20)

where \( l, m \) are horizontal wavenumbers and \( \sigma \) is the growth rate. Infinitesimal perturbations of the rest state may either dampen or grow depending on the value of the parameter \( \sigma \). Substituting Eq. (8.20) into the linearized version of Eqs. (8.16)–(8.18), we obtain

\[
\left[ \frac{\sigma}{Pr_D} + 1 - C_p (D^2 - a^2) \right] \left[ D^2 - a^2 \right] W + a^2 R a_T \Theta - a^2 R a_s \Phi = 0,
\]

(8.21)

\[
\left[ \sigma - \left( D^2 - a^2 \right) - Ri \right] \Theta + f(z) W = 0,
\]

(8.22)

\[
\left[ \lambda \sigma - L e^{-l} \left( D^2 - a^2 \right) \right] \Phi - W = 0,
\]

(8.23)

where \( D = d/dz \) and \( a^2 = l^2 + m^2 \). The boundary conditions (8.19) now becomes

\[
W = \frac{\partial^2 W}{\partial z^2} = \Theta = \Phi = 0 \text{ at } z = 0, 1.
\]

(8.24)

We assume the solutions of Eqs. (8.21)–(8.23) satisfying the boundary conditions (8.24) in the form

\[
\begin{pmatrix}
  W(z) \\
  \Theta(z) \\
  \Phi(z)
\end{pmatrix} = \begin{pmatrix}
  W_0 \\
  \Theta_0 \\
  \Phi_0
\end{pmatrix} \sin (n \pi z), \quad (n = 1, 2, 3, \ldots).
\]

(8.25)
The most unstable mode corresponds to \( n = 1 \) (fundamental mode). Therefore substituting Eq. (8.25) with \( n = 1 \) into Eqs. (8.21) – (8.23), we obtain a matrix equation

\[
\begin{pmatrix}
\sigma Pr_0^{-1} \delta^2 + \delta^2 (1 + C_p \delta^2) & -a^2 Ra_s & a^2 Ra_s \\
-2F & \sigma + \delta - Ri & 0 \\
-1 & 0 & \lambda \sigma + \delta^2 Le^{-1}
\end{pmatrix}
\begin{pmatrix}
W_0 \\
\Theta_0 \\
\Phi_0
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix},
\]  

(8.26)

where \( \delta^2 = \pi^2 + a^2 \), \( F = \frac{1}{b} \frac{dT}{dz} \) \( \sin^2(\pi z) \) \( dz \). The condition of a nontrivial solution of the above system of homogeneous linear Eq. (8.26) yields the expression for thermal Rayleigh number in the form

\[
Ra_s = \left( \frac{\sigma + \delta - Ri}{2F} \right) \left( \frac{\delta^2 (1 + C_p \delta^2)}{a^2 Pr_0} \right) + \left( \frac{Ra_s}{\lambda \sigma + \delta^2 Le^{-1}} \right).
\]

(8.27)

### 8.3.1 Marginal State

For the validity of the principle of exchange of stabilities (i.e., steady case), we have \( \sigma = 0 \) (i.e., \( \omega = \omega_l = 0 \)) at the margin of stability. Then the Rayleigh number at which the marginally stable steady mode exists becomes

\[
Ra_{St} = \left( \frac{\delta^2 - Ri}{2F} \right) \left( \frac{\delta^2 (1 + C_p \delta^2)}{a^2 Pr_0} \right) + \left( \frac{Le Ra_s}{\delta^2} \right),
\]

(8.28)

The minimum value of the Rayleigh number \( Ra_{St} \) occurs at the critical wavenumber \( a = a_{St} \) where \( a_{St} = \sqrt{h} \) satisfies the equation

\[
d_1 h^5 + d_2 h^4 + d_3 h^3 + d_4 h^2 + d_5 h + d_6 = 0,
\]

(8.29)

where

\[
d_1 = 2C_p, \ d_2 = 1 + 7 \pi^2 C_p - RiC_p, \\
d_3 = 2\pi^2 \left( 1 + 4 \pi^2 C_p - RiC_p \right), \ d_4 = 2\pi^6 C_p + \left( \pi^2 + Le Ra_s \right) Ri, \\
d_5 = -2\pi^4 \left( 1 + \pi^2 C_p \right) \left( \pi^2 - Ri \right), \ d_6 = -\pi^6 \left( 1 + \pi^2 C_p \right) \left( \pi^2 - Ri \right).
\]

In the absence of a heat source (i.e., when \( Ri = 0 \) and \( F = -1/2 \)) Eq. (8.28) reduces to
This result exactly coincides with the one given by Malashetty et al. (2010). For the absence of couple stresses; \( C_p = 0 \), the Eq. (8.30) reduces to

\[
Ra^S_t = \frac{\delta^4 \left(1 + C_p \delta^2 \right)}{a^2} + Le Ra_S, \quad (8.31)
\]

which is the classical result for double diffusive convection in a Darcy porous medium (see Nield and Bejan [2]). In case of a single component couple stress fluid system \( Ra_S = 0 \), equation (8.30) reduces to

\[
Ra^S_t = \frac{\left(\pi^2 + a^2\right)^{\frac{1}{2}} \left(\pi^2 + a^2\right)}{a^2}, \quad \tau = \frac{1}{Le}, \quad (8.32)
\]

This result was given by Siddeshwar and Pranash (2004). Further in the absence of couple stress, the expression for stationary Rayleigh number reduces to the classical result

\[
Ra^S_t = \frac{\left(\pi^2 + a^2\right)^{\frac{1}{2}}}{a^2}, \quad (8.33)
\]

which has the critical value \( Ra^S_c = 4\pi^2 \) for \( \alpha^S_c = \pi \) obtained by Horton and Rogers (1945) and Lapwood (1948).

### 8.3.2 Oscillatory State

We now set \( \omega = i \omega_1 \) in Eq. (8.27) and clear the complex quantities from the denominator, to obtain

\[
Ra_t = \Delta_1 + i \omega_1 \Delta_2, \quad (8.34)
\]

where

\[
\Delta_1 = \frac{\delta^2 \left(\delta^2 - R_i \right) \left(1 + C_p \delta^2 \right) Pr_0 - \omega^2}{2 F a^2 Pr}, \quad \Delta_2 = \frac{Ra_S Le \left(\delta^2 - R_i \right) \delta^2 + Le \lambda \omega^2}{2 F \left(\delta^4 + Le^2 \lambda^2 \omega^2 \right)}.
\]
\[ \Delta_2 = \frac{\delta^2 \left[ \left( \delta^2 - Ri \right) + \left( 1 + C_p \delta^2 \right) Pr_D \right]}{2F a^2 Pr_D} + \frac{Ra_s Le \left[ \delta^2 - Le \left( \delta^2 - Ri \right) \lambda \right]}{2F \left( \delta^4 + Le^2 \lambda^2 \omega^2 \right)}. \]

Since \( Ra_T \) is a physical quantity, it must be real. Hence, from Eq. (8.34) it follows that either \( \omega_i = 0 \) or \( \Delta_2 = 0 \) (\( \omega_i \neq 0 \), oscillatory onset). For oscillatory onset, setting \( \Delta_2 = 0 \) (\( \omega_i \neq 0 \)) gives an expression for frequency of oscillations in the form (on dropping the subscript \( i \))

\[ \omega^2 = \frac{a^2 Ra_s Pr_D \left( \lambda Le \left( \delta^2 - Ri \right) - \delta^2 \right)}{\left( \delta^2 - Ri \right) + \left( 1 + C_p \delta^2 \right) Pr_D} \frac{\delta^4}{\lambda^2 Le^2}. \]  

(8.35)

Now Eq. (8.34) with \( \Delta_2 = 0 \), gives

\[ Ra_T^{Osc} = \frac{\delta^2 \left[ \left( \delta^2 - Ri \right) \left( 1 + C_p \delta^2 \right) Pr_D - \omega^2 \right]}{2F a^2 Pr_D} + \frac{Ra_s Le \left[ \left( \delta^2 - Ri \right) \delta^2 + Le \lambda \omega^2 \right]}{2F \left( \delta^4 + Le^2 \lambda^2 \omega^2 \right)}. \]

(8.36)

The analytical expression for the oscillatory Rayleigh number given by Eq. (8.36) is minimized with respect to the wavenumber numerically, after substituting for \( \omega^2 \) (> 0) from Eq. (8.35), for various values of physical parameters in order to know their effects on the onset of stationary and oscillatory convection.

### 8.4 Finite Amplitude Analysis with Limited Representation

To obtain the information about the values of the convection amplitudes and also the rate of heat and mass transfer, we perform the nonlinear analysis, which is useful to understand the physical mechanism with a minimum amount of mathematics and is a step forward towards understanding the full nonlinear problem.

For simplicity of analysis, we confine ourselves to the two-dimensional rolls, so that all the physical quantities are independent of \( y \). We introduce stream function \( \psi \).
such that \( u = \partial \psi / \partial z \), \( w = - \partial \psi / \partial x \) into the Eq. (8.12), eliminate pressure and non-dimensionalize the resulting equation and Eqs. (8.13) and (8.14) using the transformations (8.15) to obtain

\[
\left( \frac{1}{Pr_D} \frac{\partial}{\partial t} + C_p \nabla^2 \right) \nabla^2 \psi + Ra_r \frac{\partial T}{\partial x} - Ra_s \frac{\partial S}{\partial x} = 0, \quad (8.37)
\]

\[
\left( \frac{\partial}{\partial t} - \nabla^2 - Ri \right) T - \frac{\partial (\psi_T)}{\partial (x, z)} - f(z) \frac{\partial \psi}{\partial x} = 0, \quad (8.38)
\]

\[
\lambda \frac{\partial}{\partial t} - Le^{-1} \nabla^2 S - \frac{\partial (\psi_S)}{\partial (x, z)} + \frac{\partial \psi}{\partial x} = 0. \quad (8.39)
\]

We will assume that, close to the threshold of convection, the basic circulation remains undisturbed but the temperature and concentration fields are distorted by the addition of a second harmonic with no \( x \)-dependence, so that

\[
\psi = A(t) \sin(\alpha x) \sin(\pi z), \quad (8.40)
\]

\[
T = B(t) \cos(\alpha x) \sin(\pi z) + C(t) \sin(2\pi z), \quad (8.41)
\]

\[
S = D(t) \cos(\alpha x) \sin(\pi z) + E(t) \sin(2\pi z), \quad (8.42)
\]

where the amplitudes \( A(t), B(t), C(t), D(t) \) and \( E(t) \) are to be determined from the dynamics of the system.

Substituting Eqs. (8.40)–(8.42) into Eqs. (8.37)–(8.39) and equating the coefficients of like terms we obtain the following non-linear autonomous system of differential equations

\[
\frac{dX}{dt} = L, \quad (8.43)
\]

where \( X = (A, B, C, D, E)^T \), \( L = (L_1, L_2, L_3, L_4, L_5)^T \)

with

\[
L_1 = -\frac{Pr_D}{\delta^2} \left[ \left( 1 + C_p \delta^2 \right) \delta^2 A + aRa_r B - aRa_s D \right],
\]

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The non-linear system of autonomous differential equations is not suitable to analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. However, one can make qualitative predictions as discussed below.

The system of Eq. (8.43) is uniformly bounded in time and possesses many properties of the full problem. Like the original equations (8.2)–(8.5), Eq. (8.43) must be dissipative. Thus volume in the phase space must contract. In order to prove volume contraction, we must show that velocity field has a constant negative divergence. Indeed,

\[
\frac{\partial}{\partial A} \left( \frac{dA}{dt} \right) + \frac{\partial}{\partial B} \left( \frac{dB}{dt} \right) + \frac{\partial}{\partial C} \left( \frac{dC}{dt} \right) + \frac{\partial}{\partial D} \left( \frac{dD}{dt} \right) + \frac{\partial}{\partial E} \left( \frac{dE}{dt} \right) = -\left[ Pr_{d} \left( 1 + C_{\rho} \delta^{2} \right) - 2 Ri + \delta^{2} + 4 \pi^{2} + \frac{\left( \delta^{2} + 4 \pi^{2} \right)}{\lambda Le} \right],
\]

which is always negative and therefore system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase space; in particular they may be attracted to a fixed point, a limit cycle or, perhaps, a strange attractor. From Eq. (8.44) we conclude that if a set of initial points in phase space occupies a region \( V(0) \) at time \( t = 0 \), then after some time \( t \), end points of the corresponding trajectories will fill a volume

\[
V(t) = V(0) \exp \left[ -\left( Pr_{d} \left( 1 + C_{\rho} \delta^{2} \right) - 2 Ri + \delta^{2} + 4 \pi^{2} + \frac{\left( \delta^{2} + 4 \pi^{2} \right)}{\lambda Le} \right) t \right],
\]

This expression indicates that the volume decreases exponentially with time. We can
also infer that, the large Darcy-Prandtl number, internal Rayleigh number and couple stresses and a very small Lewis number \((Le<1)\), normalized porosity tend to enhance dissipation. Finally we note that the system of Eq. (8.43) is invariant under the transformation \((A,B,D,E) \rightarrow (\pm A, \pm B, \pm D, \pm E)\).

### 8.4.1 Steady Finite Amplitude Motions

From qualitative predictions we look into the possibility of an analytical solution. In the case of steady motions, Eqs. (8.37)–(8.39) can be solved in closed form. The steady state solutions are useful because they predict that a finite amplitude solution to the system is possible for subcritical values of the Rayleigh number and that the minimum values of \(Ra_r\) for which a steady solution is possible lies below the critical values for instability to either a marginal state or an overstable infinitesimal perturbation. Setting the left hand side of Eq. (8.43) equal to zero, after eliminating all amplitudes except \(A\), gives

\[
A_1 x^2 + A_2 x + A_3 = 0,
\]

where

\[
A_1 = 4a^4 Le^2 \pi^2 \delta^2 \left(1 + \delta^2 C_p\right),
\]

\[
A_2 = a^2 \left(\frac{a^2 Le \left(4 \pi^2 \left(Ra_s - 2 FLeRa_r\right) + FLeRa_r R \right) + \delta^2}{\left(1 + \delta^2 C_p \right) \left(4 \pi^2 \delta^2 - Le^2 \left(4 \pi^2 - R \right) \left(R - \delta^2 \right) \right)}\right),
\]

\[
A_3 = \left(R - 4 \pi^2 \right) \left(\delta^4 \left(R - \delta^2 \right) \left(1 + \delta^2 C_p \right) + a^2 \left(2 FLeRa_r \delta^2 + LeRa_s \left(R - \delta^2 \right) \right) \right).
\]

The required root of Eq. (4.46) is

\[
x = \frac{1}{2 A_1} \left(-A_2 + \left(A_2^2 - 4 A_1 A_3\right)^{1/2}\right).
\]

When we let the radical in the above equation vanish, we obtain the expression for the
finite amplitude Rayleigh number $Ra^F_r$, which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form

$$Ra^F_r = \frac{1}{2B_1}\left(-B_2 + \left(B_2^2 - 4B_1B_3\right)^{1/2}\right),$$

(8.48)

where

$$B_1 = 4\alpha^8 Le^4 F^2 \left(Ri - 4\pi^2\right)^2,$$

$$B_2 = 4\alpha^8 Le^2 F \left(4\pi^2 - Ri\right) \left(-4\alpha^2 \pi^2 LeRa_s + \delta^2 \left(1 + \delta^2 C_p \right) \left(4\pi^2 \delta^2 + Le^2 \left(4\pi^2 - Ri\right) \left(Ri - \delta^2\right)\right)^2\right),$$

$$B_3 = \alpha^4 \left(4\alpha^2 Le^2 Ra_s + \delta^2 \left(1 + \delta^2 C_p \right) \left(4\pi^2 \delta^2 + Le^2 \left(4\pi^2 - Ri\right) \left(Ri - \delta^2\right)\right)\right)^2.$$

### 8.4.2 Unsteady Finite Amplitude Motions

In this section the behavior of non-linear, periodic solutions are investigated with the object of understanding the transition from periodic oscillations to behavior that is apparently chaotic (i.e. solutions are a periodic and depend sensitively on the initial conditions) and to predict the amount of heat and mass transfer. For this purpose, we have solved the autonomous system of non-linear ordinary differential equation (8.43) numerically using the Runge-Kutta method with suitable initial conditions for different values of critical finite amplitude Rayleigh number and the expressions for $Nu$ and $Sh$ are computed as the function of time $t$ and the results are discussed in section 5.

### 8.5 Heat and Mass Transport

In the study of convection in fluids, the quantification of heat and mass transport is important. This is because the onset of convection, as the Rayleigh number is increased, is more readily detected by its effect on the heat and mass transport. In the
basic state, heat and mass transport is by conduction alone. If \( H \) and \( J \) are the rate of heat and mass transport per unit area respectively, then

\[
H = -\kappa_T \left\langle \frac{\partial T_{\text{total}}}{\partial z} \right\rangle_{z=0} \quad \text{and} \quad J = -\kappa_S \left\langle \frac{\partial S_{\text{total}}}{\partial z} \right\rangle_{z=0},
\]

where the angular bracket corresponds to a horizontal average and

\[
T_{\text{total}} = T_0 - \Delta T \frac{z}{d} + T(x,z,t) \quad \text{and} \quad S_{\text{total}} = S_0 - \Delta S \frac{z}{d} + S(x,z,t).
\]

Substituting Eqs. (8.41) and (8.42) in Eq. (8.50) and using the resultant equations in Eq. (8.49), we get

\[
H = \frac{\kappa_T \Delta T}{d} (1 - 2\pi C) \quad \text{and} \quad J = \frac{\kappa_S \Delta S}{d} (1 - 2\pi E).
\]

The Nusselt and Sherwood numbers are respectively defined by

\[
Nu = \frac{H}{\kappa_T \Delta T/d} = (1 - 2\pi C),
\]

\[
Sh = \frac{J}{\kappa_S \Delta S/d} = (1 - 2\pi E).
\]

Writing \( C \) and \( E \) in terms of \( A \), we obtain

\[
Nu = 1 + \frac{16 F \pi^2 a^2 x}{\left( \delta^2 - \frac{\Delta T}{d} \right) \left( 4\pi^2 - \frac{\Delta S}{d} \right) + 4\pi^2 a^2 x^2},
\]

\[
Sh = 1 + \frac{2a^2 Le^2 x}{\left( \delta^2 + a^2 Le^2 x \right)}.
\]

The second terms on the right-hand side of Eqs. (8.54) and (8.55) represent the convective contribution to heat and mass transport respectively.

### 8.6 Results and Discussion

The onset of double diffusive convection in a porous layer saturated with couple
stress fluid in the presence of an internal heat source is analyzed using both linear and weakly nonlinear stability theories. The linear theory is based on the classical normal mode technique and the non-linear theory on the truncated Fourier series method. The expressions for stationary, oscillatory and finite-amplitude Rayleigh numbers for different values of the internal Rayleigh number, couple-stress parameter, solute Rayleigh number, normalized porosity, Darcy-Prandtl number and Lewis number are computed and the results are depicted in figures.

The neutral stability curves in the $Ra_i - a$ plane for various parameter values are as shown in figures 8.1-8.6. We fixed the values for different parameters except the varying parameter. From these figures it is clear that the neutral curves are connected in a topological sense. This connection allows the linear stability criteria to be expressed in terms of the critical Rayleigh number $Ra_{c_i}$, below which the system is stable and unstable above. Figure 8.1 depicts the effect of internal Rayleigh number $Ri$ on the neutral stability curves for stationary and oscillatory modes for the fixed values of other parameters. It is observed that increasing $Ri$ decreases the critical value of both stationary and oscillatory Rayleigh number, indicating that the effect of increasing $Ri$ is to destabilize the onset of both stationary and oscillatory convection. Since an increase in the internal Rayleigh number amounts to increase in energy supply to the system. This addition of energy in turn improves the disturbances in the layer and thus system is more unstable. Further, it is noticed that the convection sets in via oscillatory prior to stationary mode for the parameter values chosen for this figure. In figure 8.2 effect of couple stress parameter $C_p$ on the neutral stability curves for fixed values of other parameters is displayed. It is found from this figure that the minimum value of the Rayleigh number for both stationary and oscillatory modes increases with an increase in the value of $C_p$, indicating that the effect of couple stress parameter is to stabilize
The effect of the Lewis number $Le$ on the neutral stability curves for fixed values of other parameters is shown in figure 8.3. It is observed that the minimum value of Rayleigh number for stationary mode increases with increasing $Le$. From the Eq. (8.28) and also by the definition of the Lewis number, as heat diffuses more rapidly than solute one can say that with increase of $Le$ the stationary Rayleigh number increases. On the other hand, the oscillatory Rayleigh number decreases with an increase in the value of $Le$. This is because when $Le > 1$, the diffusivity of heat is more than diffusivity of solute, and therefore, solute gradient augments the onset of oscillatory convection. The result reflects the fact that the cause of oscillatory instability is the difference in the rates of diffusion of heat and solute. Thus the Lewis number has a contrasting effect on the stability of the system in the case of stationary and oscillatory modes.

Figure 8.4 displays the effect of solute Rayleigh number on the neutral stability curves for fixed values of other parameters. This figure indicates that the minimum Rayleigh number increases with an increase in the value of the solute Rayleigh number for both stationary and oscillatory mode, indicating that it has stabilizing effect on double diffusive convection in a porous layer. The effect of Darcy-Prandtl number $Pr_D$ on the oscillatory neutral stability curves for the fixed values of other parameters is shown in figure 8.5. It is observed that increasing $Pr_D$ increases the minimum of the oscillatory Rayleigh number, implying that the Darcy-Prandtl number stabilizes the system. The effect of normalized porosity $\lambda$ is depicted in figure 8.6 for fixed values of other parameters. From this figure it is found that an increase in $\lambda$ decreases the minimum of the Rayleigh number for oscillatory state, indicating that the effect of increasing $\lambda$ is to advance the onset of oscillatory convection. As normalized porosity
advective behavior in the terminology of Philips (2009)) is reduced. This makes advective heat transfer more effective and so makes it easier for the destabilizing thermal buoyancy gradient to produce convection.

The detailed behavior of critical Rayleigh number with respect to the solute Rayleigh number is analyzed in the $Ra_{tc} - Ra_s$ plane through figures 8.7-8.11. The variation of $Ra_{tc}$ with $Ra_s$ for different values of $Ri$ is depicted in the figure 8.7. It is found that with an increase in the value of $Ri$ the critical Rayleigh number decreases for both stationary and oscillatory modes. Thus the effect of $Ri$ is to destabilize the system for both stationary and oscillatory modes. The variation of $Ra_{tc}$ with $Ra_s$ for different values of couple stress parameter $C_p$ is shown in figure 8.8. It is observed that the critical Rayleigh number increases with increasing $C_p$ for both stationary and oscillatory modes, indicating that the couple stress parameter stabilizes the system. Further it is noticed that convection sets in via oscillatory prior to stationary mode.

Figure 8.9 displays the variation of $Ra_{tc}$ with $Ra_s$ for different values of the Lewis number $Le$. It is observed that increasing $Le$ increases the critical Rayleigh number for stationary mode, indicating that the effect of $Le$ is to stabilize the system in a stationary mode. On the other hand, the critical Rayleigh number for the oscillatory mode decreases with the increase of $Le$, indicating that the effect of $Le$ is to destabilize the system in the oscillatory mode. Further, this figure also reveals that the effect of $Le$ is more significant for stationary mode rather than oscillatory mode.

The variation of $Ra_{tc}$ with $Ra_s$ for different values of $Pr_D$ is presented in figure 8.10. It is seen that the critical oscillatory Rayleigh number increases with an increase in the value of $Pr_D$, indicating that the effect of the Darcy-Prandtl number is to delay the onset of oscillatory convection. Figure 8.11 shows the effect of normalized
porosity $\lambda$ on the oscillatory critical Rayleigh number, when all other parameters are kept fixed. It is noticed that the oscillatory critical Rayleigh number decreases with an increase of $\lambda$. Thus the effect of $\lambda$ is to advance the onset of oscillatory convection.

In the study of double diffusive convection the determination of heat and mass transport across the layer plays a very important role. Here, the onset of convection as the Rayleigh number is increased is more rapidly detected by its effect on the heat and mass transfer. The quantity of heat and mass transfer across the layer are given by Nusselt number $Nu$ and Sherwood number $Sh$ respectively, which represent the ratio of heat and mass transfer across the layer to the heat or mass transported by conduction alone. Figures 8.12-8.15 indicate the effect of various parameters on $Nu$ and $Sh$. In each of these cases it is observed that as Rayleigh number increases from one to four times of its critical value, the heat and mass transfer increase sharply and as Rayleigh number is increased further, they remain almost constant. It is also found that in each case the Sherwood number is greater than the Nusselt number. Figure 8.12 indicates that the effect of increasing internal Rayleigh number is to increase the value of $Nu$ and is to decrease $Sh$. Figure 8.13 shows the effect of increasing the couple-stress parameter is to decrease the values of $Nu$ and $Sh$. From figure 8.14 it is observed that the effect of the solute Rayleigh number is to enhance the heat and mass transport. Although the presence of stabilizing gradient of solute will inhibit the onset of convection, due to the strong finite-amplitude motions, which exist for large Rayleigh numbers, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. As a consequence of that the inhibiting effect of solute gradient is greatly reduced and hence fluid will convect more and more heat and mass when $Ra_s$ is increased. It is also found from figure 8.15 that increasing $Le$ increases $Nu$ as well as $Sh$, indicating that the effect of $Le$ is to enhance the heat and mass transport.
The autonomous system of unsteady finite amplitude equations is solved numerically using the Runge-Kutta method with suitable initial conditions. Then \( Nu \) and \( Sh \) are evaluated as a function of time \( t \). The unsteady transient behavior of \( Nu \) and \( Sh \) is shown graphically through figures 8.16 and 8.17 for different values of \( Ri \) and \( C_p \) respectively. It is observed that both \( Nu \) and \( Sh \) start with a conduction state value (i.e. 1) at \( t = 0 \) and then oscillate periodically about their steady state value (i.e. close to 3) for \( t > 0 \). This periodic variation of \( Nu \) and \( Sh \) is very short lived and decays as time progresses. In other words, as time progresses a steady state is reached via a transient state. Figure 8.16 indicates the effect of internal Rayleigh number \( Ri \) on transient heat and mass transfer. It is interesting to note that increasing \( Ri \) initially decreases \( Nu \) and \( Sh \) however, with the lapse of time this effect is reversed and further as time progresses it increases the amplitude of both heat and mass transfer. Figure 8.17 reveals the effect of increasing couple stress parameter is to decrease the amplitude of the oscillations of heat and mass flux.
Figure 8.1 Neutral stability curves for different values of the internal Rayleigh number $R_i$.

Figure 8.2 Neutral stability curves for different values of couple stress parameter $C_p$. 

$C_p = 3, Ra_s = 100, Le = 10, \lambda = 0.5, Pr_D = 10$
Figure 8.3 Neutral stability curves for different values of the Lewis number $Le$.

Figure 8.4 Neutral stability curves for different values of the solute Rayleigh number $Ra_s$. 

$C_p = 3, Ra_s = 100, Ri = 3, \bar{\lambda} = 0.5, Pr_D = 10$.
Figure 8.5 Neutral stability curves for different values of the Darcy Prandtl number $Pr_D$.

Figure 8.6 Neutral stability curves for different values of normalized porosity $\lambda$. 

$C_p = 3, Ra_s = 100, Le = 10, \lambda = 0.5, Ri = 3$ 

$Pr_D = 1$
Figure 8.7 Variation of the critical Rayleigh number with $Ra_s$ for different values of $Ri$.

$C_p = 3, Ra_s = 100, Le = 10, \lambda = 0.5, Pr_D = 10$

Figure 8.8 Variation of the critical Rayleigh number with $Ra_s$ for different values of $C_p$.

$Ri = 3, Ra_s = 100, Le = 10, \lambda = 0.5, Pr_D = 10$

$C_p = 2$
Figure 8.9 Variation of the critical Rayleigh number with $Ra_s$ for different values of $Le$.

Figure 8.10 Variation of the critical Rayleigh number with $Ra_s$ for different values of $Pr_D$.
Figure 8.11 Variation of the critical Rayleigh number with $Ra_s$ for different values of $\lambda$.

Figure 8.12 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $Ri$. 

$\nu = 3, Ra_s = 100, Le = 10,
Ri = 3, Pr_n = 10$

$C_p = 2, Ra_s = 100, Le = 3$
Figure 8.13 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $C_p$.

Figure 8.14 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $Ra_s$. 

$Nu$ and $Sh$
Figure 8.15 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $Le$. 

$Ra_T/Ra_{rc}$

$Nu$ and $Sh$

$Le = 3, 4, 5$

$Ri = 3, Ra_s = 100, C_p = 2$
Figure 8.16 Variation of $Nu$ and $Sh$ with time for different values of $Ri$

$C = 1, Ra_s = 100, \lambda = 0.7,$
$Pr_D = 10, Le = 2, Ra_r = 8 Ra_{rc}^F$
Figure 8.17 Variation of $Nu$ and $Sh$ with time for different values of $C_P$.

$Ri = 3$, $Ra_s = 100$, $\lambda = 0.7$, $Pr_D = 10$, $Le = 2$, $Ra_T = 8 Ra_{Fr}$.
Chapter 9
General Conclusions

In this thesis the convective instabilities in Newtonian and non-Newtonian fluid (Maxwell fluid, viscoelastic fluid and couple stress fluid) saturated porous layer under the influence of external rotation, anisotropy of porous layer, cross diffusion (Soret effect and Dufour effect) and internal heat source have been investigated.

In chapter - 3, the linear and nonlinear stability analysis of double diffusive convection in a rotating horizontal anisotropic porous layer with Soret effect is performed. The linear theory is based on the usual normal mode technique and the nonlinear theory on the truncated Fourier series analysis. The Darcy model extended to include time derivative and Coriolis terms with anisotropic permeability is used to describe the flow through porous media and the following conclusions are drawn:

- The effect of increasing the values of mechanical and thermal anisotropy parameters in the presence of rotation and Soret effect is to stabilize the onset of convection to be oscillatory rather than stationary. The effect of the mechanical anisotropy parameter is more pronounced compared to that of the thermal anisotropy.

- The effect of rotation is to stabilize the onset of both stationary and oscillatory convection. However, the oscillatory mode is most favorable for a system with moderate and high values of the Taylor number.

- The solute Rayleigh number stabilizes the system for both stationary and oscillatory mode. The Lewis number stabilizes the system for stationary mode and destabilizes the system for oscillatory mode.
The positive Soret parameter destabilizes the system while negative Soret parameter stabilizes the system in both stationary and oscillatory convection.

The effect of increasing the Darcy-Prandtl number is to advance the onset of oscillatory convection.

The effect of increasing the normalized porosity parameter is to inhibit the onset of oscillatory convection.

The thermal Nusselt number and the Sherwood number decrease with an increase of mechanical anisotropy parameter and Taylor number whereas the thermal Nusselt thermal and the Sherwood number increase with an increase of the thermal anisotropy parameter and the solute Rayleigh number.

The Soret parameter suppresses the heat transport while the mass transport is reinforced by it.

In chapter - 4, the cross diffusion effects namely Soret and Dufour effects on the onset of double diffusive convection in a rotating anisotropic porous layer is studied using both linear and non-linear stability analyses. The normal mode technique is used in the linear analysis, while non-linear analysis is based on a minimal representation of the double Fourier series and the following conclusions are drawn:

The effect of increasing the values of mechanical anisotropy parameter in the presence of rotation and cross-diffusion is to destabilize the onset of the oscillatory convection.

The effect of Taylor number is to stabilize the onset of the oscillatory convection.

The solute Rayleigh number stabilizes the system for the oscillatory mode.
The effect of increasing the Darcy-Prandtl number and normalized porosity parameter is to inhibit the onset of an oscillatory convection.

The positive as well as negative Soret parameter stabilizes oscillatory convection.

The Dufour parameter has stabilizing effect on the double diffusive convection in a rotating anisotropic porous medium.

The thermal Nusselt number and the Sherwood number decrease with an increase of mechanical anisotropic parameter and Taylor number while both increase with an increase of solute Rayleigh number.

The Soret parameter suppresses the heat transport while the mass transport is reinforced by it.

The effect of Dufour parameter is to enhance the heat and mass transport.

In chapter - 5, the effect of rotation on the onset of double-diffusive convection in a sparsely packed anisotropic porous layer, in the presence of Soret effect is investigated analytically using the linear and nonlinear stability theories. The usual normal mode technique is used to solve the linear problem. The truncated Fourier series method is used to make the finite amplitude analysis. The following conclusions are drawn:

- The mechanical anisotropy parameter has stabilizing effect on stationary and oscillatory modes. However, the convection sets in as oscillatory mode prior to the stationary mode.

- The effect of thermal anisotropy parameter is to inhibit the onset of stationary and oscillatory convection.
The Taylor number has a stabilizing effect on the double diffusive convection in sparsely packed anisotropic porous medium.

The effect of Darcy number is to inhibit the onset of stationary convection while it has dual effect on oscillatory convection.

The effect of solute Rayleigh number is to delay both stationary and oscillatory convection. And the effect of Lewis number is to delay the onset of stationary convection while it advances the oscillatory convection.

The effect of normalized porosity is to advance the onset of oscillatory convection. And the Darcy Prandtl number has a dual effect on the oscillatory mode.

The Soret parameter has stabilizing effect on oscillatory convection and destabilizing effect on stationary convection.

The effect of mechanical anisotropy parameter and Taylor number is to reduce the heat and mass transport.

The heat and mass transport is reinforced by the thermal anisotropy parameter. The Darcy number and Soret parameter suppresses the heat transport while mass transport is reinforced by it.

In chapter - 6, the onset of double diffusive convection in a Maxwell fluid saturated anisotropic porous layer in the presence of the Soret effect is investigated analytically using both linear and nonlinear theories. The normal mode technique is used to solve the linear problem. The truncated Fourier series method is used to carry out the finite amplitude analysis and the important conclusions are summarized as follows:
The effect of relaxation parameter is to destabilize the system for oscillatory mode.

The effect of increasing both the mechanical and thermal anisotropy parameters is to advance the oscillatory convection.

The negative Soret coefficient has destabilizing effect, whereas the positive Soret coefficient has a stabilizing effect.

The heat transfer decreases with an increase of the Soret parameter and thermal anisotropy parameter while mass transfer increases with an increase of the Soret parameter and thermal anisotropy parameter.

The effect of mechanical anisotropy parameter and Lewis number is to enhance the heat and mass transport.

The transient behavior of the Nusselt and Sherwood numbers approach the steady state values as time progresses.

In chapter - 7 the onset of Darcy-Brinkman convection in a horizontal, sparsely packed porous layer saturated with a binary viscoelastic fluid with internal heat source is studied analytically using both linear and non-linear stability theories. The usual normal mode technique is used to solve the linear problem. The truncated Fourier series method is used to make the finite amplitude analysis. The following important conclusions are drawn:

The effect of increasing relaxation parameter is to advance the onset of oscillatory convection while increasing retardation parameter delays the onset of oscillatory convection.

The internal Rayleigh number has a destabilizing effect on the double diffusive convection in a porous medium.
The Darcy number and solute Rayleigh number have stabilizing effect on the system for stationary and oscillatory modes.

The Darcy-Prandtl number and normalized porosity have destabilizing effect on the system for oscillatory mode.

The effect of Lewis number is to advance the onset of oscillatory convection whereas its effect is to inhibit the stationary onset.

The effect of Darcy number is to suppress the heat and mass transport.

The heat transport is reinforced while the mass transport is suppressed by internal Rayleigh number.

The effect of solute Rayleigh number is to enhance the heat and mass transport.

The transient behavior of Nusselt and Sherwood numbers approach the steady state values (that of the Newtonian binary fluid) as time progresses.

In chapter - 8, the onset of double diffusive convection in a couple stress fluid saturated horizontal porous layer with an internal heat source is studied analytically using linear and weak non-linear stability theories. The classical normal mode technique is used to solve the linear problem. The truncated Fourier series method is used to make the finite amplitude analysis. The effect of various parameters on neutral stability curves as well as heat and mass transfer are discussed. The following conclusions are drawn:

- The internal Rayleigh number has a destabilizing effect on the double diffusive convection in a porous medium.
- The couple stress parameter and solute Rayleigh number have a stabilizing effect on the stationary and oscillatory convection.
The effect of the Lewis number is to inhibit the onset of stationary convection whereas its effect is to advance the oscillatory onset.

The Darcy-Prandtl number has stabilizing effect while normalized porosity has destabilizing effect on the system for oscillatory mode.

The effect of couple stress parameter is to suppress the heat and mass transport.

The heat transport is reinforced while the mass transport is suppressed by the internal Rayleigh number.

The effect of the solute Rayleigh number and the Lewis number is to enhance the heat and mass transport.

The transient behavior of the Nusselt and Sherwood numbers approach the steady state values as time progresses.