CHAPTER - V

An Inventory Model for Decaying Items with Variable Storage Cost, Stock Dependent Demand Rate and Shortages under Inflationary Environment

5.1 INTRODUCTION

Most of the classical inventory models did not take into account the effects of inflation and time value of money. Perhaps, it was believed that inflation would not influence the cost and price components to any significant degree. However, in the last several years, most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money. As a result, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored.

It has been observed in supermarkets that the demand rate is usually influenced by the amount of stock level, that is, the demand rate may go up or down to the on-hand stock level. As pointed out by **Levin *et al.* (1972)** at times, the presence of inventory has a motivational effect on the people around it. It is common belief that large piles of goods displayed in a supermarket will lead the customers to buy more. In the last several years, many researchers have given considerable attention to the situation where the demand rate is dependent on the level of the on-hand inventory. **Gupta** and **Vrat (1986)** were the first to develop models for stock-dependent consumption rate. **Mandal** and **Phaujdar (1989)** then developed an economic production quantity model for deteriorating items with stock dependent consumption rate.

The control and maintenance of production inventories of worsening items with shortages have attracted more attention in inventory analysis. For developing inventory models deterioration plays an important role, since it is a natural process in many cases. Deterioration is normally identified as decay or damage to goods. Foods, drugs, pharmaceuticals, radioactive substances are examples of items in which sufficient deterioration can take place during the normal storage period and thus it plays an important role in analyzing the system. Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items with stock dependent demand rate and shortages have received much attention of several researches in the recent year because most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and foodstuffs from depletion by direct spoilage wile kept in store. **Ghare** and **Schrader (1963)** developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by **Shah** and **Jaiswal (1976)**. Inventory models with a time dependent rate of deterioration were considered by **Covert** and **Philip (1973)**. Some of the significant recent work in this field has been done by **Chung** and **Ting (1993), Fujiwara (1993)**, **Hariga** and **Benkherouf (1994), Wee (1995), Giri** and **Chaudhuri (1997), Jalan** and **Chaudhuri (1999)**, Structural properties of an inventory system with deterioration and trended demand. **Burwell (1997)** developed economic lot size model for price-dependent demand under quantity and freight discounts. Inventory model for ameliorating items for price dependent demand rate was proposed by **Mondal *et.al.* (2003)** and inventory model with price and time dependent demand was developed by **You (2005)**. In general holding cost is assumed to be known and constant. But in realistic condition, holding cost may not **Goh (1994)** considered various functions to describe holding cost. **M. Mandal** and **M. Maiti (1999)** considered inventory of damageable items with the variable replenishment rate, stock dependent demand and some units in hand. Inventory models for a single deteriorating item with stock dependent demand rate have been studied extensively in the last decade by **Datta** and **Pal, Giri** and **Chaudhuri (1998), Padmanabhan** and **Vrat (1995)** Presented inventory models for deteriorating items with stock dependent selling rate and derived the profit functions for both without backlogging and complete backlogging cases. They assumed the selling rate as a function of current inventory level and rate of deterioration as a constant. They assumed the total average cost, warehouse space, inventory cost, purchase and selling prices to be vague and imprecise.

In this paper, we developed an economic order quantity (EOQ) in which deterioration rate is a linear function of time and shortages are considered as completely backlogged. Demand rate is stock dependent in linear form as $D(t) = \alpha + \beta I(t)$. We solve the model to optimize the total profit, which is maximized. The model is illustrated with numerical examples.

5.2 ASSUMPTIONS AND NOTATIONS

The fundamental Notations are used to develop the model.

I(t)	the inventory level at time t ($0 \leq t < T$).
T	the cycle length.
q	the ordering quantity is.
A_0	the ordering cost.
s	the selling price per unit item .
c_1	the inventory holding cost per unit item per unit time.
c_2	the shortage cost per unit item per unit time.
c_3	the deterioration cost per unit item per unit time.
D (t)	the Demand rate.
θ (t)	the time dependent deterioration rate
$h(t)$	The variable storage cost.
Q	the optimal inventory level.
R	the inflation.

The fundamental assumptions are used to develop the model.

For the present Model, it is assumed that

- Demand D(t) is dependent on the inventory level or stock dependent in linear form: $D(t) = \alpha + \beta I(t)$, where $\alpha > 0$, $0 \le \beta \le 1$, β is the stock dependent demand rate parameter.
- $h(t)$ is the variable storage cost. $h(t) = h + \mu t$ Where $\mu > 0$
- The deterioration rate θ(t) is dependent on time in linear form $\theta(t) = bt$.
- The ordering cost A_0 is constant.
- Inflation is also considered in this model.
- The cycle length is assumed $0 < t < T$.

5.3 FORMULATION AND SOLUTION

The length of the cycle is T. Due to the demand and deterioration of the items the inventory is depleted during time t_1. At the time t_1 the inventory level becomes zero and shortages occurring in the period (t_1, T) which is completely backlogged. Let I(t) be the inventory level at time t ($0 \le t < T$).

The differential equation can be expressed when the instantaneous state over (0,T) are given by

$$I_1'(t) + btI_1(t) = -[\alpha + \beta I_1(t)], \qquad 0 \le t \le t_1 \qquad \dots (1)$$

$$I_2'(t) = -[\alpha + \beta I_2(t)], \qquad t_1 \le t \le T \qquad \dots (2)$$

With $I_1(0) = 0, I_2(t_1) = 0$

From equation (1), we get

$$I_1(t) = -\alpha\left[t - \frac{bt^3}{3} - \frac{\beta t^2}{2} - \frac{b^2t^5}{12} - \frac{2b\beta t^4}{3} - \frac{\beta^2 t^3}{2}\right], \quad 0 \le t \le t_1 \qquad \dots (3)$$

From equation (2), we get

$$I_2(t) = (1 - \beta t)\left\{\alpha(t_1 - t) + \frac{\beta}{2}(t_1^2 - t^2)\right\} \qquad t_1 \le t \le T \qquad \dots (4)$$

Present worth of holding costs during the time period 0 to t_1 with variable storage cost.

$$H = C_1\int_0^{t_1} h(t).I_1(t)e^{-Rt}dt$$

$$= -C_1\alpha\left[h\left\{\frac{t_1^2}{2} - \frac{Rt_1^3}{3} - \frac{\beta t_1^3}{6} - \frac{bt_1^4}{12}\right\} + \mu\left\{\frac{t_1^3}{3} - \frac{Rt_1^4}{4} - \frac{\beta t_1^4}{8} - \frac{bt_1^5}{15}\right\}\right], \quad \dots (5)$$

The total deterioration cost during the time period 0 to t_1 is given by

$$D = C_2 \int_0^{t_1} \theta(t).I_1(t) e^{-rt} dt$$

$$= -\alpha C_2 \beta \left[\frac{t_1^3}{3} - \frac{bt_1^5}{15} - \frac{\beta t_1^4}{8} - \frac{\beta^2 t_1^5}{10} - \frac{Rt_1^4}{4} \right] \quad \ldots (6)$$

The total shortage cost during the time period t_1 to T is given by

$$S = C_3 \int_{t_1}^{T} -I_2(t) e^{-rt} dt$$

$$= C_3 \left[\alpha \left(\frac{t_1^2}{2} + \frac{T^2}{2} \right) + \frac{\beta}{2} \left(\frac{2t_1^3}{3} + \frac{T^3}{3} \right) + \alpha\beta R \left(\frac{t_1^4}{12} + \frac{T^4}{4} \right) + \frac{\beta^2 R}{2} \left(\frac{2t_1^5}{15} + \frac{T^5}{5} \right) \right] \ldots (7)$$

From equation (5), (6) and (7) the total profit per unit time can define

$$U(T) = S(\alpha + \beta I(t)) - \frac{1}{T}[A_0 + H + D + S] \quad \ldots (8)$$

Our main objective to maximize the profit function U (T), the necessary condition for maximizing the profit are

$$\frac{\partial U(T, t_1)}{\partial T} = 0, \frac{\partial U(T, t_1)}{\partial t_1} = 0 \quad \ldots (9)$$

With the software Mathematica-8.0, we can compute the optimal value of T* and t_1* by equation (9). The optimal value of the profit function U* (T,) is determined by equation (8). The optimal value of T* and t_1* satisfies the sufficient conditions for maximizing profit function U* (T, t_1) are

$$\frac{\partial^2 U(T,t_1)}{\partial T^2} < 0, \frac{\partial^2 U(T,t_1)}{\partial t_1^2} < 0 \qquad \ldots (10)$$

5.4 NUMERICAL EXAMPLE

Example-1. Let us consider A = 40^0, α = 100, β = 0.2, b = 0.5, h = 1.6, C_1 = 1.2, C_2 = 0.06, C_3 = 2, R = 0.04

Based on above input data and Using the software Mathematica-5.1, we calculate the optimal value of U*(T),

U* (T) = 3794.62, T* = 3.0831, t_1* = 1.8482

Example - 2. Let us consider A = 500, α = 150, β = 0.21, b = 0.17, h = 1.1, C_1 = 1.4, C_2 = 0.08, C_3 = 2.5, R = 0.02, U* (T) = 6467.50, T* = 2.2141, t_1* = 1.8592

5.5 SENSITIVITY ANALYSIS

We have studied the effects of changes of the parameters on the optimal values of S (T, t_1), T* and t_1* derived by the proposed method. The sensitivity analysis is performed in view of the numerical example given above. We have executed sensitivity analysis by changing the parameters a, b, α and β as + 20%, + 50%, - 20% and - 50%. All remaining parameters have original values with respect to these changes. The corresponding changes in $U^*(T, t_1)$, T^* and t_1^* are shown in below table.

Parameters	% change	T*	t_1*	U* (T, t_1)
a	-50	3.05571	1.8215	3978.1315
	-20	3.03069	1.78240	4071.315
	+20 +50	3.04993	1.70750	3508.6869
		3.06009	1.81768	3110.4955
α	-50	3.95645	2.02388	3889.5347
	-20	3.87390	2.12810	3970.8666
	+20 +50	3.67173	1.93818	3765.3933
		3.76248	1.94990	3745.7257
	-50	3.39729	2.22046	3869.6804

	-20	3.30004	1.86548	3900.3134
β	+20 +50	3.20408	1.91124	3950.1047
		3.10878	1.75736	3953.5795
	-50	5.01628	1.99738	3842.3532
	-20	4.87102	1.87911	3967.1480
b	+20 +50	3.00856	1.75506	3998.8802
		3.19306	1.63501	3599.5976

Our study of the above table brings out the following:

We observed that as parameter *a* and *b* increase, optimal values of T* and t_1* decrease while the average total profit U (T, t_1) of an inventory system increases, whereas when parameter *a* and *b* decrease, optimal values of T* and t_1* increase while the average total profit S (T, t_1) of an inventory system decreases. It is interesting to observe that if deterioration parameter α increases, the optimal values of T* and t_1* increase while the average total profit S (T, t_1) of an inventory system decreases. If deterioration parameter α decreases, optimal values of T* and t_1* decrease while the average total profit S (T, t_1) of an inventory system increases. If the second deterioration parameter β increases, the optimal values of T* and t_1*

decrease while the average total profit S (T, t_1) of an inventory system increases. If deterioration parameter β decreases, the optimal values of T* and t_1* increase while the average total profit S (T, t_1) of an inventory system decreases.

5.5 GRAPHICAL REPRESENTATION OF CONCAVITY OF THE PROFIT FUNCTION

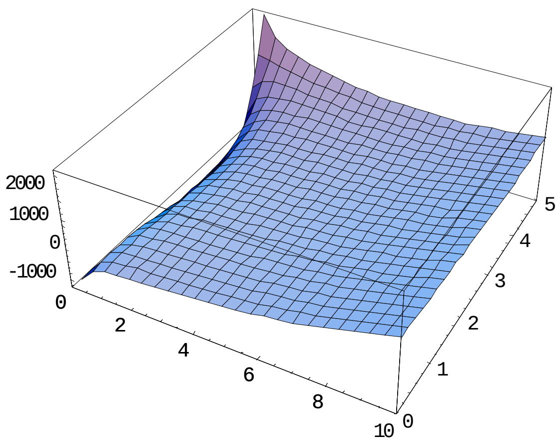

Fig. 5.1 Graphical representation of concavity of the profit function

5.6 CONCLUSION

In this chapter, we have developed an inventory lot size model for deteriorating items with, such as fruits, vegetables and foodstuffs from depletion by direct spoilage while kept in store. The rate of deterioration is a linear function of time. The demand rate is assumed of time dependent. The shortages are allowed and shortages are completely backlogged. Inflation is considered in this paper. The deterioration cost, inventory holding cost and, shortage cost are considered in this model. The numerical examples are given to illustrate the model developed. The model is solved analytically by maximizing the total profit. In the numerical examples we found the maximum value of profit.