SYMBOLS

\( A \): Ordering cost per order
\( H \): Holding cost per unit time
\( D \): Deterioration cost per unit per item
\( S \): Shortage cost per unit per item
\( O \): Cost of lost sales per unit
\( D(t) \): Demand rate at any time \( t \)
\( T \): Total Cycle length
\( t_2 \): The inventory becomes zero at that time
\( I_{\text{max}} \): Size of initial inventory
\( Q \): Total ordering quantity
\( I_a(t) \): Positive inventory at time \( t \) during \( 0 \leq t \leq t_1 \)
\( I_b(t) \): Positive inventory at time \( t \) during \( t_1 \leq t \leq t_2 \)
\( I_c(t) \): Negative inventory at time \( t \) during \( t_2 \leq t \leq T \)
\( B(t) = \frac{1}{1 + \delta(T-t)} \): The partial backlogging rate with \( \delta > 0 \) denote the backlogging parameter during \( t_1 \leq t \leq T \)
\( \theta \): Rate of deterioration.
\( \mu \): The parameter of the ramp type demand function
\( t_1 \): The time when inventory level reaches zero
\( t_1^* \): The optimal point for the replenishment policy
\( I(t) \): On hand inventory level at time \( t \) over the ordering cycle \([0, T]\)

The demand rate \( F(t) \) is defined as ramp type function of time and is defined as:

\[
F(t) = \begin{cases} 
Dt, & t < \mu, \\
D\mu, & t \geq \mu, 
\end{cases}
\]
\(Dt\) is positive and continuous for \(t \in [0, T]\)

We considered \(i = 1, 2, 3, ..., n\) for suppliers, \(j = 1, 2, 3, ..., m\) for products and \(k = 1, 2, 3, ..., l\) for retailers

\[P(t) = \lambda D(t),\] where \(D(t) = be^{at}\) and \(\lambda > 1\), production rate is demand dependent

\(D(t)\): Demand rate at any time \(t\)

\(\lambda be^{at}\): Producer’s production rate that is equal to supplier’s demand rate

\(\lambda be_{e}^{at}\): Producer’s demand rate for multi-item

\(\alpha_s\): Supplier’s percentage of raw material to produce products

\(\alpha\): Supplier’s proportional probability of imperfect items with probability density

Function \(f(\alpha)\)

\(T_r\): Retailer’s cycle length for multi products

\(T\): Retailer’s cycle length

\(\beta\): Producer’s proportional probability of imperfect items with probability density

function \(g(\beta)\)

\(\beta_p\): Product’s demand percentage for multi-item to fulfill the demand of the retailers

\(T_s\): Supplier’s cycle length

\(\lambda be_{e}^{a}T\): Production rate of multi products

\(T_{p_r}\): Producer’s cycle length for multi products

\(T_p\): Manufacturing run-time of multi products

\(C(P)\): Per unit item production cost for multi-item for the producer

\(T_k\): Time for collecting multi products from producer for the retailers

\(n\): Multiple suppliers

\(l\): Multiple retailers

\(m\): Multiple items

\(Q_s(t)\): Supplier’s on-hand inventory of good items at time \(t\)
\( Q_p(t) \): Producer’s on-hand inventory of good items at time \( t \)

\( Q_{pr}(t) \): Producer’s on-hand inventory of defective items which would be reworked.

\( Q_r(t) \): Retailer’s on-hand inventory of good items at time \( t \)

\( R \): Supplier’s replenishment lot size

\( \lambda \): Producer’s production rate that is equal to supplier’s demand rate

\( A_s \): Supplier’s set up cost

\( r_s \): Supplier’s screening rate per unit time

\( S_s \): Supplier’s screening cost per unit item

\( h_s \): Supplier’s holding cost per unit per unit time

\( I_s \): Supplier’s cost per unit idle time

\( C_s \): Supplier’s purchasing cost per unit item

\( w_s \): Supplier’s selling price per unit perfect items

\( \overline{w}_s \): Supplier’s selling price per unit imperfect items

\( E(x) \): Expectation of variable \( x \)

\( SAP \): Supplier’s average profit

\( ESAP \): Supplier’s expected average profit

\( \beta \): Producer’s proportional probability of imperfect items with probability density function \( g(\beta) \)

\( \lambda \beta e^{-\lambda t} \): Reworking rate per unit time

\( \tau \): Random time with mean \( \frac{1}{\theta} \) after which the production system becomes uncontrolable

\( F(\tau) \): Probability distributions function of \( \tau \)

\( f(\tau) \): Probability density functions of \( \tau \)

\( A_p \): Producer’s set up cost

\( r_p \): Producer’s screening rate per unit time

\( S_p \): Producer’s screening cost per unit item

\( h_p \): Producer’s holding cost per unit per unit time for perfect items
\( h_p' \) : Producer’s holding cost per unit per unit time for defective items which would be reworked

\( r_p \) : Cost to rework for imperfect item of producer

\( L \) : Fixed cost like labor, energy and technology cost

\( \gamma \) : The variation constant of tool/die costs

\( p_i \) : Producer’s cost per unit idle time

\( C(P) \) : Per unit item production cost

\( N \) : Numbers of imperfect objects in the production cycle

\( w_p \) : Producer’s selling price per unit perfect item

\( \overline{w_p} \) : Producer’s selling price per unit imperfect item

\( PAP \) : Producer’s average profit

\( EPAP \) : Producer’s expected average profit

\( be_{c}^{\ast} \) : Customer’s demand rate

\( be_{r}^{\ast} \) : Retailer’s demand rate

\( A_r \) : Retailer’s set up cost

\( h_r \) : Retailer’s holding cost per unit per unit time

\( w_r \) : Retailer’s selling price per unit item

\( RAP \) : Retailer’s average profit

\( ERAP \) : Retailer’s expected average profit