Chapter 6

Soret Effect on Convection in a Binary Maxwell Fluid Saturated Anisotropic Porous Layer

6.1 Introduction

Natural convection in porous media is a subject of considerable interest in contemporary fluid flow and heat transfer research. Its importance stems from a wide range of occurrences in industrial applications and geological systems. Among the applications in engineering discipline one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. From a purely scientific point of view, porous convection is also of great interest because it is one of the simplest systems exhibiting nonlinear instability.

As some new technologically significant materials are discovered acting like non-Newtonian fluids therefore mathematicians, physicists and engineers are actively conducting research in rheology. Maxwell fluids also can be considered as a special case of a Jeffreys-Oldroyd B fluid, which contain relaxation and retardation time coefficients. Maxwell’s constitutive relation can be recovered from that corresponding to Jeffreys-Oldroyd B fluids by setting the retardation time to be zero. Several fluids such as glycerin, crude oils or some polymeric solutions, behave as Maxwell fluids.

The studies of non-Newtonian fluids have received considerable attention

Part of this chapter has been published in the International Journal of Applied Mathematics and Mechanics, 10 (11): 37-64, 2014.
because of numerous applications in industry, geophysics and engineering. Some investigations are notably important in industries such as food stuff, personal care products, textile coating and suspension solutions. A great number of applications in geophysics may be found in the books by Phillips (2009), Ingham and Pop (1998), Nield and Bejan (2013). Interest in viscoelastic fluid flows through porous media has also grown considerably, due to the large demands of such diverse areas as biorheology, geophysics, chemical and petroleum industries, see in Hayat et al. (2007), Khaled and Vafai (2003), Younes (2003), Kumar and Mohan (2012). The mathematical model of Maxwell fluid has been served as a simplified description of dilute polymeric solutions/fluids by Raikher and Rusakov (1996) and Speziale (2000). Cui et al. (2010) studied Borehole guided waves non-Newtonian (Maxwell) fluid-saturated porous medium. On the linear stability of a Maxwell fluid with double-diffusive convection has been given by Awad et al. (2010). An extended Maxwell fluid model in terms of dimensionless relaxation time in polymeric non-Newtonian liquids motion is given by Boubaker (2012). Recently Vieru and Zafar (2013) made an attempt to some Couette flows of a Maxwell fluid with wall slip condition.

Further, in isotropic media there are two well defined relaxation functions, describing pure dilatational and shear deformations of the medium. The problem in anisotropic media is to obtain the time dependence of the relaxation components with a relatively reduced number of parameters. Most of the studies have usually been concerned with homogeneous isotropic porous structures. In a porous medium, due to the structure of the solid material in which the fluid flows, there can be a pronounced anisotropy in such parameters as permeability or thermal diffusivity. The geological and pedological processes rarely form isotropic medium as is usually assumed in transport studies. In geothermal system with a ground structure composed of many
strata of different permeabilities, the overall horizontal permeability may be up to ten times as large as the vertical component. Process such as sedimentation, compaction, frost action and reorientation of the solid matrix are responsible for the creation of anisotropic natural porous medium. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process, fiber materials used in insulating purposes. The novelties introduced by anisotropy have only recently been studied. Rees and Postelnicu (2001) investigated the onset of convection in an inclined anisotropic porous layer. Rees and Storesletten (2002) reported that transverse anisotropy has no effect on the identity of the preferred mode of convection whenever the anisotropy parameter is less than unity, while for greater than unity, there always exists a range of surface inclinations where transverse rolls are preferred. Malashetty and Swamy (2010) reported the onset of convection in binary fluid saturated anisotropic porous layer. The onset of double diffusive reaction-convection in anisotropic porous layer is well documented by Malashetty and Biradar (2011).

The mass fluxes can be created by temperature gradient that is called as the Soret effect. Barten et al. (1995) have observed the non-linear traveling wave and stationary onset for the negative values of the Soret coefficient in the mixtures of binary fluids. They observed the oscillatory convection in a finite container containing the binary fluid systems such as He$_3$-He$_4$ and water-ethanol with realistic boundary conditions. They used the linear stability analysis to find the criteria for the onset of oscillatory convection. A study of convective instability in a fluid mixture heated from above with negative separation ratio (Soret coefficient) was performed experimentally by La Porta and Surko (1998). Although the linear analysis predicts that the instability occurs at a zero wavenumber, a large wavenumber pattern is observed. The onset is supercritical and convection amplitude exhibits damped oscillations for sudden change
in the forcing parameters. Bourich et al. (2004) have given an exhaustive review of literature on the Soret effect convection either in fluid or porous media. An analytical study of linear and non-linear double diffusive convection with the Soret effect, in couple stress liquids is given by Malashetty et al. (2006) and in a fluid saturated anisotropic porous layer is given by Gaikwad et al. (2009). Soret effect on thermosolutal convection developed in a horizontal shallow porous layer salted from below and subject to cross fluxes of heat is studied by Mansour et al. (2008).

Recently, Wang and Tan (2011) studied the stability analysis of Soret-driven double diffusive convection of Maxwell fluid in a porous medium. Cross-diffusion effects namely the Soret and the Dufour effects on heat and mass transfer in a composite porous media is investigated by Bahadori (2012). Also Gaikwad and Biradar (2013) studied the onset of double diffusive convection in a Maxwell fluid saturated porous layer. Although some literature on double diffusive convection in a porous medium saturated by ordinary fluid with or without anisotropy is available, but attention has not been given to the Soret effect in a Maxwell fluid saturated porous medium. Therefore in the present chapter, we intend to perform a linear and weakly non-linear stability analysis of a binary Maxwell fluid saturated anisotropic porous layer in the presence of the Soret effect. The objective of this unit is to study how the onset criterion for oscillatory convection is affected by the Soret parameter, relaxation parameter, and anisotropy parameters and also to know the effect of these parameters on heat and mass transfer.

6.2 Mathematical Formulation

Consider a horizontal anisotropic porous layer saturated with binary Maxwell fluid of an infinite extent confined between the planes \( z = 0 \) and \( z = d \), with the
vertically downward gravity force $g$ acting on it. A constant temperatures $\Delta T + T_0$ and $T_0$ with stabilizing concentrations $\Delta S + S_0$ and $S_0$ respectively are maintained between the lower and upper surfaces. A Cartesian frame of reference is chosen with the origin in the lower boundary and the $z$-axis vertically upwards. The porous medium is assumed to possess horizontal isotropy in both thermal and mechanical properties. The generalized Darcy–Maxwell model is employed as a momentum equation. We also note that we are restricting our study to liquids and hence the Dufour effect is negligible. With the Oberbeck–Boussinesq approximation, the basic governing equations are

$$\nabla \cdot \mathbf{q} = 0, \quad (6.1)$$

$$\left(1 + \frac{\lambda_1}{\bar{\lambda}_1} \frac{\partial}{\partial t}\right) \left(\frac{\rho_0}{\varepsilon} \frac{\partial \mathbf{q}}{\partial t} + \nabla p - \rho g\right) = -\mu \mathbf{K} \cdot \mathbf{q}, \quad (6.2)$$

$$\gamma \frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{q} \cdot \nabla) T = \nabla \cdot \left(\mathbf{k}_T \cdot \nabla T\right), \quad (6.3)$$

$$\varepsilon \frac{\partial S}{\partial t} + \nabla \cdot (\mathbf{q} \cdot \nabla) S = \kappa_S \nabla^2 S + \kappa_{ST} \nabla^2 T, \quad (6.4)$$

$$\rho = \rho_0 \left[1 - \beta_T (T - T_0) + \beta_s (S - S_0)\right], \quad (6.5)$$

where $\mathbf{q}$ is the velocity, $\bar{\lambda}_1$ is the stress-relaxation time, $p$ is the pressure, $\mu$ is the fluid viscosity, $\mathbf{K}$ is the inverse anisotropic permeability tensor, $T$ is the temperature, $S$ is the solute concentration, $\kappa_{ST}$ is the Soret coefficient, $\mathbf{k}_T$ is the anisotropic thermal diffusion tensor. $\varepsilon, \rho, \beta_T$ and $\beta_s$ denote porosity, density, thermal and solute expansion coefficients respectively, and $\kappa_s$ is the solute diffusivity. It is hereby stated that permeability and thermal diffusion are most strongly anisotropic than solute diffusivity. Therefore, we ignore the solute anisotropy. Unfortunately, we have no experimental support for this because measurement of anisotropic diffusivity is lacking.
Further, \( \gamma = \frac{(\rho c)_m}{(\rho c)_f} \), \( (\rho c)_m = (1-\varepsilon)(\rho c)_f + \varepsilon(\rho c)_p \), \( c \) is the specific heat of the solid, \( c_p \) is the specific heat of the fluid at constant pressure, the subscripts \( f, s \) and \( m \) denote fluid, solid and porous medium values respectively.

6.2.1 Basic State

The basic state of the fluid is assumed to be quiescent and is given by,

\[ q_b = (0, 0, 0), \quad p = p_b(z), \quad T = T_b(z), \quad S = S_b(z), \quad \rho = \rho_b(z). \quad (6.6) \]

Using Eq. (6.6) into Eqs. (6.1)–(6.5) one can obtain

\[ \frac{dp_b}{dz} = -\rho_b g, \quad \frac{dT_b}{dz} = 0, \quad \frac{d^2 S_b}{dz^2} = 0, \quad \rho_b = \rho_0 \left[1 - \beta_T (T_b - T_0) + \beta_S (S_b - S_0)\right]. \quad (6.7) \]

Then the conduction state temperature and concentration are given by

\[ T_b(z) = \Delta T \left(1 - \frac{z}{d}\right) + T_0, \quad S_b = \Delta S \left(1 - \frac{z}{d}\right) + S_0. \quad (6.8) \]

6.2.2 Perturbed State

On the basic state we superpose infinitesimal perturbations in the form

\[ q = q_b + q', \quad p = p_b(z) + p', \quad T = T_b(z) + T', \quad S = S_b(z) + S', \quad (6.9) \]

where primes indicate perturbations. Substituting Eq. (6.9) into Eqs. (6.1)–(6.5) and using Eqs. (6.6)–(6.8), the perturbed equations are given by

\[ \nabla \cdot q' = 0, \quad (6.10) \]

\[ \left(1 + \frac{\gamma}{\varepsilon} \frac{\partial}{\partial t}\right) \left(\frac{\rho_b}{\varepsilon} \frac{\partial q'}{\partial t} + \nabla p' + \rho_b \left(\beta_T T' - \beta_S S'\right) \mathbf{g}\right) = -\mu \mathbf{K} \cdot q', \quad (6.11) \]

\[ \gamma \frac{\partial T'}{\partial t} + (q' \cdot \nabla) T' + w' \frac{dT_b}{dz} - \mathbf{K} \cdot \nabla T', \quad (6.12) \]

\[ \varepsilon \frac{\partial S'}{\partial t} + (q' \cdot \nabla) S' + w' \frac{dS_b}{dz} = \kappa_T \nabla^2 S' + \kappa_{ST} \nabla^2 T', \quad (6.13) \]

By operating curl twice on Eq. (6.11), we eliminate \( p' \) from it and then render the
resulting equation and the Eqs. (6.12) and (6.13) dimensionless using the following transformations

\[
(x', y', z') = (x^*, y^*, z^*) d, \quad t' = t^*(\gamma d^2 / \kappa_T), \quad (u', v', w') = (\kappa_T / d)(u^*, v^*, w^*),
\]

\[
T' = (\Delta T) T^*, \quad S' = (\Delta S) S^*,
\]

(6.14)

to obtain non-dimensional equations as (on dropping the asterisks for simplicity),

\[
\left(1 + \lambda_i \frac{\partial}{\partial t}\right) \left(1 + \frac{\partial}{\partial t} \frac{\partial}{\partial z^2}\right) + \nabla^2 + \frac{\partial^2}{\partial z^2} \right) w + \left(1 + \lambda_i \frac{\partial}{\partial t}\right) \left(-RaT \nabla^2 T + RaS \nabla^2 S \right) = 0,
\]

(6.15)

\[
\left[ \frac{\partial}{\partial t} - \left(\eta \nabla^2 + \frac{\partial^2}{\partial z^2} \right) + \mathbf{q} \cdot \nabla \right] T - w = 0,
\]

(6.16)

\[
\left[ \lambda_i \frac{\partial}{\partial t} + \frac{1}{Le} \nabla^2 + \mathbf{q} \cdot \nabla \right] S - w = S_T \frac{RaT}{RaS} \nabla^2 T,
\]

(6.17)

where

\[
\lambda_i = \lambda_i \kappa_i / \gamma d^2, \text{ relaxation parameter,}
\]

\[
Pr_D = \gamma \varepsilon d^2 / K_z \kappa_T, \text{ Darcy-Prandtl number,}
\]

\[
Ra_T = \beta_r g \Delta T d K_z / \nu \kappa_T, \text{ thermal Rayleigh number,}
\]

\[
Ra_S = \beta_s g \Delta S d K_z / \nu \kappa_T, \text{ solute Rayleigh number,}
\]

\[
\xi = K_x / K_z, \text{ mechanical anisotropy parameter,}
\]

\[
\eta = \kappa_T / \kappa_T, \text{ thermal anisotropy parameter,}
\]

\[
\lambda = \epsilon / \gamma, \text{ normalized porosity,}
\]

\[
Le = \kappa_T / \kappa_S, \text{ Lewis number,}
\]

\[
S_T = \kappa_T \beta_s / \kappa_T \beta_T, \text{ Soret parameter.}
\]

We however assume that the Soret effect is weak and hence assume moderate values for the Soret coefficient. The viscoelastic character of the liquid mixture appears in the
relaxation parameter $\lambda_1$ (which is also known as the Deborah number) and retardation parameter $\lambda_2$. The Deborah number is a ratio of the relaxation time of the material to a characteristic time of the process. The parameter $\lambda_2 = 0$ for a Maxwell fluid while $\lambda_1 = \lambda_2 = 0$ for a Newtonian fluid. For dilute polymeric solutions the value of the Deborah number is most likely in the range $[0.1, 2]$. It is worth mentioning here that the Darcy-Prandtl number depends on the properties of the fluid and on the nature of the porous matrix. It affects the stability of the porous system through this combined dimensionless group and a typical value for the Prandtl number for a viscoelastic fluid is $Pr = 10$. The normalized porosity $\lambda$ is expressed in terms of the porosity of the porous medium $\varepsilon$ and the solid to fluid heat capacity ratio, $\gamma$. Since $0 < \varepsilon < 1$, it is clear that $0 < \lambda < 1$.

The boundary conditions for dimensionless perturbation quantities are given by

$$w = T = S = 0 \text{ at } z = 0, 1.$$  \hfill (6.18)

### 6.3 Linear Stability Analysis

The thresholds of both stationary and oscillatory convections are predicted using linear theory. The Eigenvalue problem defined by Eqs. (6.15)–(6.17) subject to the boundary conditions (6.18) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that the amplitudes of the perturbations are very small, we write

$$\begin{pmatrix} w \\ T \\ S \end{pmatrix} = \begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} \exp\left[ i (lx + my) + \sigma t \right],$$  \hfill (6.19)

where $l, m$ are horizontal wavenumbers and $\sigma$ is the growth rate which is in general
a complex quantity such that \( \sigma = (\omega_r + i \omega_i) \). The system with \( \omega_r < 0 \) is always stable, while for \( \omega_r > 0 \) it will become unstable. For neutral stability state \( \omega_r = 0 \).

Substituting Eq. (6.19) into the linearized version of Eqs. (6.15) – (6.17), we obtain

\[
\left[ (1 + \lambda_i \sigma) \frac{\sigma}{Pr_D} \left( D^2 - a^2 \right) + \frac{D^2}{\bar{\varepsilon}} - a^2 \right] W + (1 + \lambda_i \sigma) \left( a^2 Ra_T \Theta - a^2 Ra_s \Phi \right) = 0, \quad (6.20)
\]

\[
-W + \left[ \sigma - \left( D^2 - \eta a^2 \right) \right] \Theta = 0, \quad (6.21)
\]

\[
-W - S_T \frac{Ra_T}{Ra_s} (D^2 - a^2) \Theta + \left[ \lambda_\sigma - \frac{1}{Le} \left( D^2 - a^2 \right) \right] \Phi = 0, \quad (6.22)
\]

where \( D = d/dz \) and wavenumber \( a^2 = l^2 + m^2 \).

The boundary conditions (6.18) now read as

\[
W = \Theta = \Phi = 0 \text{ at } z=0, 1. \quad (6.23)
\]

We assume the solutions of Eqs. (6.20) – (6.22) satisfying the boundary conditions (6.23) in the form

\[
\begin{pmatrix}
W(z) \\
\Theta(z) \\
\Phi(z)
\end{pmatrix} = 
\begin{pmatrix} W_0 \\
\Theta_0 \\
\Phi_0
\end{pmatrix} \sin(n \pi z) \quad (n = 1, 2, 3, \ldots). \quad (6.24)
\]

The most unstable mode corresponds to \( n = 1 \) (fundamental mode). Therefore, substituting Eq. (6.24) with \( n = 1 \) into Eqs. (6.20) – (6.22), we obtain a matrix equation

\[
\begin{pmatrix}
\sigma \delta^2 + \frac{\delta^2}{Pr_D (1 + \lambda_i \sigma)} & -a^2 Ra_T & a^2 Ra_s \\
-1 & \sigma + \delta^2 & 0 \\
-1 & S_T \frac{Ra_T}{Ra_s} & \lambda_\sigma + \delta^2/Le
\end{pmatrix}
\begin{pmatrix} W_0 \\
\Theta_0 \\
\Phi_0
\end{pmatrix} = 
\begin{pmatrix} 0 \\
0 \\
0
\end{pmatrix}, \quad (6.25)
\]

where \( \delta^2 = \pi^2 + a^2 \), \( \delta_1^2 = \pi^2 \bar{\varepsilon}^{-1} + a^2 \), and \( \delta_2^2 = \pi^2 + \eta a^2 \).

The condition of nontrivial solution of above system of homogeneous linear equations (6.25) yields the expression for thermal Rayleigh number in the form
\[ Ra_T = \left( \frac{\sigma + \delta^2}{a^2} \right) \left[ \frac{\sigma \delta^2 + \delta^2}{\left( Pr_0 + \frac{\sigma^2}{(1 + \lambda \sigma)} \right) \left( \lambda \sigma + \delta^2 \left( \text{Le}^{-1} + S_T \right) \right)} + \frac{a^2 Ra_s}{\left( \lambda \sigma + \delta^2 \left( \text{Le}^{-1} + S_T \right) \right)} \right]. \]  

(6.26)

### 6.3.1 Marginal Stationary State

For validity of the principle of exchange of stabilities (i.e., steady case), we have \( \sigma = 0 \) (i.e. \( \omega_r = \omega_i = 0 \)) at the margin of stability. Then the Rayleigh number at which marginally stable steady mode exists becomes

\[ Ra_T^S = \left( \eta a^2 + \pi^2 \right) \left[ \frac{\left( a^2 + \pi^2 \xi^{-1} \right)}{a^2} + \frac{\text{Le} Ra_s}{\left( a^2 + \pi^2 \right)} \right]. \]  

(6.27)

The minimum value of the stationary Rayleigh number \( Ra_T^S \) occurs at the critical wavenumber \( a = a_c^S \) where stationary critical wavenumber \( a_c^S = \sqrt{h} \) satisfies a polynomial equation of degree four in \( h \).

\[ t_1 h^4 + t_2 h^3 + t_3 h^2 + t_4 h + t_5 = 0, \]  

(6.28)

where \( t_1 = \xi \eta, t_2 = 2 \pi^2 \xi \eta, t_3 = \pi^2 \left( \text{Le} Ra_s \left( \eta - 1 \right) \xi + \pi^2 \left( \eta \xi - 1 \right) \right), t_4 = -2 \pi^6, t_5 = -\pi^8. \)

It is important to note that the result (6.27) exactly coincides with the one given by Gaikwad et al. (2009a). In the absence of the Soret effect i.e., \( S_T = 0 \) Eq. (6.27) implies

\[ Ra_T^S = \left( \eta a^2 + \pi^2 \right) \left[ \frac{\left( a^2 + \pi^2 \xi^{-1} \right)}{a^2} + \frac{\text{Le} Ra_s}{\left( a^2 + \pi^2 \right)} \right], \]  

(6.29)

which is the exactly one given by Malashetty and Swamy (2010). Further, for an isotropic porous media, that is, when \( \xi = \eta = 1 \), Eq. (6.29) gives

\[ Ra_T^S = \frac{\left( \pi^2 + a^2 \right)^2}{a^2} + Ra_s \text{Le}, \]  

(6.30)

which is classical result obtained by Nield and Bejan (2006). For a single component
fluid \((Ra_s = 0)\), the expression for the stationary Rayleigh number given by Eq. (6.29) reduces to

\[ Ra_s^* = \left( \eta a^2 + \pi^2 \right) \left( \frac{a^2 + \pi^2 \xi^{-1}}{a^2} \right), \quad (6.31) \]

which is the one obtained by Storesletten (1998) for the case of a single component fluid saturated anisotropic porous layer. Further, for an isotropic porous medium, \(\xi = \eta = 1\), the above reduces to the classical result which has critical value \(Ra_c^* = 4\pi^2\), for \(a_c^* = \pi^2\).

### 6.3.2 Marginal Oscillatory State

Now we set \(\sigma = i\omega_i\) in Eq. (6.26) and clear the complex quantities from the denominator, to obtain

\[ Ra_f = \Delta_i + i\omega_i \Delta_2, \quad (6.32) \]

where

\[
\Delta_i = \frac{Ra_s Le \left( (1 + LeS_r) \delta^2 \delta_2^2 + Le \omega^2 \lambda \right)}{\left( (1 + LeS_r)^2 \delta^4 + Le^2 \omega^2 \lambda^2 \right)} + \\
\left( \frac{Pr_d \delta_1^2 \left( (1 + LeS_r) \delta^4 + Le^2 \omega^2 \lambda \right) \delta_2^2 \delta_2^2 \lambda + Le^2 \omega^2 \lambda^2 \right)}{\left( (1 + LeS_r)^2 \delta^4 + Le^2 \omega^2 \lambda^2 \right) (1 + \omega^2 \lambda_i^2)} - \omega^2 \\
\left( -Pr_d \delta_1^2 \delta_1^2 \lambda + \delta^2 \left( 1 + \omega^2 \lambda_i^2 \right) \right) \\
\left( a^2 Pr_d \left( (1 + LeS_r)^2 \delta^4 + Le^2 \omega^2 \lambda^2 \right) (1 + \omega^2 \lambda_i^2) \right),
\]

and

\[
\Delta_2 = \left( \frac{Pr_d \delta_1^2 \left( (1 + LeS_r) \delta^4 + Le^2 \omega^2 \lambda \right) \delta_2^2 \delta_2^2 \lambda + Le^2 \omega^2 \lambda^2 \right)}{a^2 Pr_d \left( (1 + LeS_r)^2 \delta^4 + Le^2 \omega^2 \lambda^2 \right) (1 + \omega^2 \lambda_i^2)} + a^2 LePr_d Ra_s \\
\left( (1 + LeS_r) \delta^2 - Le^2 \lambda \right) (1 + \omega^2 \lambda_i^2) + \left( (1 + LeS_r) \delta^4 \delta_2^2 - Le^2 S_r \omega^2 \delta^2 \lambda \right) \\
\left( +Le^2 \omega^2 \delta_2^2 \lambda \right) \\
\left( -Pr_d \delta_1^2 \lambda + \delta^2 \left( 1 + \omega^2 \lambda_i^2 \right) \right) \\
\left( a^2 Pr_d \left( (1 + LeS_r)^2 \delta^4 + Le^2 \omega^2 \lambda^2 \right) (1 + \omega^2 \lambda_i^2) \right),
\]

Since \(Ra_f\) is a physical quantity, it must be real. Hence, from Eq. (6.32) it follows that
either \( \omega_i = 0 \) or \( \Delta_2 = 0 \). For oscillatory onset \( \Delta_2 = 0 \) \( (\omega_i \neq 0) \) and this gives a dispersion relation of the form (on dropping the subscript \( i \))

\[
a_0 \left( \omega^2 \right)^2 + a_1 \left( \omega^2 \right) + a_2 = 0,
\]

where the coefficients \( a_0, a_1 \) and \( a_2 \) are

\[
a_0 = Le^2 \delta^2 \lambda \left( \delta_z^2 \lambda - \delta^2 S_T \right) \lambda^2,
\]

\[
a_1 = Le^2 \lambda \left( -S_T \delta^4 + Pr_0 \delta^4 \lambda + \delta^2 \delta_z^2 \lambda \right) - Le^2 Pr_0 \delta^2 \lambda \left( -S_T \delta^2 + \delta_z^2 \lambda \right) \lambda_i + \left( \left( 1 + Le S_T \right) \delta^2 \delta_i^2 + a^2 Le Pr_0 Ra_s \left( \left( 1 + Le S_T \right) \delta^2 - Le \delta_z^2 \lambda \right) \right) \lambda_i^2,
\]

\[
a_2 = \left( 1 + Le S_T \right) \delta^2 \delta_i^2 + a^2 Le Pr_0 Ra_s \left( \left( 1 + Le S_T \right) \delta^2 - Le \delta_z^2 \lambda \right) + Pr_0 \delta^2 \lambda_i \left( Le S_T \delta^2 \lambda - \left( 1 + Le S_T \right) \delta^2 \left( \delta_z^2 \lambda_i - 1 \right) \right).
\]

Now Eq. (6.32) with \( \Delta_2 = 0 \), gives

\[
Ra_{_T}^{\omega_c} = \frac{Ra_s Le \left( \left( 1 + Le S_T \right) \delta^2 \delta_i^2 + Le \omega^2 \lambda \right)}{\left( \left( 1 + Le S_T \right) \delta^4 + Le^2 \omega^2 \lambda^2 \right)} + \left[ \frac{Pr_0 \delta^2 \left( \left( 1 + Le S_T \right) \delta^4 \delta_i^2 - Le^2 S_T \omega^2 \delta^2 \lambda + Le^2 \omega^2 \delta_z^2 \lambda^2 \right) - \omega^2 \left( \left( 1 + Le S_T \right) \delta^4 + Le^2 \omega^2 \lambda^2 \right) \left( -Pr_0 \delta^2 \lambda_i + \delta^2 \left( 1 + \omega^2 \lambda_i \right) \right) \right]}{\left( \left( 1 + Le S_T \right) \delta^4 + Le^2 \omega^2 \lambda^2 \left( 1 + \omega^2 \lambda_i \right) \right)}.
\]

(6.34)

The analytical expression for oscillatory Rayleigh number given by Eq. (6.33) is minimized with respect to the wavenumber numerically, after substituting for \( \omega^2 (> 0) \) from Eq. (6.33), for various values of the physical parameters in order to know their effects on the onset of oscillatory convection.

### 6.4 Finite Amplitude Analysis with Limited Representation

The nonlinear analysis is considered using a truncated representation of the Fourier series with only two terms. Although the linear stability analysis is sufficient
for obtaining the stability condition of the motionless solution and the corresponding
Eigen functions describing qualitatively the convective flow, it can neither provide
information about the values of the convection amplitudes, nor regarding the rate of
heat and mass transfer. To obtain this additional information, we perform the nonlinear
analysis, which is useful to understand the physical mechanism with minimum amount
of mathematics and is a step forward towards understanding the full nonlinear problem.

For simplicity of analysis, we confine ourselves to the two-dimensional rolls,
so that all the physical quantities are independent of \( y \). We introduce stream function \( \psi \)
such that \( u = \partial \psi / \partial z, \ w = -\partial \psi / \partial x \) into the Eq. (6.11), eliminate pressure and non-
dimensionalize the resulting equation and Eqs. (6.12)–(6.13) using the transformations
(6.14) to obtain

\[
\left[ \left( 1 + \lambda_s \frac{\partial}{\partial t} \right) \left( \frac{1}{Pr_t} \frac{\partial}{\partial t} \nabla^2 \right) + \frac{\partial^2}{\partial x^2} + \frac{1}{\zeta} \frac{\partial^2}{\partial z^2} \right] \psi + \left( 1 + \lambda_s \frac{\partial}{\partial t} \right) \left( Ra_r \frac{\partial T}{\partial x} - Ra_s \frac{\partial S}{\partial x} \right) = 0,
\]

(6.35)

\[
\left( \frac{\partial}{\partial t} - \eta \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2} \right) T - \frac{\partial (\psi, T)}{\partial (x, z)} + \frac{\partial \psi}{\partial x} = 0,
\]

(6.36)

\[
\left( \lambda \frac{\partial}{\partial t} - \frac{1}{Le} \frac{\partial^2}{\partial x^2} - \frac{1}{Le} \frac{\partial^2}{\partial z^2} \right) S - S \left( \frac{Ra_r}{Ra_s} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) T - \frac{\partial (\psi, S)}{\partial (x, z)} \right) \frac{\partial \psi}{\partial x} = 0.
\]

(6.37)

The first effect of non-linearity is to distort the temperature and concentration fields
through the interaction of \( \Psi, T \) and also \( \Psi, S \). The distortion of these fields will
corresponds to a change in the horizontal mean, i.e. a component of the form \( \sin(2\pi z) \)
will be generated. Thus a minimal Fourier series which describes the finite amplitude
free convection is given by,

\[
\Psi = A(t) \sin(ax) \sin(\pi z),
\]

(6.38)

\[
T = B(t) \cos(ax) \sin(\pi z) + C(t) \sin(2\pi z),
\]

(6.39)
\[ S = D(t) \cos(\alpha x) \sin(\pi z) + E(t) \sin(2\pi z), \quad (6.40) \]

where the amplitudes \( A(t), B(t), C(t), D(t) \) and \( E(t) \) are real and to be determined from the dynamics of the system. Substituting Eqs. (6.38)–(6.40) into Eqs. (6.35)–(6.37) and equating the coefficients of like terms, we obtain the following non-linear autonomous system of differential equations

\[
\frac{dX}{dt} = \mathbf{M}, \quad (6.41)
\]

where

\[
\begin{aligned}
\mathbf{X} &= \left( A, B, C, D, E, F \right)^T, \\
\mathbf{M} &= \left( M_1, M_2, M_3, M_4, M_5, M_6 \right)^T \text{ with}
\end{aligned}
\]

\[
M_1 = F,
\]

\[
M_2 = -\left( a A + \delta^2 \delta B + \pi a AC \right),
\]

\[
M_3 = \left( \frac{\pi a}{2} AB - 4 \pi^2 C \right),
\]

\[
M_4 = -\frac{1}{\lambda} \left( a A + \frac{\delta^2 D}{Le} + \pi a AE + S_T \frac{Ra}{Ra_s} \delta^2 B \right),
\]

\[
M_5 = \frac{1}{\lambda} \left( \frac{\pi a AD}{2} - \frac{4 \pi^2 E}{Le} - 4 \pi^2 CS_T \frac{Ra}{Ra_s} \right),
\]

\[
M_6 = \left[ \frac{F}{\lambda} + \frac{Pr_T \delta^2}{\lambda_4 \delta^2} A + \frac{Pr_T a}{\lambda_4 \delta^2} \left( Ra_B - Ra_s D \right) - \frac{Pr_T a}{\delta^2} \left( Ra_s \frac{dB}{dt} - Ra_s \frac{dD}{dt} \right) \right].
\]

The non-linear system of autonomous differential equations is not suitable for analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. However, one can make qualitative predictions as discussed below. The system of equations (6.41) is uniformly bounded in time and possesses many properties of the full problem. Thus volume in the phase space must contract. In order to prove volume contraction, we must show that velocity field has a constant
negative divergence. Indeed,
\[
\frac{\partial}{\partial A} \left( \frac{dA}{dt} \right) + \frac{\partial}{\partial B} \left( \frac{dB}{dt} \right) + \frac{\partial}{\partial C} \left( \frac{dC}{dt} \right) + \frac{\partial}{\partial E} \left( \frac{dE}{dt} \right) + \frac{\partial}{\partial F} \left( \frac{dF}{dt} \right) = -\left( \frac{Pr_D \delta^2}{\lambda a^2} + \delta^2 + 4\pi^2 + \left( \frac{\delta^2 + 4\pi^2}{Le \lambda} + \frac{1}{\lambda a^2} \right) \right),
\] (6.42)

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase space; in particular they may be attracted to a fixed point, a limit cycle or, perhaps, a strange attractor. From Eq. (6.42) we conclude that if a set of initial points in phase space occupies a region $V(0)$ at time $t = 0$, then after some time $t$, the end points of the corresponding trajectories will fill a volume
\[
V(t) = V(0) \exp \left[ -\left( \frac{Pr_D \delta^2}{\lambda a^2} + \delta^2 + 4\pi^2 + \left( \frac{\delta^2 + 4\pi^2}{Le \lambda} + \frac{1}{\lambda a^2} \right) \right) t \right].
\] (6.43)

This expression indicates that the volume decreases exponentially with time. We can also infer that, the large Darcy Prandtl number, very small Lewis number and relaxation parameters tend to enhance dissipation.

### 6.4.1 Steady Finite Amplitude Motions

The simplified model represented by Eq. (6.41) has the great advantage that steady finite amplitude solutions can be obtained at once and their stability can be investigated analytically. From qualitative predictions, we look into the possibility of an analytical solution. In the case of steady motions, setting the left hand side of Eq. (6.41) equal to zero, writing the amplitudes $B$, $C$, $D$ and $E$ in terms of $A$, with $A^2 + 8 = x$, we get
\[
A_1 x^2 + A_2 x + A_3 = 0,
\] (6.44)

where
\[
A_i = \delta^2 a^i Le,
\]
The required root of Eq. (6.44) is,

\[ x = \frac{-A_2 + (A_2^2 - 4A_4A_3)^{1/2}}{2A_1} \]  

(6.45)

When we let the radical in the above equation to vanish, we obtain the expression for finite amplitude Rayleigh number \( Ra_F^x \), which characterizes the onset of finite amplitude steady motions, in the form

\[ Ra_F^x = -\frac{B_2 + (B_2^2 - 4B_1B_3)^{1/2}}{2B_1} \]  

(6.46)

where

\[ B_1 = a^8 Le^2 (S_T - 1)^2, \]

\[ B_2 = 2a^6 \left( a^2 LeRa_s (S_T - 1) + \delta^2 \left( 1 + S_T + 2LeS_T \right) \delta^2 \right. + \left. Le^2 (S_T - 1) \delta^2 \right), \]

\[ B_3 = \frac{a^4 \left( a^2 LeRa_s + \delta^2 \left( \delta - Le\delta_s \right) \left( \delta + Le\delta_s \right) \right)^2}{Le^2}. \]

6.5 Heat and Mass Transport

In the study of convection in fluids, the quantification of heat and mass transport is important. This is because the onset of convection, as Rayleigh number is increased, is more readily detected by its effect on the heat and mass transport. In the basic state, heat and mass transport is by conduction alone. If \( H \) and \( J \) are the rate of heat and mass transport per unit area respectively, then

\[ H = -\kappa_T \left( \frac{\partial T_{total}}{\partial z} \right)_{z=0}, \quad J = -\kappa_S \left( \frac{\partial S_{total}}{\partial z} \right)_{z=0} - \kappa_{ST} \left( \frac{\partial T_{total}}{\partial z} \right)_{z=0}, \]  

(6.47)
where the angular bracket corresponds to a horizontal average and

\[ T_{total} = T_0 - \Delta T \frac{z}{d} + T(x, z, t), \quad S_{total} = S_0 - \Delta S \frac{z}{d} + S(x, z, t). \]  

(6.48)

Substituting Eqs. (6.39) and (6.40) into Eq. (6.48) and using resultant equations in Eq. (6.47), we get

\[ H = \frac{k_{r_s} \Delta T}{d} (1 - 2\pi C), \quad J = \frac{k_{s_s} \Delta S}{d} \left[ (1 - 2\pi E) + Le S_{r_s} \frac{Ra_r}{Ra_s} (1 - 2\pi C) \right]. \]  

(6.49)

The Nusselt and Sherwood numbers are then defined respectively by

\[ Nu = \frac{H}{k_{r_s} \Delta T/d} = (1 - 2\pi C), \]  

(6.50)

\[ Sh = \frac{J}{k_{s_s} \Delta S/d} = \left[ (1 - 2\pi E) + S_{r_s} Le \frac{Ra_r}{Ra_s} (1 - 2\pi C) \right]. \]  

(6.51)

Writing \( C \) and \( E \) in terms of \( A \), and substituting in Eqs. (6.50) and (6.51), we obtain

\[ Nu = 1 + \frac{2x}{\left( \frac{\delta^2}{a^2} + x \right)}, \]  

(6.52)

\[ Sh = 1 + \left\{ \frac{2x}{\left( \frac{\delta^2}{a^2 Le^2} + x \right)} + Le S_{r_s} \frac{Ra_r}{Ra_s} \left[ 1 + \frac{2x}{\left( \frac{\delta^2}{a^2} + x \right)} \left( \frac{\delta^2}{a^2 Le} + x \right) \right] \right\}. \]  

(6.53)

It is important to know that our finite amplitude analysis, which is based on the truncated Fourier series, is valid for thermal Rayleigh number around the convection threshold. Therefore the Nusselt number and Sherwood number are limited by an upper bound value 3. Better results can only be obtained by including more number of terms in the Fourier series representation, which allows the variation of wave number as the value of the thermal Rayleigh number varies. As the viscoelastic effect does not appear in the steady finite amplitude analysis, the autonomous system of nonlinear ordinary
differential equations (6.41) is solved numerically using the Runge-Kutta method with suitable initial conditions for different values of the parameters. The time evolution of \( Nu \) and \( Sh \) are generated and the results are discussed.

### 6.6 Results and Discussion

The onset of double diffusive convection in a two-component Maxwell fluid saturated, anisotropic porous layer, in the presence of the Soret effect is investigated analytically using both linear and nonlinear theories. In the linear stability theory the expressions for both the stationary and oscillatory Rayleigh numbers are derived analytically along with a expression for frequency of oscillation. The stationary critical Rayleigh number is found to be independent of the relaxation parameter because of the absence of base flow in the present case. The critical Rayleigh number for the oscillatory mode is derived as a function of the Soret parameter, relaxation parameter, solute Rayleigh number, normalized porosity, the Darcy-Prandtl number and the Lewis number, mechanical and thermal anisotropy parameters. The nonlinear theory provides the quantification of heat and mass transport and also explains the possibility of the finite amplitude motions.

The neutral stability curves in the \( Ra_{\text{osc}} - a \) plane for various parameter values are as shown in figures 6.1-6.4. We fixed the values for the parameters except the varying parameter. From these figures it is clear that the neutral curves are connected in a topological sense. This connection allows the linear stability criteria to be expressed in terms of the critical Rayleigh number \( Ra_{c} \), below which the system is stable and unstable above.

Figure 6.1 depicts the influence of relaxation parameter \( \lambda \) on the marginal oscillatory stability curves for the fixed values of other parameters. It is found that
increasing $\lambda_i$ decreases the oscillatory critical Rayleigh number, indicating that the effect of increasing $\lambda_i$ is to advance the onset of oscillatory convection. In figure 6.2, the marginal oscillatory stability curves for different values of the Soret parameter $S_r$ are drawn. We observed that the negative Soret coefficient has destabilizing effect, whereas the positive Soret coefficient has a stabilizing effect.

Figure 6.3 shows the effect of mechanical anisotropy parameter $\xi$ for the fixed values of other parameters on the marginal oscillatory stability curves. It can be observed that with an increase of $\xi$ decreases the minimum of the Rayleigh number for oscillatory convection, indicating that the effect of increasing mechanical anisotropy parameter is to advance the onset of oscillatory convection. The effect of mechanical anisotropy can be understood as follows; let us keep the vertical permeability $K_z$ fixed and then an increased horizontal permeability $K_x$, reduces the critical Rayleigh number. This is due to the fact that increased permeability enhances the fluid mobility in the vertical direction and hence convection sets in early. On the other hand keep horizontal permeability $K_x$ fixed and then increased vertical permeability $K_z$, increases the critical Rayleigh number. Further, we find that the minimum of Rayleigh number shift towards the smaller values of the wavenumber with increasing mechanical anisotropy parameter. This indicates that the cell width increases with an increase of mechanical anisotropy parameter. The effect of thermal anisotropy parameter $\eta$ on the oscillatory neutral curves for the fixed values of other parameters is displayed in figure 6.4. It is found that increasing $\eta$ decreases the oscillatory critical Rayleigh number and the corresponding wavenumber, indicating that its effect is to destabilize the system.

The detailed behavior of critical Rayleigh number for stationary, oscillatory and finite amplitude modes with respect to the solute Rayleigh number is analyzed in the
$Ra_{fc} - Ra_s$ plane through figures 6.5-6.9. We find that the quantities namely, the critical Rayleigh number for stationary, oscillatory and finite amplitude modes is an increasing function of the solute Rayleigh number. It is clear that for the parameters chosen for these figures, the oscillatory convection sets in prior to the stationary and finite amplitude convection. Further, it is important to note that convection first sets in via oscillatory mode for small and moderate values of solute Rayleigh number and with further an increase of solute Rayleigh number, the convection first sets in via finite amplitude mode.

The variation of $Ra_{fc}$ for all the cases namely, stationary, oscillatory and finite amplitude with $Ra_s$ for different values of mechanical anisotropy parameter $\xi$ for the fixed values of the other parameters is displayed in figure 6.5. It is important to note that critical Rayleigh number increases with an increase of $\xi$ for the oscillatory mode, whereas it decreases for the stationary and finite amplitude modes, indicating that the effect of increasing mechanical anisotropy parameter is to delay the onset of oscillatory convection and is to advance the onset of stationary and finite amplitude convection as compared to the isotropic case. In figure 6.6 the variation of $Ra_{fc}$ with $Ra_s$ for different values of thermal anisotropy parameter $\eta$ is presented. It is noticed that with an increase of $\eta$ the critical Rayleigh number decreases for oscillatory mode, whereas it increases for the stationary and finite amplitude modes implying that the effect of increasing thermal anisotropy parameter is to advance the onset of oscillatory convection and is to inhibit the onset of oscillatory and finite amplitude convection as compared to the isotropic case.

Figure 6.7 indicates the variation of $Ra_{fc}$ with $Ra_s$ for different values of relaxation parameter $\lambda_1$ for oscillatory mode. It is observed that with an increase of $\lambda_1$
decreases the critical Rayleigh number for oscillatory mode. In figure 6.8 the variation of $Ra_{cr}$ for all the cases namely, stationary, oscillatory and finite amplitude with $Ra_s$ for different values of the Soret parameter $S_r$ is shown. This figure reveals that with an increase of $S_r$ decreases the critical Rayleigh number for stationary mode whereas it increases the critical Rayleigh number for oscillatory and finite amplitude modes. Figure 6.9 depicts the variation of $Ra_{cr}$ for all the cases for different values of the Lewis number $Le$. It is seen from this figure that with an increase of $Le$ the critical Rayleigh number increases for stationary mode whereas it decreases for oscillatory and finite amplitude modes.

The weak nonlinear analysis provides not only the onset threshold of finite amplitude motions but also the information on the heat and mass transport in terms of the Nusselt number $Nu$ and the Sherwood number $Sh$. The effect of various parameters on steady heat and mass transport is shown in figures 6.10-6.13. The effect of the Soret parameter on heat and mass transport is displayed in figure 6.10. It is found that heat transport is suppressed (almost insignificant) while mass transport is reinforced by the Soret parameter. The effect of increasing thermal anisotropy parameter $\eta$ reduces the heat transfer and enhances the mass transport and is shown in figure 6.11. In the figures 6.12 and 6.13 the effects of mechanical anisotropy parameter $\xi$ and the Lewis number $Le$ on heat and mass transport are presented. It is observed that in both the cases $Nu$ and $Sh$ increase with an increase of $\xi$ and $Le$ and hence the heat and mass transport are enhanced by these parameters.

The autonomous system of ordinary differential equations is solved numerically using the Runge-Kutta method with suitable initial conditions. Then $Nu$ and $Sh$ are evaluated as a function of time. The transient behavior of $Nu$ and $Sh$ is shown
graphically through the figures 6.14-6.18. It is observed that both \( Nu \) and \( Sh \) start with a conduction state value i.e., \( 1 \) at \( t = 0 \) and then oscillate periodically about their steady state value, i.e., close to \( 3 \) for \( t > 0 \). This periodic variation of \( Nu \) and \( Sh \) is very short lived and decays as time progresses. In other words, as time progresses a steady state is reached via a transient state. Figure 6.14 indicates the effect of relaxation parameter \( \lambda_1 \) on transient heat and mass transfer. It is found that with an increase of \( \lambda_1 \) both heat and mass transfer increase. On the other hand the effect of increasing mechanical anisotropy parameter is to suppress the heat and mass transfer and is shown in figure 6.15. The effect of the Lewis number is to decrease the amplitude of the oscillations of heat flux while it increases the amplitude of the oscillations of mass flux and is shown in figure 6.16. The figures 6.17 and 6.18 respectively indicate that the effect of increasing \( S_r \) and \( \eta \) is to increase the amplitudes of both heat and mass transfer.
Figure 6.1 Neutral oscillatory stability curves for different values of relaxation parameter $\lambda_i$.

Figure 6.2 Neutral oscillatory stability curves for different values of the Soret parameter $S_T$. 

$\eta = 0.7$, $\zeta = 0.5$, $S_T = 0.001$
$Ra_s = 100$, $Le = 10$
$Pr_D = 10$, $\lambda = 0.3$

$\xi = 0.5$, $\eta = 0.7$
$Ra_s = 100$, $\lambda_i = 0.5$
$Pr_D = 10$, $\lambda = 0.3$, $Le = 10$
Figure 6.3 Neutral oscillatory stability curves for different values of mechanical anisotropy parameter $\xi$.

Figure 6.4 Neutral oscillatory stability curves for different values of thermal anisotropy parameter $\eta$. 

$\xi = 0.4, 0.6, 1, 1.2$

$\eta = 0.7, S_c = 0.001, \lambda = 0.3,$
$Ra_s = 100, \lambda_i = 0.5,$
$Pr_D = 10, Le = 10$
Figure 6.5 Variation of the critical Rayleigh number with $R_a_s$ for different values of $\xi$.

Figure 6.6 Variation of the critical Rayleigh number with $R_a_s$ for different values of $\eta$. 

ξ = 0.5, $S_r = 0.001$, $Le = 10$
λ = 0.3, $Pr = 10$, $\lambda_s = 0.5$, $Pr_\eta = 10$, $Le = 10$
ξ = 0.1, 0.6, 1, 1.2
η = 0.1, 0.7, 1, 1.2
Figure 6.7 Variation of the critical Rayleigh number with $Ra_S$ for different values of $\lambda_i$.

Figure 6.8 Variation of the critical Rayleigh number with $Ra_S$ for different values of $S_T$. 

$\eta = 0.7, \xi = 0.5, S_T = 0.001, Le = 10, Pr_D = 10, \lambda = 0.3$

$\xi = 0.5, \eta = 0.7, \lambda_i = 0.5, Pr_D = 10, \lambda = 0.3, Le = 10$

$S_T = 0.01, 0.05, 0.07$
Figure 6.9 Variation of the critical Rayleigh number with $Ra_s$ for different values of $Le$.

Figure 6.10 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $S_T$. 

$x = 0.5$, $\eta = 0.7$, $\lambda = 0.5$, $Pr = 10$, $\lambda = 0.3$, $S_T = 0.001$

$Le = 10, 50, 100, 1000$

$S_T = 0.001, 0.0015, 0.002$

$\xi = 0.5, \eta = 0.7, Le = 2, Ra_s = 100$
Figure 6.11 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $\eta$.

Figure 6.12 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $\xi$. 
Figure 6.13 Variation of the Nusselt number and the Sherwood number with the critical Rayleigh number for different values of $Le$. 

$\xi = 0.5$, $\eta = 0.7$, $S_T = 0.001$, $Ra_T = 100$
Figure 6.14 Variation of $Nu$ and $Sh$ with time for different values of $\lambda_i$. 

$\xi = 0.5$, $\eta = 0.7$, $S_T = 0.001$, $Le = 2$, $\lambda = 0.4$, $Pr_D = 10$, $Ra_S = 100$, $Ra_T = 4 Ra_{Te}^c$. 

Nu and Sh with time for different values of $\lambda_i$. 

$\lambda_i = 0.2$ and $\lambda_i = 0.8$.
\( \lambda_\gamma = 0.3, \ \eta = 0.7, S_T = 0.001, \)
\( Le = 2, \ \lambda = 0.4, Pr_D = 10, \)
\( Ra_s = 100, Ra_T = 4 Ra_{fc}^F \)

Figure 6.15 Variation of \( Nu \) and \( Sh \) with time for different values of \( \xi \).
$\zeta = 0.5, \eta = 0.7, S_r = 0.001,$
$\lambda_1 = 0.3 \lambda = 0.4, Pr_D = 10,$
$Ra_S = 100, Ra_T = 4 Ra_{Tc}$.

Figure 6.16 Variation of $Nu$ and $Sh$ with time for different values of $Le$. 

189
Figure 6.17 Variation of $Nu$ and $Sh$ with time for different values of $S_T$.

\[ \xi = 0.5, \eta = 0.7, \lambda_1 = 0.3, \]
\[ Le = 2, \lambda = 0.4, Pr_D = 10, \]
\[ Ra_S = 100, Ra_T = 4 Ra_{fc}. \]
Figure 6.18 Variation of $Nu$ and $Sh$ with time for different values of $\eta$.

$\xi = 0.5$, $\lambda_i = 0.3$, $S_r = 0.001$, $Le = 2$, $\lambda = 0.4$, $Pr_D = 10$, $Ra_s = 100$, $Ra_r = 4Ra_{rc}$.